

Physics of Massive Neutrinos

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See [hep-ph/0308123](https://arxiv.org/abs/hep-ph/0308123) for a
review



1998–2003 : The Neutrino Revolution

Neutrino flavors oscillate \Rightarrow Neutrinos have mass

Post 2003

Era of further discovery and precision lies ahead


Fundamental properties of neutrinos within reach

Experimental pathways falling in place

- Reactors
- Off-axis beams
- Superbeams
- New detector technologies

Ultimately and inevitably lead to neutrino factories

Goal: unravel the enigma of flavor physics



Neutrino Oscillations

flavor states : ν_α $\alpha = e, \mu, \tau, \dots$

mass states : ν_i $i = 1, 2, 3, \dots$

Vacuum oscillations: $P(\nu_\alpha \rightarrow \nu_\beta) \cong \left| \sum_{j=1}^n V_{\beta j} e^{-i \frac{m_j^2 L}{2E}} V_{\alpha j}^* \right|^2$

For 3 neutrino mixing

$$V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c_a & s_a \\ 0 & -s_a & c_a \end{bmatrix} \begin{bmatrix} c_x & 0 & s_x e^{-i\delta} \\ 0 & 1 & 0 \\ -s_x e^{i\delta} & 0 & c_x \end{bmatrix} \begin{bmatrix} c_s & s_s & 0 \\ -s_s & c_s & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & e^{i(\frac{1}{2}\phi_2)} & 0 \\ 0 & 0 & e^{i(\frac{1}{2}\phi_3 + \delta)} \end{bmatrix}$$

atm

unknown

solar

Majorana phases

$$c \equiv \cos\theta \quad s \equiv \sin\theta$$

- 3 mixing angles $\theta_a, \theta_s, \theta_x$
- 3 complex phases δ, ϕ_2, ϕ_3 (CP)

Oscillation probabilities do not depend on ϕ_2, ϕ_3

Empirically, the observed oscillations have very different δm^2 scales and are nearly decoupled

Useful effective 2-neutrino approximation when one δm^2 is dominant

$$\Delta \equiv \frac{\delta m^2 L}{4E}$$

$$P(\nu_\alpha \rightarrow \nu_\beta) \cong \sin^2 2\theta \sin^2 \Delta$$

$$P(\nu_\alpha \rightarrow \nu_a) \cong 1 - \sin^2 2\theta \sin^2 \Delta$$

independent $\delta m^2 = N_\nu - 1$

Matter effects on ν_e oscillations

ν_e scattering on electrons modifies ν_e oscillation amplitudes and wavelengths in matter

$$\tan 2\theta^m = \frac{\tan 2\theta}{1 - \frac{2\sqrt{2}G_F N_e E_\nu}{\delta m^2 \cos 2\theta}}$$

Enhancement for $\delta m^2 > 0$

Suppression for $\delta m^2 < 0$

Crucial for:

- solar neutrinos (N_e varies)
- long-baselines through Earth (E_ν varies)

Where we stand today: Evidence of oscillations

Atmospheric neutrinos

ν_μ and $\bar{\nu}_\mu$ disappear

ν_e and $\bar{\nu}_e$ do not

$\theta_a \sim 45^\circ$, θ_x small

SuperKamiokande, Macro, Soudan

$$\delta m_a^2 \sim 2 \times 10^{-3} \text{eV}^2$$

Solar neutrinos

ν_e disappear

$\theta_s \sim 33^\circ$

SNO, SuperK, Gallium, Chlorine

$$\delta m_s^2 \sim 6 \times 10^{-5} \text{eV}^2$$

LMA solution ($\delta m_s^2 > 0$)

matter enhancement,
but not resonant

Reactor antineutrinos

$\bar{\nu}_e$ disappear

KamLAND, $L \approx 175 \text{ km}$

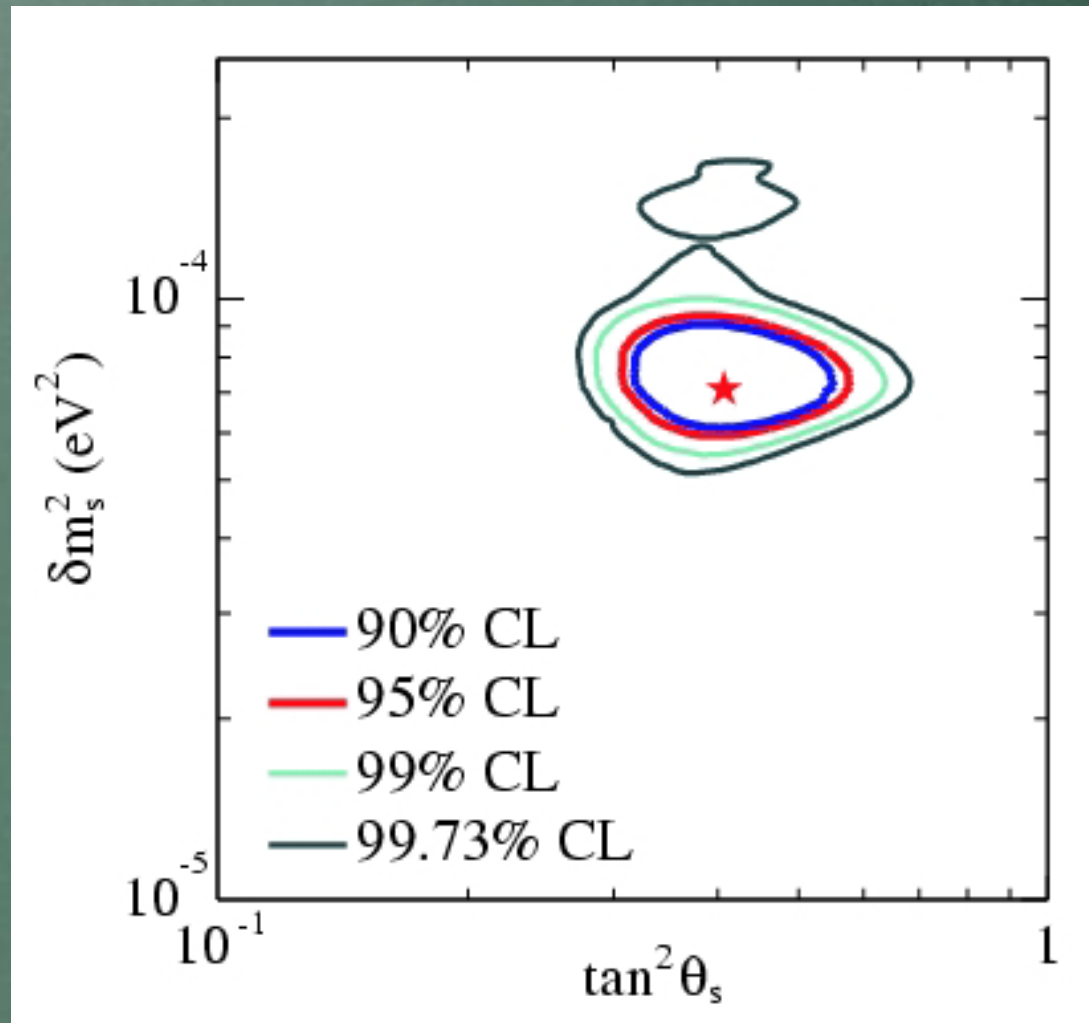
$$\delta m_s^2 \sim 7 \times 10^{-5} \text{eV}^2$$

Confirms LMA

KamLAND + Solar further
constrains δm_s^2

KamLAND massacre: all other solar solutions killed

Solar + KamLAND

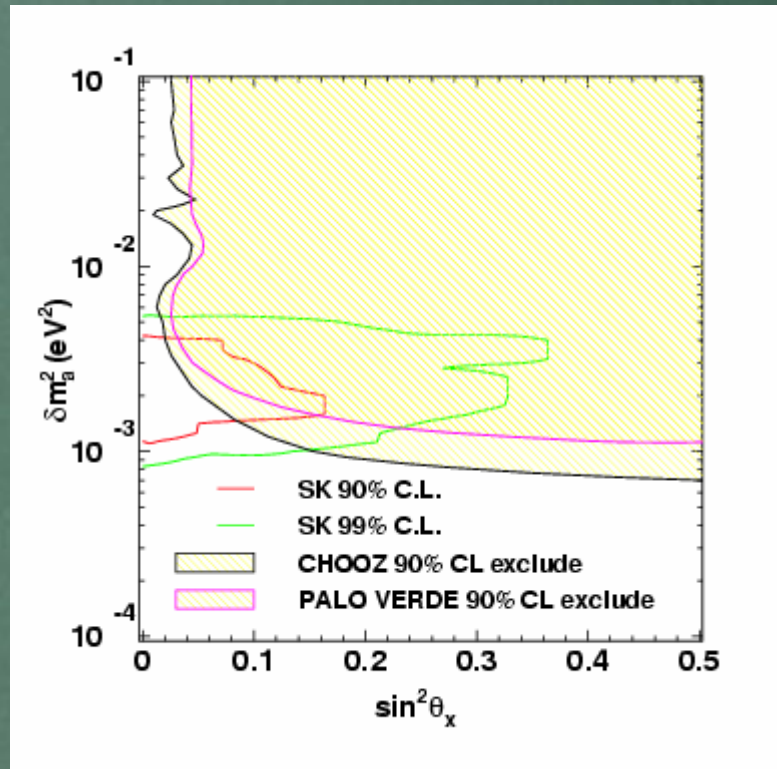


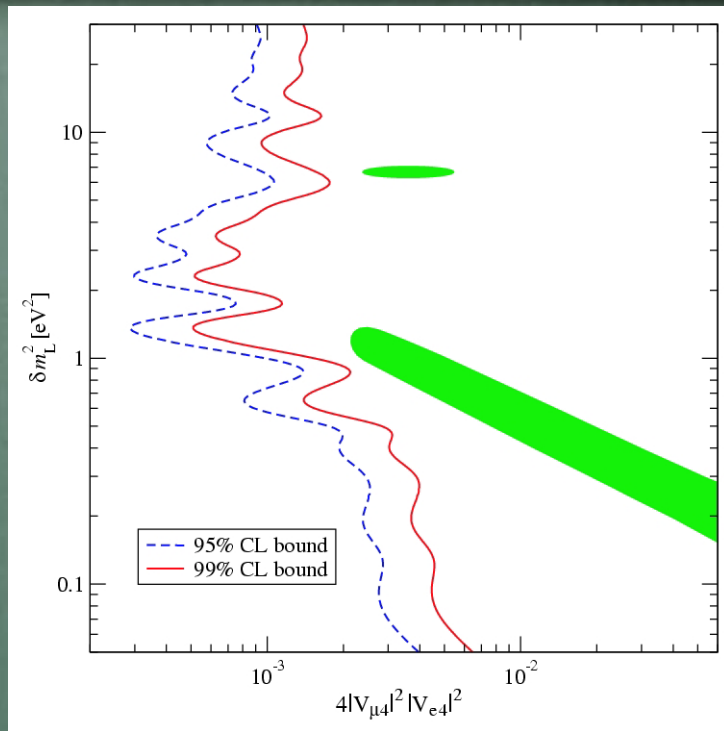
Reactor antineutrinos

CHOOZ, $L \approx 1$ km

$\bar{\nu}_e$ do not disappear

$$\theta_x \leq 13^\circ \text{ for } \delta m_a^2 = 2.0 \times 10^{-3} \text{ eV}^2$$





3+1 spectra ruled out by reactor and accelerator expts.

$$\text{gof} = 5.6 \times 10^{-3}$$

2+2 spectra imply significant participation of ν_s in atm or solar OSC.

Strong limits on

$$\nu_\mu \rightarrow \nu_s \quad \text{atm}$$

rule out 2+2 spectra

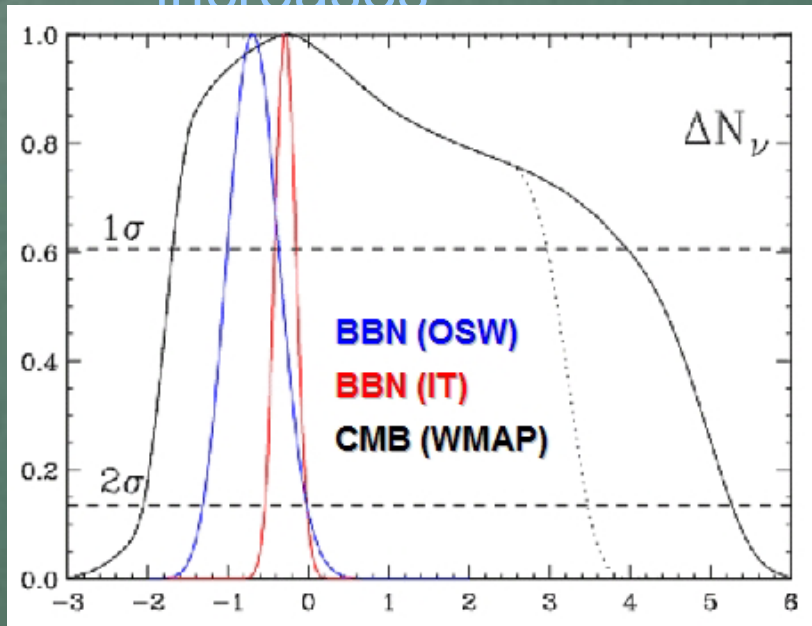
$$\nu_e \rightarrow \nu_s \quad \text{solar}$$

$$\text{gof} = 1.6 \times 10^{-6}$$

Steriles and BBN

Extra neutrinos speed up the expansion of the Universe and neutron-proton freezeout occurs earlier

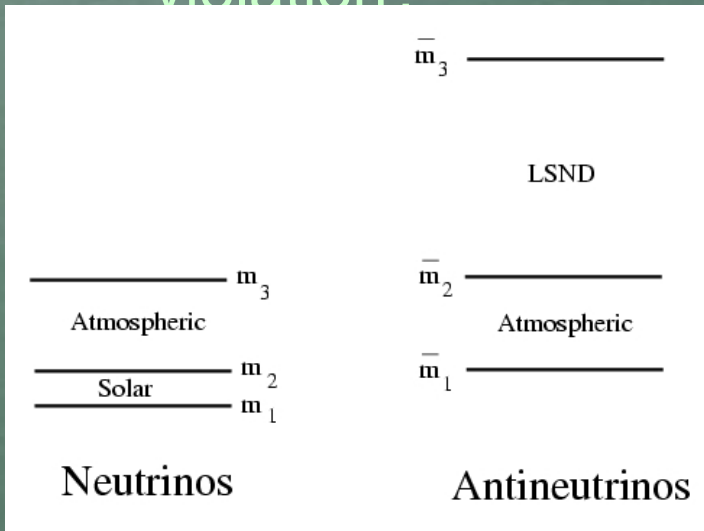
\mathcal{M}_ν ^4He abundance increases



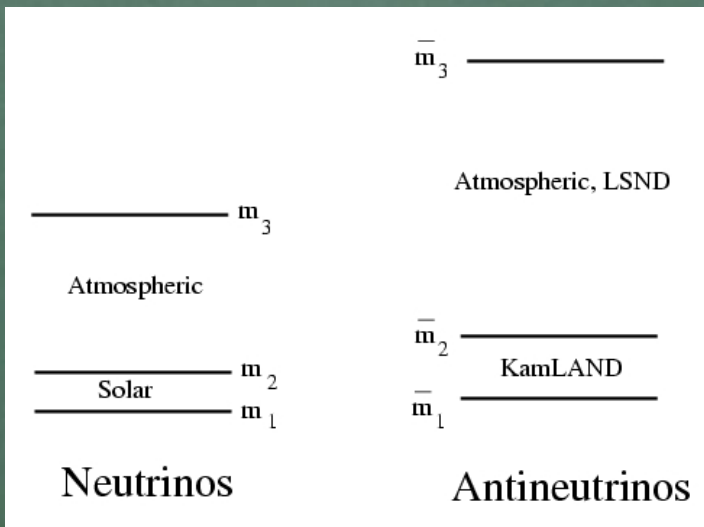
BBN requires $N_\nu \leq 3.0$ at 2σ

LSND sterile neutrino would be fully thermalized by BBN era.
Standard BBN cosmology rejects it.

LSND anomaly from CPT violation?



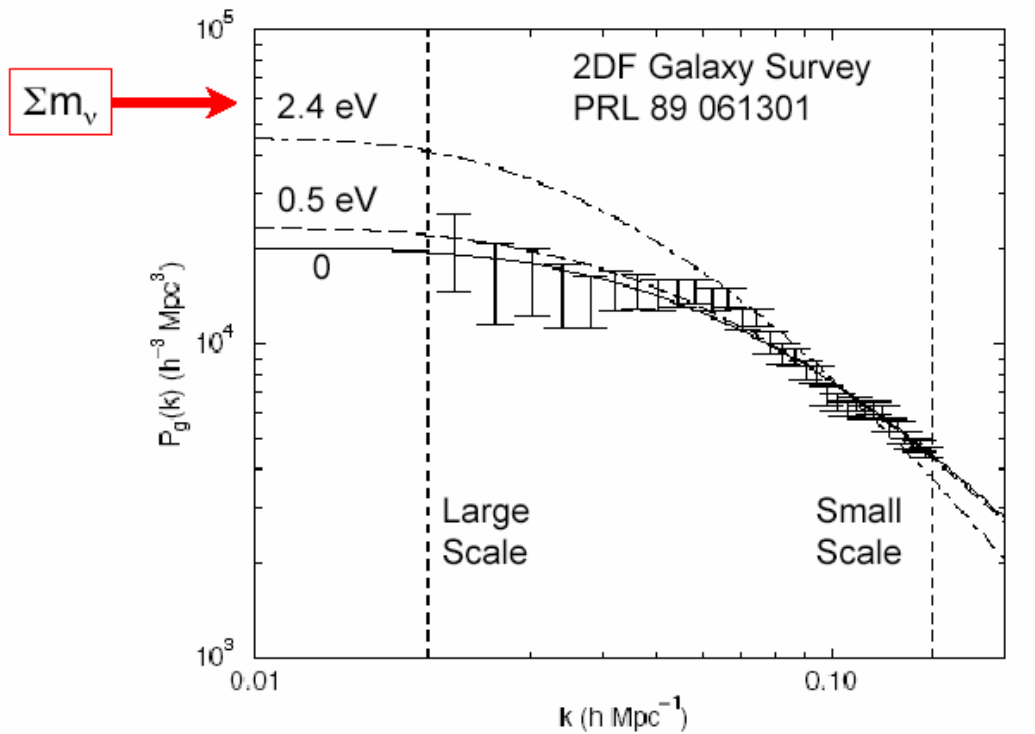
Ruled out by KamLAND at 3σ



Also excluded at 3σ



Neutrino mass and Large Scale Structure in the Universe



Σm_ν influences the matter power spectrum in 2 ways:

- Lighter neutrinos cause the power suppression to begin at larger scales
- Lighter neutrinos suppress power less on smaller scales

$$\Sigma m_\nu \leq 0.71 \text{ eV} \text{ 2dF} + \text{Ly}\alpha \text{ Forest} + \text{WMAP}$$

$$\Sigma m_\nu \leq 0.63 \text{ eV} \text{ 2dF} + \text{WMAP} \quad (\text{Others} \quad \Sigma m_\nu \leq 1 \text{ eV})$$

$$\text{LSND} \quad \Sigma m_\nu \geq \sqrt{\text{find}} \frac{\Delta m_{LSND}^2}{\Delta m_{LSND}^2} \geq 0.3 \text{ eV}$$

Just escapes LSS bounds

Final resolution of LSND sterile neutrino awaits MiniBooNE

3-neutrino observables	Present knowledge ($\sim 95\%$ C. L.)	Near future
θ_a	$45^\circ \pm 10^\circ$	$P(\nu_\mu \rightarrow \nu_\mu)$ MINOS, CNGS
θ_s	$32.5^\circ \pm 3.6^\circ$	SNO NC, KamLAND
θ_x	$\leq 13^\circ$ (for $ \delta m_a^2 = 2.0 \times 10^{-3} \text{ eV}^2$)	$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ Reactor, $P(\nu_\mu \rightarrow \nu_e)$ LBL
$ \delta m_a^2 $	$(2.0_{-0.8}^{+1.2}) \times 10^{-3} \text{ eV}^2$	$P(\nu_\mu \rightarrow \nu_\mu)$ MINOS, CNGS
$\text{sgn}(\delta m_a^2)$	unknown	$P(\nu_\mu \rightarrow \nu_e), P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ LBL
$ \delta m_s^2 $	$(7.1_{-1.1}^{+1.8}) \times 10^{-5} \text{ eV}^2$	$P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ KamLAND
$\text{sgn}(\delta m_s^2)$	+ (MSW)	done
δ	unknown	$P(\nu_\mu \rightarrow \nu_e), P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ LBL
Majorana	unknown	$0\nu\beta\beta$
ϕ_2	unknown	$0\nu\beta\beta$ (if $\simeq 0, \pi$)
ϕ_3	unknown	hopeless
m_ν	$\sum m_\nu < 1 \text{ eV}$	LSS, $0\nu\beta\beta$, β -decay

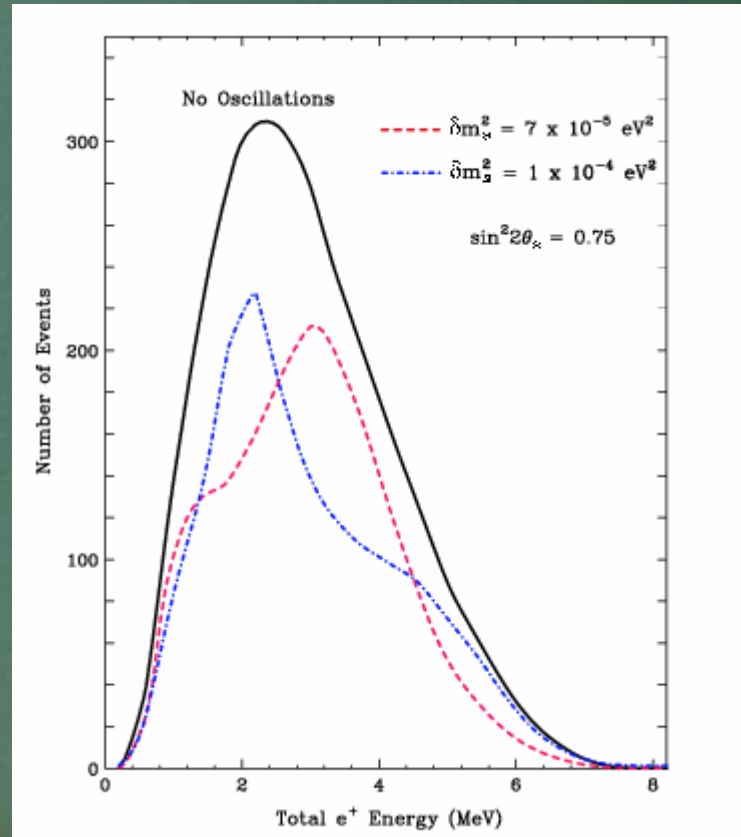
Key Neutrino Issues and how they are being/can be resolved

KEY ISSUE #1: VERIFY OSCILLATIONS / PRECISION

“See” the oscillation wiggles versus energy, not just average suppressions

$$P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \quad \delta m_s^2$$

KamLAND



MINOS (simulation)

$$P(\nu_\mu \rightarrow \nu_\mu) \quad \delta m_a^2$$

K2K (250 km)

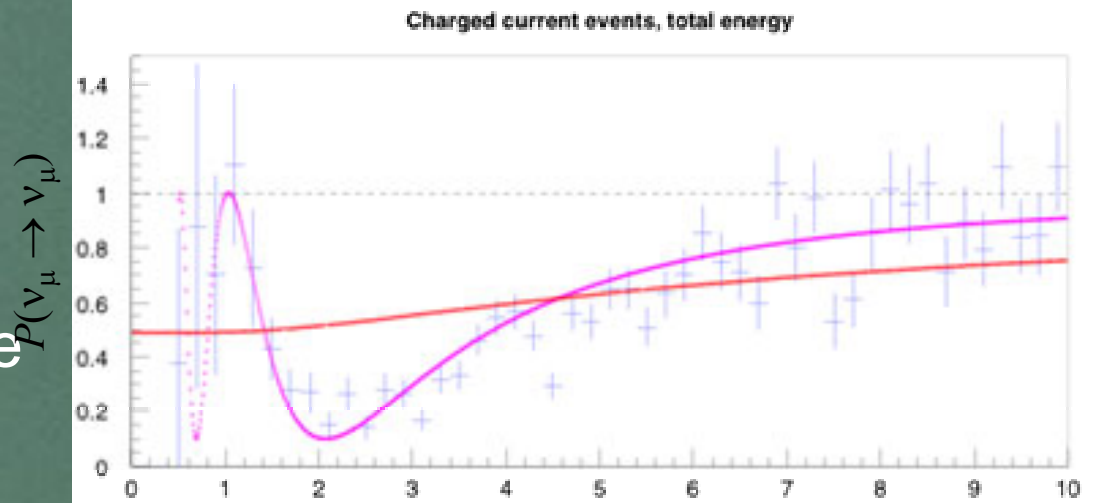
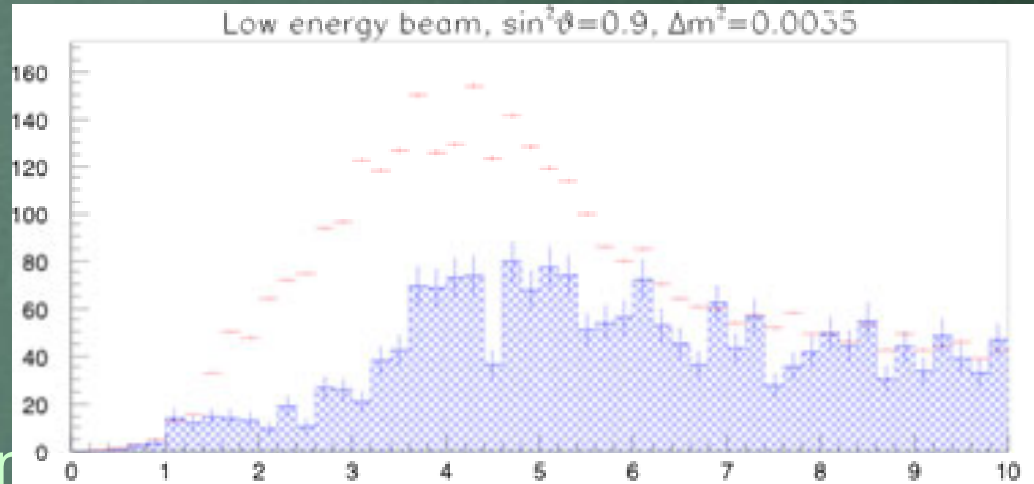
MINOS (730 km)

OPERA, ICARUS (730 km)

Observe ν_τ appearance

$$P(\nu_\mu \rightarrow \nu_\tau) \quad \delta m_a^2$$

OPERA,
ICARUS



KEY ISSUE #2: HOW SMALL IS θ_x ?

Proposed reactor experiments with two detectors
Short L ($< \text{few km}$)



Measure θ_x from wiggles in $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$ vs energy

Sensitivity limit: $\sin^2 2\theta_x \approx 0.01$

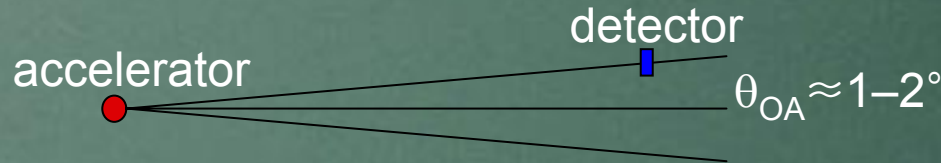
	$\frac{L_1}{0.1 \text{ km}}$	$\frac{L_2}{1}$
Krasnoyarsk km		
Kashiwazaki km	0.3 km	1.7
Diablo Canyon km	0.15 km	1.2

Future accelerator experiments

Measure θ_x via appearance:

$$P(\nu_\mu \rightarrow \nu_e) \text{ or } P(\nu_e \rightarrow \nu_\mu) \approx \sin^2 2\theta_x \sin^2 \Delta_a$$

- Off-axis “magic” (J-PARC, FNAL)



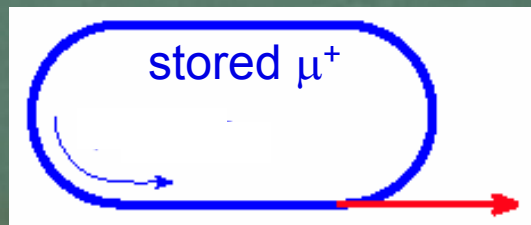
~ monochromatic E_ν , lower backgrounds

- Superbeams (upgrades $\times 4-5$)

Off-axis or Wide-band* (BNL)

*binning quasi-elastic events gives equivalent of many narrow-band beams

- Neutrino factory



$\nu_e, \bar{\nu}_\mu$

Golden channel: $\nu_e \rightarrow \nu_\mu$

- New detector technologies with 50–500 kton sizes
 - low-Z calorimeter
 - liquid Argon
 - water Cherenkov
 - iron scintillator

Approximate discovery reaches in $\sin^2 2\theta_x$

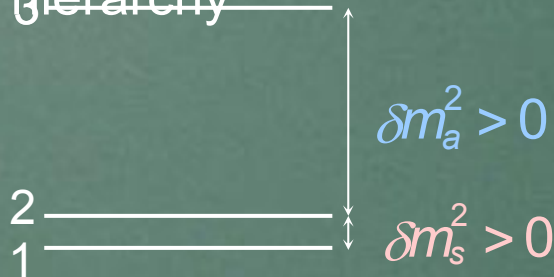
Current limit	10^{-1}
Reactor	10^{-2}
Conventional μ -beam	10^{-2}
Superbeam	3×10^{-3}
NuFact (entry level)	5×10^{-4}
NuFact (high performance)	5×10^{-5}

How low in $\sin^2 2\theta_x$ will we need to go?

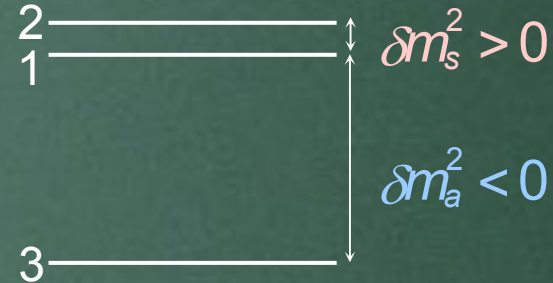
KEY ISSUE #3: MASS HIERARCHY?

Present data allow 2 mass orderings

normal
hierarchy



inverted hierarchy



Earth matter effects

- enhance $P(\nu_\mu \rightarrow \nu_e)$ and suppress $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ or vice-versa, depending on sign of δm_a^2
- increase with distance
- long baselines needed ($L > 900$ km) to determine hierarchy

KEY ISSUE #4: CP VIOLATION?

Is $P(\nu_\mu \rightarrow \nu_e) \neq P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$? (intrinsic)

$$\Delta P \propto \left(\frac{\delta m_s^2}{\delta m_a^2} \right) \sin \delta \sin^2 2\theta_x \quad \frac{\Delta P}{P} \sim \frac{\sin \delta \Delta_s}{\theta_x}$$

- δ measurement depends on θ_x ($\sin \theta_x e^{-i\delta}$ in \mathbf{V})
- Both δm_s^2 and δm_a^2 oscillations must contribute
- Must distinguish intrinsic CP-violation from fake CP-violation due to matter effects

Magic baselines $P(\nu_\mu \rightarrow \nu_e)$

$L \approx 600$ km depends only on $\sin \delta$ (not

$L \approx 7600$ km $\cos \delta$)
no δ -dependence (no CP-violation)
— matter oscillation wavelength

Approximate discovery reaches in $\sin^2 2\theta_x$

	<u>$\text{sgn}(\delta m_a^2)$</u>	<u>CP-violation</u>
Superbeam	1×10^{-2}	3×10^{-2}
NuFact (entry level)	1×10^{-3}	2×10^{-3}
NuFact (high performance)	1×10^{-4}	5×10^{-4}

Must resolve degeneracies that can confuse CP-violating and CP-conserving solutions

Parameter sets that give same

$$P(\nu_\mu \rightarrow \nu_e) \text{ and } P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \text{ at one } L \text{ and } E$$

Eight-fold degeneracy

$$(\delta, \theta_x)$$

$$\text{sgn}(\delta m_a^2) = \pm$$

$$\left(\theta_a, \frac{\pi}{2} - \theta_a \right) \text{ if } \theta_a \neq \frac{\pi}{4}$$

Best strategies:

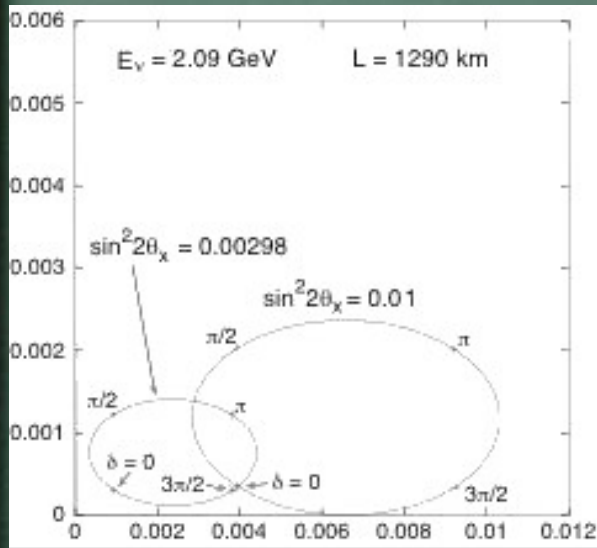
- 1) detector at first oscillation peak
- 2) long L (>1000 km)
- 3) 2 distances

8-fold parameter degeneracy

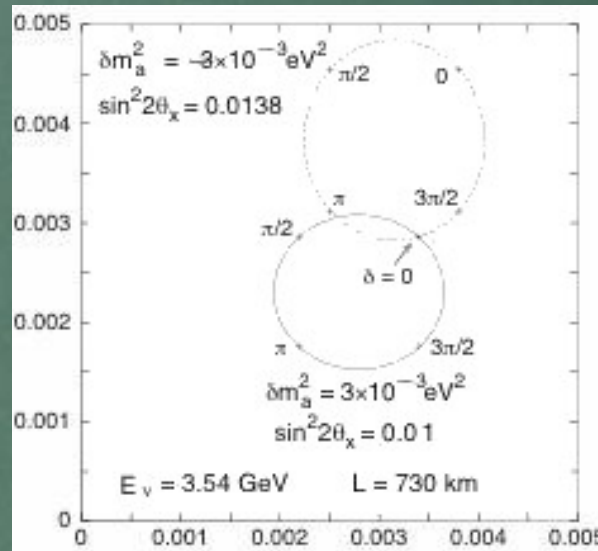
$$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \text{ vs } P(\nu_\mu \rightarrow \nu_e)$$

θ_x fixed, δ varied

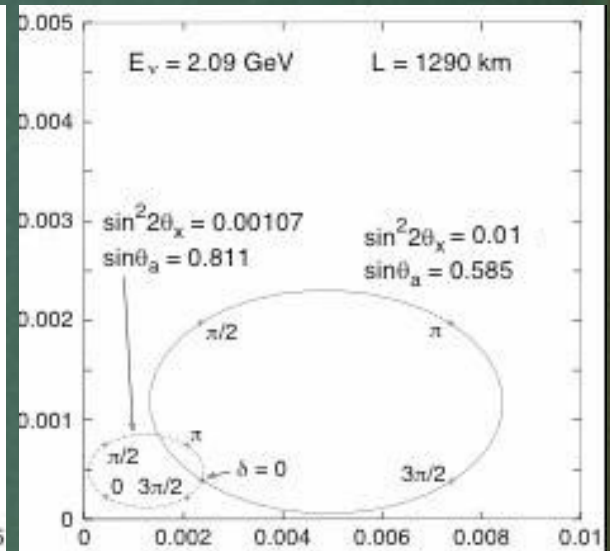
(δ, θ_x) degeneracy



$\text{sgn}(\delta m_a^2)$ degeneracy



$\left(\theta_a, \frac{\pi}{2} - \theta_a\right)$ degeneracy

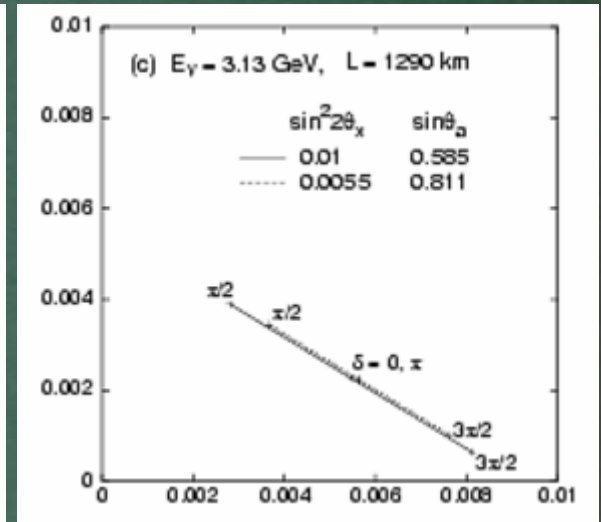
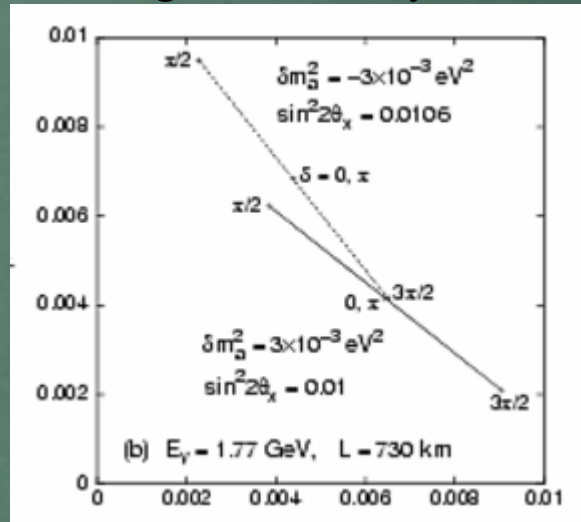
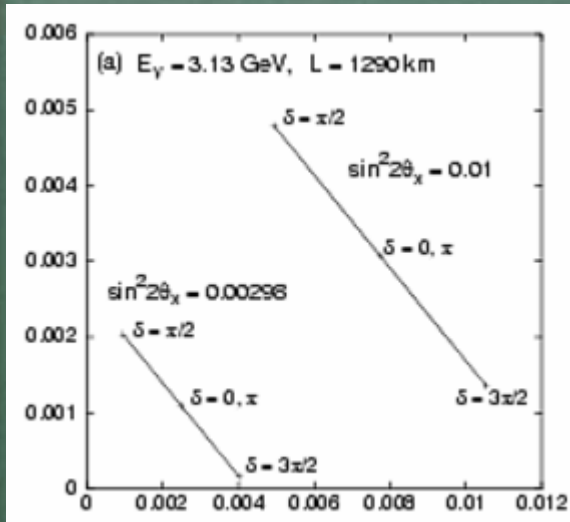


Remaining ambiguities when $\theta_a = \pi/2$

(δ, θ_x) degeneracy resolved

$\text{sgn}(\delta m_a^2)$ degeneracy remains,
large uncertainty in δ

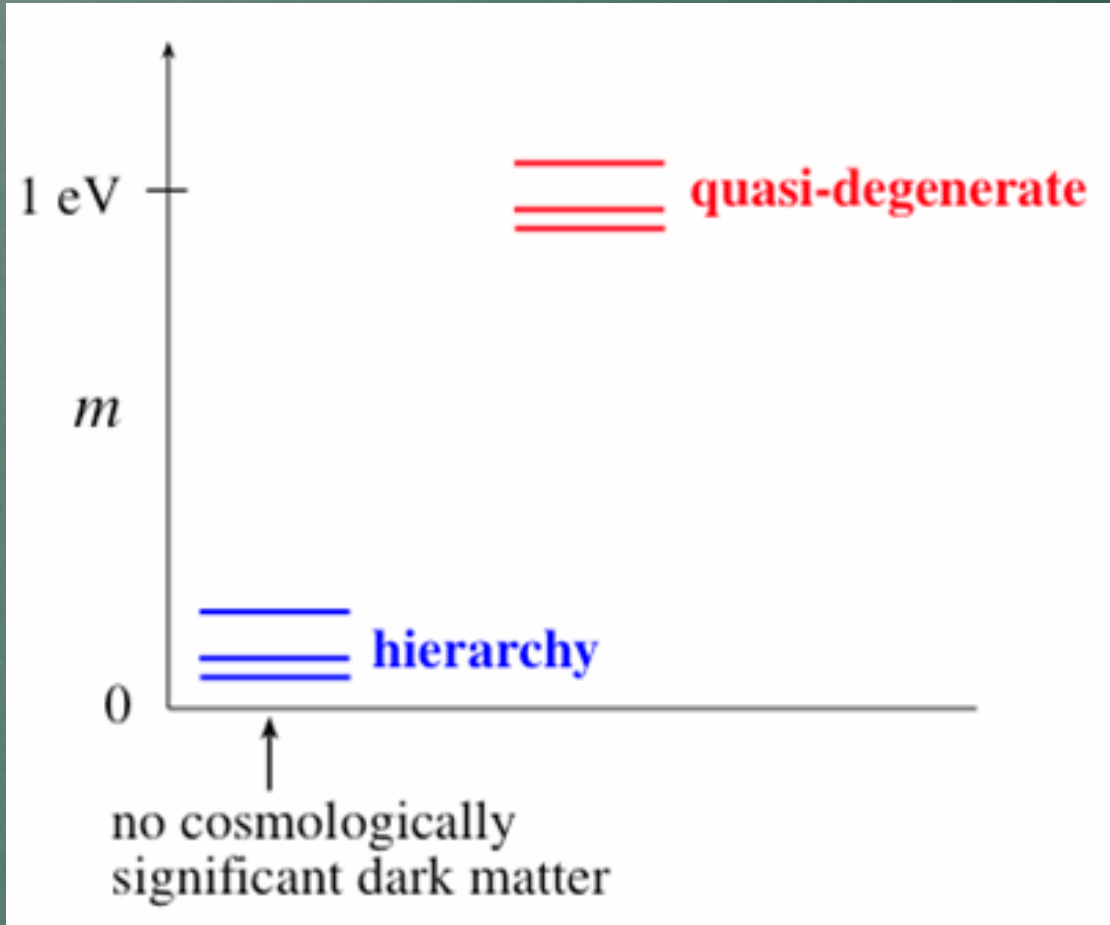
$\theta_a, \frac{\pi}{2} - \theta_a$ still degenerate,
large uncertainty in $\sin^2 2\theta_x$



In all cases, a $(\delta, \pi - \delta)$ ambiguity remains

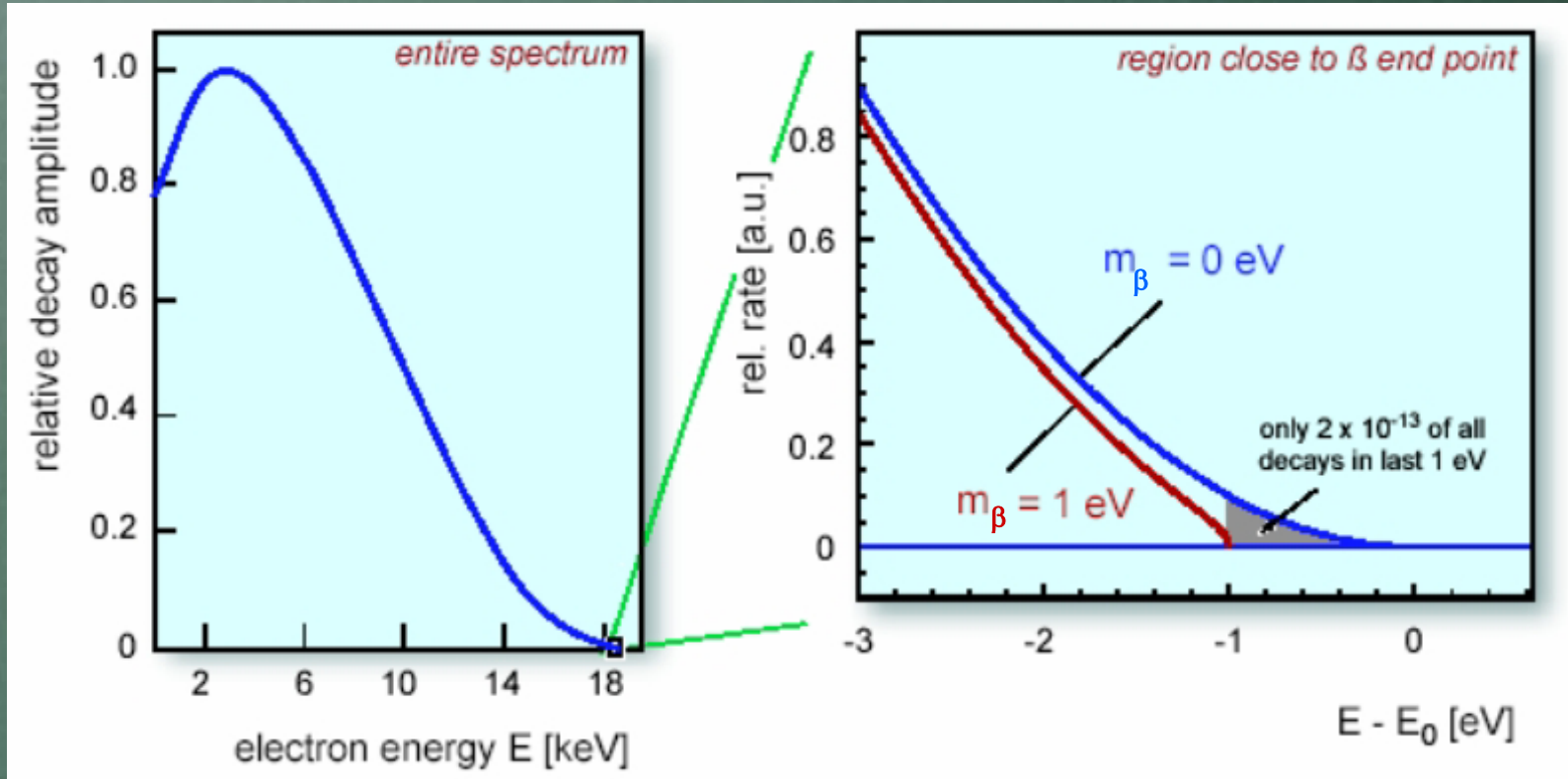


KEY ISSUE #5: ABSOLUTE NEUTRINO MASS SCALE



β -decay endpoint measurement

ν rest mass cuts off the β -spectrum in the endpoint energy region



$$m_\beta^2 = \sum |V_{ei}|^2 m_i^2$$

Present limits:

$$m_\beta \leq 2.2 \text{ eV}$$

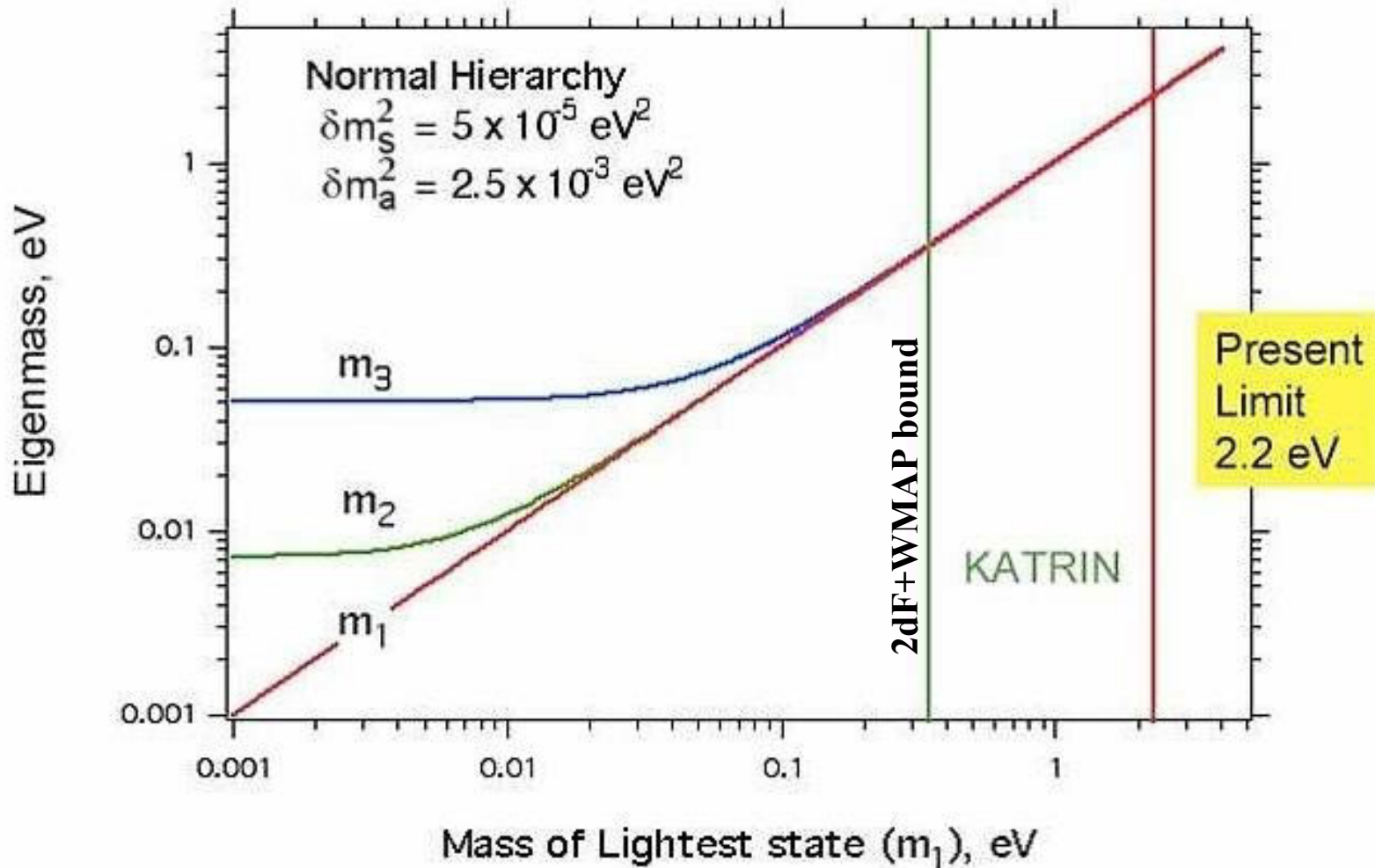
Troitsk, Mainz

Future sensitivity:

$$m_\beta \geq 0.35 \text{ eV}$$

KATRIN (>2007)

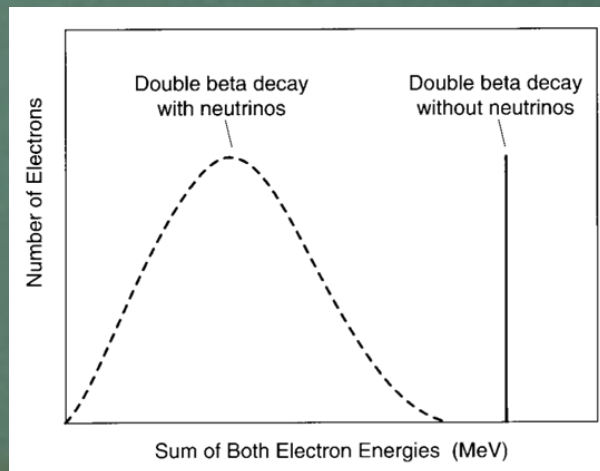
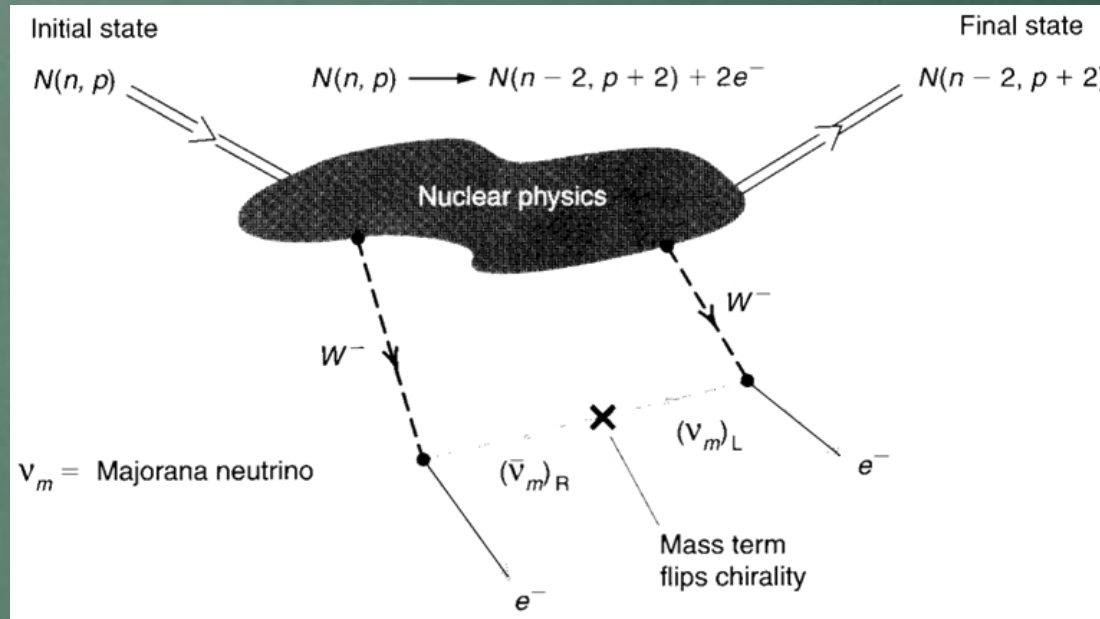
Important that KATRIN confirms cosmological



Near future sensitivity: SDSS+WMAP probe $\sum m_\nu > 0.1 \text{ eV}$

KEY ISSUE #6: DIRAC OR MAJORANA?

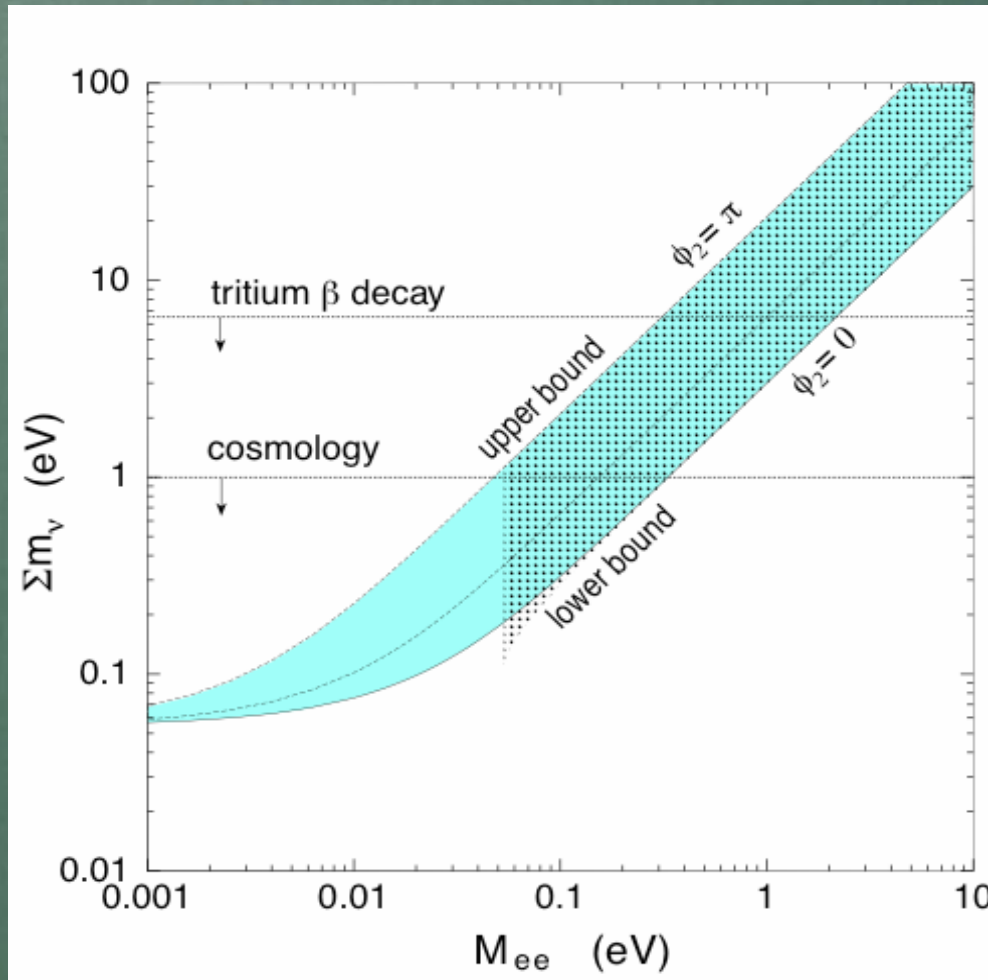
Neutrinoless double- β decay only if neutrinos are Majorana



$0\nu\beta\beta$ experimental limit

$$M_{ee} \leq 0.35 - 1.24 \text{ eV}$$

$$2M_{ee} + \sqrt{M_{ee}^2 + \delta m_a^2} \leq \Sigma m_\nu \leq 2 \frac{M_{ee}}{\cos 2\theta_s} + \sqrt{\left(\frac{M_{ee}}{\cos 2\theta_s}\right)^2 + \delta m_a^2}$$



Neutrinoless double- β decay can constrain Σm_ν (upper and lower bounds)

Detect CP violation via $0\nu\beta\beta$ decay?

Optimum conditions

- mass spectrum not hierarchical
- $\theta_x = 0$ (minimizes CPV, CPC confusion)

For a measurement $M_{ee}^{1+}(\frac{x}{y})$

) the necessary condition for CPV detectability is

$$\sin^2 2\theta_s \geq 1 - \left(\frac{1-y}{1+x} \right)^2 \cong 0.99$$

for the present factor of 3 uncertainty in nuclear matrix elements

For realistic improvements in uncertainties, it is unlikely that the solar oscillation amplitude is sufficiently large to allow detection of CP violation via $0\nu\beta\beta$

So neutrino oscillations only way to probe CP violation in the lepton sector

KEY ISSUE #7: 3×3 MIXING MATRIX UNITARITY?

Need to measure all elements

ν_e beams required: only at a neutrino factory

<u>channel</u>	<u>detect</u>
$\nu_\mu \rightarrow \nu_\mu$	μ^-
$\nu_\mu \rightarrow \nu_e$	e^-
$\nu_\mu \rightarrow \nu_\tau$	τ^-
$\bar{\nu}_e \rightarrow \bar{\nu}_e$	e^+
$\bar{\nu}_e \rightarrow \bar{\nu}_\mu$	μ^+
$\bar{\nu}_e \rightarrow \bar{\nu}_\tau$	τ^+

With ν_e beams can also test time reversal violation

$$P(\nu_e \rightarrow \nu_\mu) \neq P(\nu_\mu \rightarrow \nu_e)$$

KEY ISSUE #8: WHAT THEORY?

Seesaw mechanism favored

$$m_\nu \sim \frac{m_D^2}{M_N}$$

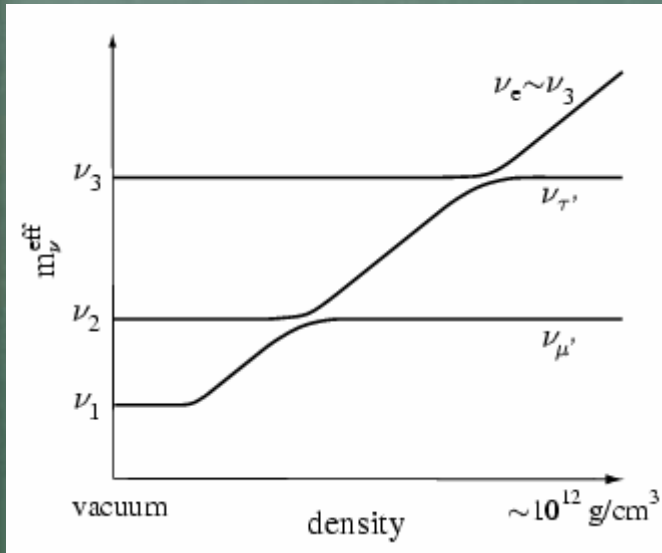
N : singlets in GUT representations

GUT models can accommodate
all quark and lepton data

Make differing predictions for θ_x and CP violation



Miscellaneous #1: Galactic supernovae?



$$\delta m_a^2, \sin^2 2\theta_x$$

$$\delta m_s^2, \sin^2 2\theta_s$$

jumping prob. depends
on θ_x
adiabati
c

Virtue- directly probe $\text{sgn}(\delta m_a^2)$ and θ_x (no 8-fold degeneracy)

Vices- need to assume knowledge of initial neutrino spectra

- only three per century



Key parameter $\tau = \frac{\langle E_{\text{nonelectron}} \rangle}{\langle E_e \rangle}$ - Larger the better
 Unfortunately, recent SN models find τ

↪ 1.1

Safe deductions for SNO + SK
 (HyperK):

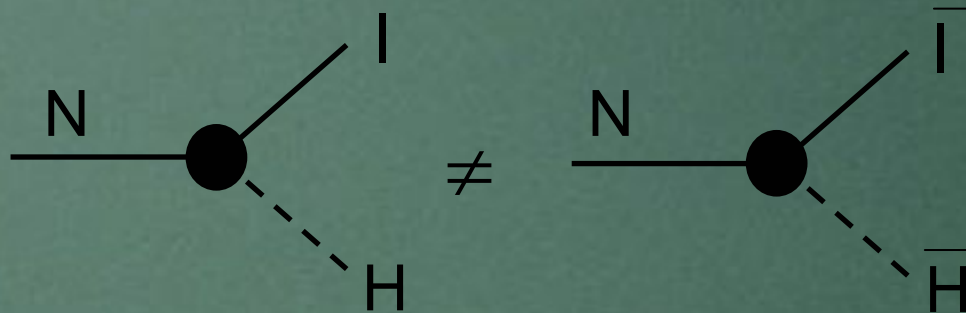
- If $\delta m_a^2 < 0$, upper or lower bound on θ_x and $\sin^2 2\theta_x \text{sgn}(\delta m_a^2) \text{ } \textcircled{9} 10^{-3}$, determined
- If $\delta m_a^2 > 0$, $\theta_x \text{sgn}(\delta m_a^2)$ undetermined

If $\sin^2 2\theta_x \text{sgn}(\delta m_a^2) \text{ } \textcircled{9} 0.01$ from reactors/accelerators, determined

Miscellaneous #2: Leptogenesis?

Matter-antimatter asymmetry from processes that violate CP in the early universe

Lepton asymmetry from decays of heavy right-handed neutrinos can lead to the baryon asymmetry



In some models, sign of cosmological baryon number is related to the CP phase in neutrino oscillations

These models make testable low energy predictions


SUMMARY

Neutrino mass is the first discovery of physics beyond the Standard Model.

Oscillation experiments “on the table” have great potential for another breakthrough in measuring θ_x .

The future of oscillation physics is very bright, with Superbeams and longer baselines as the next horizon.


Whatever experiments accomplish over the next decade, Neutrino Factories will be essential to reconstruct all neutrino mixings with high precision. Combine Neutrino Factory and Superbeam data.

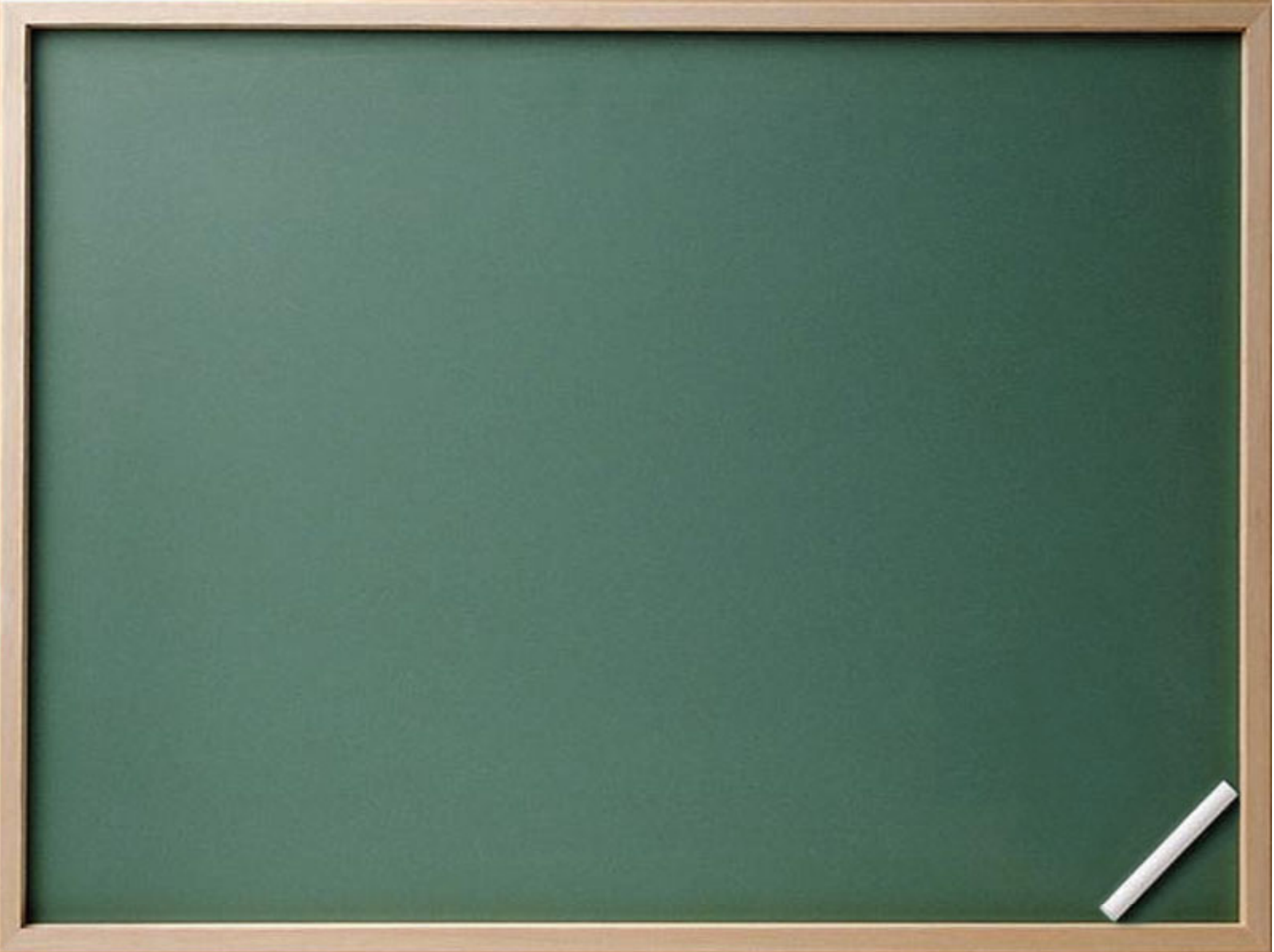


If theoretical prejudices for Grand Unified Theories are correct, neutrino mass owes its origin to right-handed neutrinos with masses near the GUT scale.
Leptogenesis could be a consequence.

These and other ideas can soon be “put to the test” at least in the context of models, by measuring θ_x , $\text{sgn}(\delta m_a^2)$ and δ .

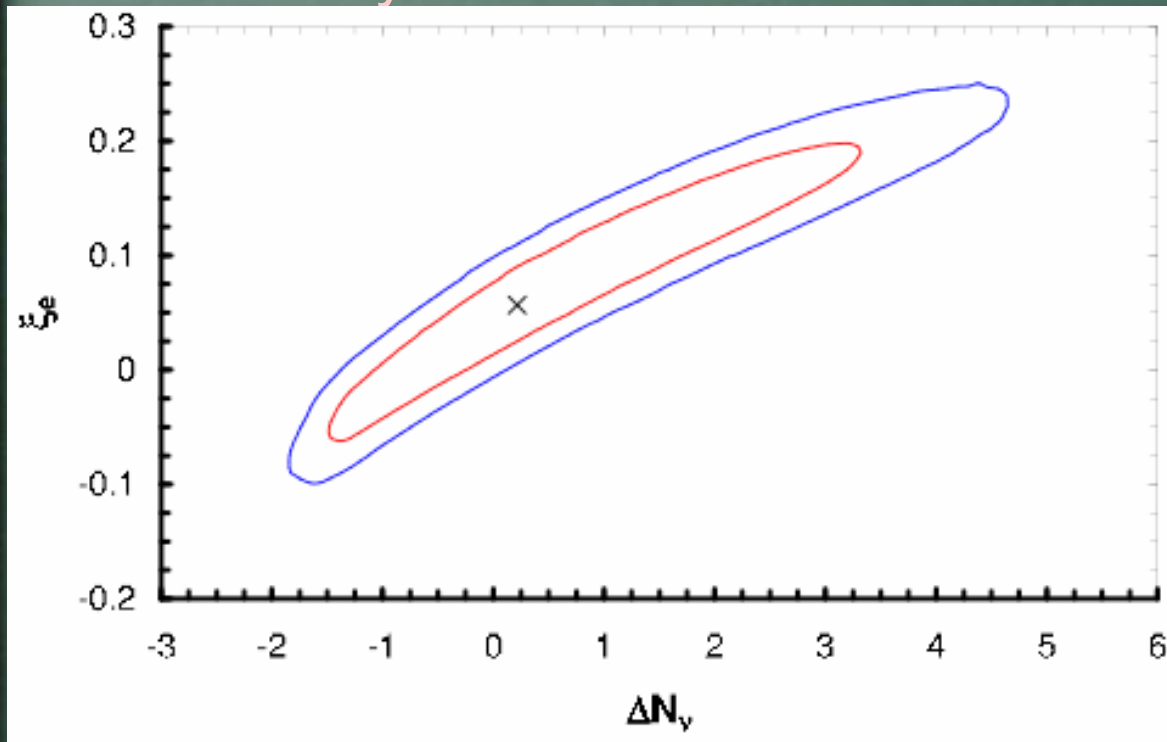
Neutrino physics has always been full of surprises. There will likely be more surprises to come!





Houdini's escape from the BBN constraints

A large asymmetry between numbers of ν_e^- and ν_e in the early universe allows extra neutrinos



$$L_e = \frac{n_{\nu_e} - \bar{n}_{\nu_e}}{n_\gamma} \cong 0.7 \xi_e$$

degeneracy parameter

$$\xi_e \equiv \frac{\mu_e}{T}$$

$$\left(\frac{n}{p}\right)_{equil} = \exp\left(-\frac{\Delta m_{np}}{T} - \xi_e\right)$$

ξ_e reconciles LSND neutrino with BBN by suppressing its thermalization prior to BBN

LSND sterile neutrino implies $\frac{n_{\nu_e} - n_{\bar{\nu}_e}}{n_\gamma} \sim 0.01 - 0.1$

$$\frac{n_B}{n_\gamma} \sim 10^{-9}$$

Huge compared to baryon asymmetry

