

Subleading Corrections to 2-body Hadronic B Decays

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- ◆ Brief review on PP, PV modes
- ◆ Can we understand why $\omega_K/\omega_\pi \sim 1$?
- ◆ Corrections from 3-parton distribution amplitudes of the final state mesons
- ◆ impact of subleading corrections
- ◆ Summary *Ref.: hep-ph/0308005*

Brief review on PP, VP modes

QCD Factorization vs. PQCD: (hep-ph/0207228, M.Beneke; hep-ph/0303116, H.N.Li)

| Decay Mode | Exp. Average | Default fit (QCDF) | Fit2 (QCDF) | PQCD |
|-----------------------------------|-------------------|--------------------|-------------|----------------------|
| $B^0 \rightarrow \pi^+ \pi^-$ | 5.15 ± 0.61 | 5.12 | 5.24 | $7.0^{+2.0}_{-1.5}$ |
| $B^\pm \rightarrow \pi^\pm \pi^0$ | 4.88 ± 1.06 | 5.00 | 4.57 | $3.7^{+1.3}_{-1.1}$ |
| $B^0 \rightarrow \pi^0 \pi^0$ | (2.07 ± 0.57) | 0.78 | 0.94 | 0.3 ± 0.1 |
| $B^0 \rightarrow \pi^\mp K^\pm$ | 18.56 ± 1.08 | 17.99 | 18.47 | $15.5^{+3.1}_{-2.5}$ |
| $B^\pm \rightarrow \pi^0 K^\pm$ | 11.49 ± 1.26 | 12.07 | 11.83 | $9.1^{+1.9}_{-1.5}$ |
| $B^\pm \rightarrow \pi^\pm K^0$ | 17.93 ± 1.70 | 15.65 | 17.88 | 17.3 ± 2.7 |
| $B^0 \rightarrow \pi^0 K^0$ | 8.82 ± 2.20 | 5.55 | 6.87 | 8.6 ± 0.3 |

Table 1: (in units of 10^{-6}): The default fit (QCDF): $\gamma = 116^\circ$ with $\chi^2 = 4.5$. “Fit2” (QCDF): $\gamma = 97^\circ$, $\chi^2 = 1.0$ refers to a fit without annihilation contributions and chirally enhanced spectator corrections but with $m_s = 80$ MeV. PQCD predictions for the $B \rightarrow \pi\pi$ and $K\pi$ branching ratios in unit of 10^{-6} with $\phi_3 = 80^\circ$.

QCDF: annihilation effects are probably not large in PP modes, yet they constitute the largest uncertainty in the calculation of this amplitude.

PQCD: dynamical penguin enhancement (to Brs) + imaginary annihilation penguins (to ACPs)



Problems in QCD factorization approaches

- ▶ The recent QCDF fitting result (by D.S.Du *et al*) hints that, to account for the large $\omega\bar{K}^0$ rate, the annihilation contributions to modes $\phi K^{-,0}, K^-\rho^+, \omega K^{-,0}$ may be **over 80%**.
- ▶ In case the annihilation effects dominate in Brs of $B \rightarrow Kh^*$, experiments should observe a simple relation among these modes, for instance $\rho^+ K^{-,0} : \rho^0 K^{-,0} : \omega K^{-,0} \approx 1 : (1/\sqrt{2})^2 : (1/\sqrt{2})^2 : (1/\sqrt{2})^2$, the same as their annihilation ratios squared. It should be easily checked from experimental measurements.
- ▶ Recently a global fit for all measured $B \rightarrow PV$ rates was performed by R.Aleksan et al. They conclude that it is impossible to reach a reliable best fit even considering the charming-penguin effects.
- ▶ Note that Charming penguin effects are shown to be negligible by A. Khodjamiriam. T. Mannel and B. Melić

Problems in QCD factorization approaches

Two possible ways to solve the ratio problem for $\pi^+\pi^-/\pi^-\pi^0 \lesssim 1$:

- (i) large $\gamma > 90^\circ$;
- (ii) a smaller form factor $F_0^{B\pi} = 0.25 + \text{large ratio } a_2/a_1$ (which is parametrized in the analysis).

Beneke and Neubert (hep-ph/0308039) showed that

(ii) together with $m_s = 80$ MeV and $\rho_A = 1$ is favored by the data.

- ◆ They chose $a_2(\pi\pi)/a_1(\pi\pi) = 0.48/0.88$, $a_2(\pi\rho)/a_1(\pi\rho) = 0.41/0.90$.
But why?
- ◆ Large a_2 and small γ are good for producing the large $K\omega$ rates.
- ◆ ϕK Brs are sensitive to ρ_A , the annihilation contributions. **(H.Y. Cheng, KCY)**

However they obtained $\pi^-\omega/\overline{K}^0\omega \simeq 8.4/4.9 \approx 1.7$, but the data is ~ 1

Can we understand why $\omega K/\omega\pi \sim 1$?

◆ *Data* (in units of 10^{-6}):

BaBar

$$\mathcal{B}(B^\pm \rightarrow \omega K^\pm) = (5.0 \pm 1.0 \pm 0.4)$$

$$\mathcal{B}(B^0 \rightarrow \omega K^0) = (5.3 \pm 1.3 \pm 0.5)$$

$$\mathcal{B}(B^\pm \rightarrow \omega\pi^\pm) = (5.4 \pm 1.0 \pm 0.5),$$

Belle

$$\mathcal{B}(B^\pm \rightarrow \omega K^\pm) = (6.7_{-1.2}^{+1.3} \pm 0.6)$$

$$\mathcal{B}(B^0 \rightarrow \omega K^0) = (4.0 \pm_{1.6}^{1.9} \pm 0.5)$$

$$\mathcal{B}(B^\pm \rightarrow \omega\pi^\pm) = (4.2_{-1.8}^{+2.0} \pm 0.5),$$

◆ *Theoretical results* (in units of 10^{-6}):

(Generalized) factorization: $N_c=2\sim 3$:

$$\omega\pi^\pm : 11 - 5, \omega K^\pm : 0.9 - 1.5, \omega K^0 : 0.6 - 0.3$$

$$\omega K/\omega\pi^\pm \lesssim 0.2 \text{ by Cheng et al. } (< 0.5 \text{ by Ali et al.})$$

Earlier QCDF calculation:

$$\omega\pi^\pm : 6 - 7, \omega K^\pm : 0.4 - 0.5, \omega K^0 : \sim 0.01 \text{ (D.S.Du et al)}$$

$$\Rightarrow \omega K/\omega\pi^\pm < 0.1$$

PQCD: $\omega\pi^\pm : 8 \pm 2$ for $\gamma = 75^\circ - 180^\circ$ (C.D.Lu & M.Z. Yang)

$$\omega K^\pm : 3.2, \omega K^0 \sim 2.2 \text{ (C.H. Chen)}$$

$$\Rightarrow \omega K/\omega\pi^\pm < 0.5$$

theoretically, why the ratio $\omega K/\omega\pi$ is small?

$$M(B^- \rightarrow \pi^- \omega) \simeq G_F m_\omega (\epsilon \cdot p_\pi) \left(f_\pi A_0^{B \rightarrow \omega}(m_\pi^2) \{V_{ub} V_{ud}^* a_1 - V_{tb} V_{td}^* [a_4 - a_6 r_\chi^\pi]\} \right. \\ \left. + f_\omega F_1^{B \rightarrow \pi}(m_\omega^2) \{V_{ub} V_{ud}^* a_2 - V_{tb} V_{td}^* \left[a_4 + 2(a_3 + a_5) + \frac{1}{2} a_9 \right]\} \right)$$

$$M(\bar{B}^0 \rightarrow \bar{K}^0 \omega) \simeq G_F m_\omega (\epsilon \cdot p_K) \left[-f_K A_0^{B \rightarrow \omega}(m_{K^0}^2) V_{tb} V_{ts}^* (a_4 - a_6 r_\chi^K) \right. \\ \left. + f_\omega F_1^{B \rightarrow K}(m_\omega^2) \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left(2(a_3 + a_5) + \frac{1}{2} a_9 \right) \right\} \right]$$

$$M(B^- \rightarrow K^- \omega) = G_F m_\omega (\epsilon \cdot p_K) \left[f_K A_0^{B \rightarrow \omega}(m_K^2) \{V_{ub} V_{us}^* a_1 - V_{tb} V_{ts}^* (a_4 - a_6 r_\chi^K)\} \right. \\ \left. + f_\omega F_1^{B \rightarrow K}(m_\omega^2) \left\{ V_{ub} V_{us}^* a_2 - V_{tb} V_{ts}^* \left(2(a_3 + a_5) + \frac{1}{2} a_9 \right) \right\} \right].$$

where $r_\chi^\pi \approx r_\chi^K = \frac{2m_K^2}{(m_b+m_u)(m_u+m_s)} \approx \mathbf{1}$,

$a_{2i} = c_{2i} + c_{2i-1}/N_c, a_{2i-1} = c_{2i-1} + c_{2i}/N_c$

$a_1 \approx 1, a_4 \approx -0.0041 - 0.0036i, a_6 = -0.0548 - 0.0036i$

- ◆ $\omega\pi$: is not small and tree dominated $\propto a_1$. \rightarrow Annihilation is negligible.
- ◆ ωK : should be small because a_4, a_6 terms are destructive

The ratio $\overline{K}^0 \omega / \pi^- \omega$ reads

$$\frac{\overline{K}^0 \omega}{\pi^- \omega} \approx \left| \frac{V_{cb}}{V_{ub}} \right|^2 \left(\frac{f_K}{f_\pi} \right)^2 \left| \frac{a_4 - a_6 r_\chi^K + (F_1^{BK} f_\pi) / (F_1^{B\pi} f_K) r_1 a_9 / 2 + f_B f_K b_3(K, \omega)}{a_1 + r_1 a_2} \right|^2,$$

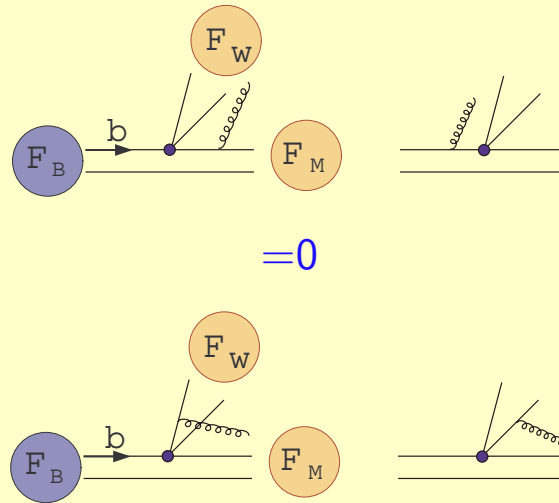
where $r_1 = f_\omega F_1^{B\pi} / f_\pi A_0^{B\omega}$.

Without annihilation contributions, because in the $\overline{K}^0 \omega$ amplitude a_4 and $a_6 r_\chi^K$ terms are opposite in sign, **the ratio $\overline{K}^0 \omega / \pi^- \omega$ should be very small.**

- ◆ Choosing **smaller m_s can enlarge the ratio but it is still $\lesssim 0.2$ for $m_s \gtrsim 80$ MeV.**
- ◆ QCDF:VP modes may be dominated by annihilation contribution (fit by D.S.Du et al.) but this possibility seems to be ruled out in the recent fitting analysis in which $CL < 0.1\%$ (by R.Aleksan et al.)
- ◆ PQCD cannot offer the explanation. **the annihilation contribution, b_3 , is imaginary.**
- ◆ Can the contribution arising from tree-parton distribution amplitudes (DAs) of the final state mesons improve the results?

Answer is YES.

3-parton distribution amplitudes



$$M(B^- \rightarrow \pi^- \omega) = \cdots + G_F m_\omega (\epsilon \cdot p_\pi) f_\omega F_1^{B \rightarrow \pi}(m_\omega^2) (V_{ub} V_{ud}^* c_1 - V_{tb} V_{td}^* (2c_4 - 2c_6 + c_3)) f_3$$

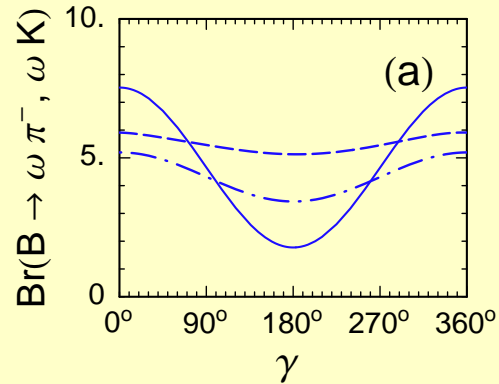
$$M(\bar{B}^0 \rightarrow \bar{K}^0 \omega) = \cdots + G_F m_\omega (\epsilon \cdot p_K) f_\omega F_1^{B \rightarrow K}(m_\omega^2) (V_{ub} V_{us}^* c_1 - V_{tb} V_{ts}^* (2c_4 - 2c_6)) f_3$$

$$f_3 = \frac{\sqrt{2}}{m_B^2 f_\omega F_1^{B\pi}(m_\omega^2)} \langle \omega \pi^- | O_1 | B^- \rangle_{\text{qqg}} = -\frac{4}{\bar{\alpha}_g m_B^4 F_1^{B\pi}(m_\omega^2)} p_\omega^\alpha \langle \pi^- | \bar{d} \gamma^\mu \gamma_5 g_s \tilde{G}_{\alpha\mu} b | B^- \rangle$$

$$= 0.12$$

◆ Result: in units of 10^{-6}

(We have adopted LC sum rule results for form factors but with $A_0^{B\omega} = 0.28$)



For $\gamma \approx 90^\circ$, $\omega \pi^\pm \approx 5.5$ (dashes line), sensitive to $A_0^{B\omega}$,
 $\omega K^\pm \approx 4.5$ (solid line)
 $\omega K^0 \approx 4.3$ (dash-dotted line),

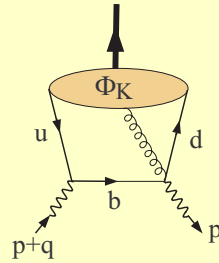
◆ Data: $\mathcal{B}(B^\pm \rightarrow \omega \pi^\pm) = (5.9 \pm 0.9)$
 $\mathcal{B}(B^\pm \rightarrow \omega K^\pm) = (5.3 \pm 0.8)$
 $\mathcal{B}(B^0 \rightarrow \omega K^0) = (5.1 \pm 1.1)$

Note that

(i) without the contributions from 3-parton Fock states of mesons,

$\phi K^-, \omega \pi^-, \omega K^-, \omega \bar{K}^0 \simeq 11, 3.9, 3.1, 2.9$. The 3-parton Fock state effects give constructive contributions to $\omega \pi, \omega K$ modes, but destructive one to the ϕK mode;

(ii) the annihilation parameter can be determined from the fit of ϕK Brs $\sim 8.5 \times 10^{-6}$.



$$\begin{aligned}
 T_\alpha(p, q) &= i \int d^4x e^{ipx} \langle \pi^-(q) | \bar{d}(x) g_s \tilde{G}_{\alpha\mu}(x) \gamma^\mu \gamma_5 b(x) \bar{b}(0) i \gamma_5 u(0) | 0 \rangle \\
 &= \frac{m_B^2 f_B}{m_b} \frac{1}{m_B^2 - (p+q)^2} \langle \pi^-(q) | \bar{d} \gamma^\mu \gamma_5 g_s \tilde{G}_{\alpha\mu} b | B^-(p+q) \rangle + \dots
 \end{aligned}$$

$$\langle \pi^-(q) | \bar{d} \gamma^\mu \gamma_5 g_s \tilde{G}_{\alpha\mu} b | B^-(p+q) \rangle = p_\alpha f_-(p^2) + (p_\alpha + 2q_\alpha) f_+(p^2)$$

Under the Borel transformation $-(p+q)^2 \rightarrow M^2$, the final sum rule takes the form

$$\begin{aligned}
 f_-(p^2) &= -f_+(p^2) = -\frac{m_b}{2m_B^2 f_B} \int_0^1 du \int_0^u d\alpha_g \exp\left(\frac{m_B^2}{M^2} - \frac{m_b^2 - \bar{u}p^2}{uM^2}\right) \Theta\left(u - \frac{m_b^2 - p^2}{s_0 - p^2}\right) \\
 &\times \left[-f_{3\pi} \frac{m_b^2 - p^2}{u^2} \phi_{3\pi} + f_\pi \frac{m_b}{u} (\tilde{\phi}_\parallel - 2\tilde{\phi}_\perp) \right]
 \end{aligned}$$

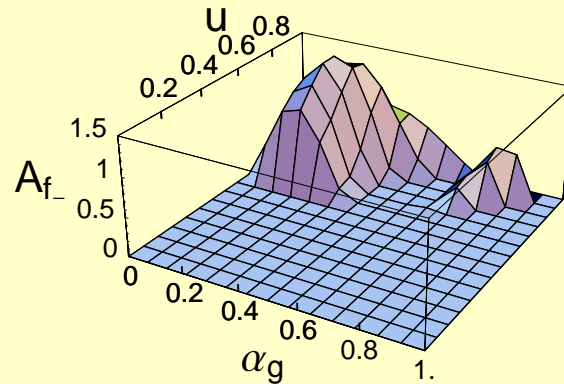
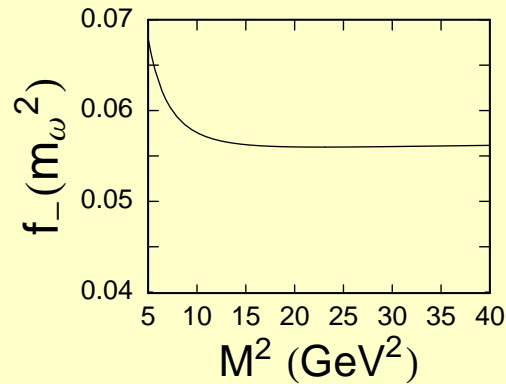
$$\Rightarrow f_\mp(\omega^2) = -(0.057 \pm 0.005) \text{ for Borel window: } 9 \text{ GeV}^2 < M^2 < 20 \text{ GeV}^2$$

($s_0 \simeq 37 \text{ GeV}^2$: the threshold of resonances)

$$\begin{aligned}
& \langle \pi(q) | \bar{d}(x) g_s G_{\mu\nu}(vx) \sigma_{\alpha\beta} \gamma_5 u(0) | 0 \rangle \\
&= if_{3\pi} [q_\beta (q_\mu g_{\nu\alpha} - q_\nu g_{\mu\alpha}) - q_\alpha (q_\mu g_{\nu\beta} - q_\nu g_{\mu\beta})] \int \mathcal{D}\alpha \phi_{3\pi} e^{iqx(\alpha_d + v\alpha_g)}, \\
& \langle \pi(q) | \bar{d}(x) \gamma_\mu g_s \tilde{G}_{\alpha\beta}(vx) u(0) | 0 \rangle = if_\pi (q_\alpha g_{\beta\mu} - q_\beta g_{\alpha\mu}) \\
& \times \int \mathcal{D}\alpha \tilde{\phi}_\perp e^{iqx(\alpha_d + v\alpha_g)} - if_\pi \frac{q_\mu}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) \int \mathcal{D}\alpha (\tilde{\phi}_\parallel + \tilde{\phi}_\perp) e^{iqx(\alpha_d + v\alpha_g)}.
\end{aligned}$$

where $\tilde{G}_{\alpha\beta} = \frac{1}{2} \epsilon_{\alpha\beta\sigma\tau} G^{\sigma\tau}$ and $\mathcal{D}\alpha_i = d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3)$. While $\phi_{3\pi}(\alpha_i)$ is a twist 3 wave function, the remaining functions $\tilde{\phi}_\perp$ and $\tilde{\phi}_\parallel$ are all of twist 4.

$$\begin{aligned}
\phi_{3\pi}(\alpha_i) &= 360\alpha_d\alpha_{\bar{u}}\alpha_g^2 \left[1 + \omega_{1,0} \frac{1}{2} (7\alpha_g - 3) + \omega_{2,0} (2 - 4\alpha_d\alpha_{\bar{u}} - 8\alpha_g + 8\alpha_g^2) \right. \\
&\quad \left. + \omega_{1,1} (3\alpha_d\alpha_{\bar{u}} - 2\alpha_g + 3\alpha_g^2) \right], \\
\tilde{\phi}_\perp(\alpha_i) &= 30\delta^2 \alpha_g^2 (1 - \alpha_g) \left[\frac{1}{3} + 2\varepsilon(1 - 2\alpha_g) \right], \\
\tilde{\phi}_\parallel(\alpha_i) &= -120\delta^2 \alpha_d\alpha_{\bar{u}}\alpha_g \left[\frac{1}{3} + \varepsilon(1 - 3\alpha_g) \right].
\end{aligned}$$



$f_-(m_\omega^2) = \int_0^1 du \int_0^u d\alpha_g A_{f_-}$ with $M^2=10 \text{ GeV}^2$. The volume in the plot is equal to $f_-(m_\omega^2)$. Here $u = \alpha_d + \alpha_g$.

In π^- , the averaged *gluon* momentum fraction is:

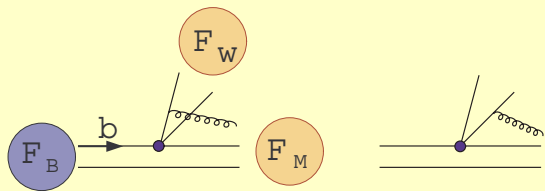
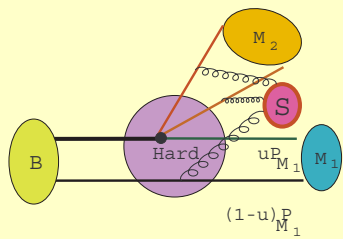
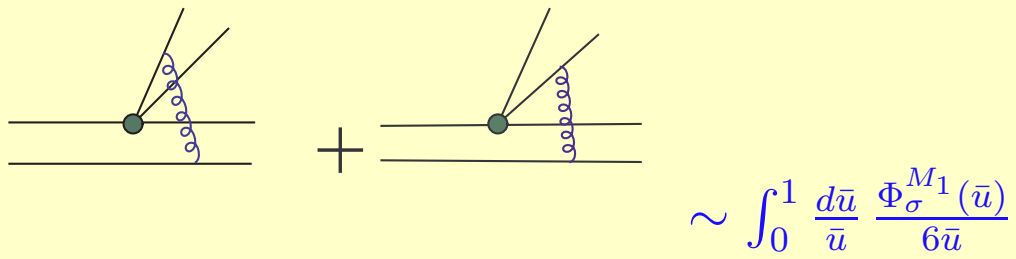
$$\bar{\alpha}_g^\pi = (\int_0^1 du \int_0^u d\alpha_g \times \alpha_g A_{f_1}) / f_1 \simeq 0.23;$$

the averaged momentum fraction of *gluon + d quark*:

$$u = (\int_0^1 du \int_0^u d\alpha_g \times u A_{f_1}) / f_1 \simeq 0.83.$$

\Rightarrow to this order the gluon and d quark can account for 83 % momentum fraction of the final state π^- meson.

Impact of the subleading corrections



Impact of the subleading corrections

Effective coefficients a_i^{SL} with the subleading corrections for PP, VP modes:

$$\begin{aligned} a_{2i}^{\text{SL}} &= a_{2i} + [1 + (-1)^{\delta_{3i} + \delta_{4i}}] c_{2i-1} f_3/2, \\ a_{2i-1}^{\text{SL}} &= a_{2i-1} + (-1)^{\delta_{3i} + \delta_{4i}} c_{2i} f_3, \end{aligned}$$

where $i = 1, \dots, 5$, and c_i are Wilson coefficients defined at the scale $\mu_h = \sqrt{\Lambda_\chi m_B/2} \simeq 1.4 \text{ GeV}$ with Λ_χ the momentum of the emitted gluon.

Table 2: Values for a_i for charmless B decay processes without (first row) and with (second row) 3-parton Fock state contributions of final state mesons, where a_{3-10} are in units of 10^{-4} and the annihilation effects are not included.

| a_1 | a_2 | a_3 | a_4 | a_5 | a_6 |
|------------------|----------------|-------------|--------------|-------------|--------------|
| $1.02 + 0.014i$ | $0.10 - 0.08i$ | $26 + 26i$ | $-328 - 91i$ | $1.2 - 30i$ | $-487 - 72i$ |
| $0.974 + 0.014i$ | $0.25 - 0.08i$ | $-55 + 26i$ | $-291 - 91i$ | $112 - 30i$ | $-487 - 72i$ |

Note that

(i) 3DAs give significant corrections to a_2, a_3, a_5 , i.e., they are important in consideration of color suppressed modes and decay channels with singlet meson(s).

Impact of the subleading corrections

◆ $B \rightarrow J/\psi K$ Decay

$$A(B \rightarrow J/\psi K) \cong \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* a_2 f_{J/\psi} m_{J/\psi} F_1^{BK}(m_{J/\psi}^2) (2\varepsilon^* \cdot p_B),$$

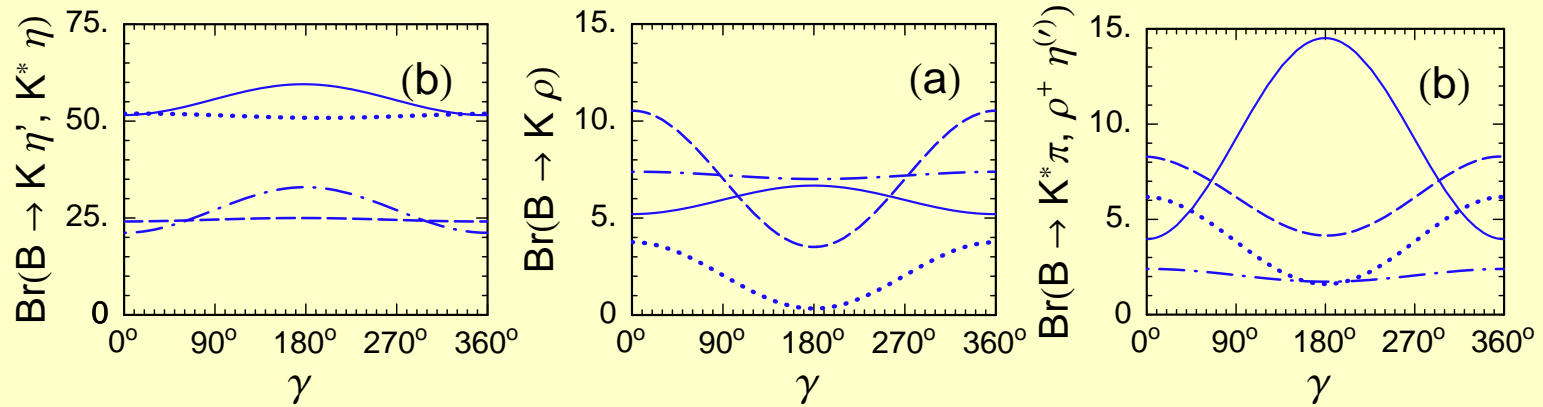
◆ extraction from the data, $|a_2| \simeq 0.25 - 0.3$

$$◆ a_2^{\text{SL}} = a_2^{\text{t2}} + c_1 \times f_3 = 0.10 + 0.05i + c_1(\mu_h)0.14 = 0.27 + 0.05i$$

◆ This solves the long-standing sign ambiguity of $a_2(J/\psi K)$ which turns out to be positive for its real part. Note that if $\text{Re}(a_2)$ were negative, f_3 would have to be ~ -0.3 which in turns would lead to

- i) $\phi K \sim 20 \times 10^{-6}$,
- ii) $\omega\pi^-, \omega K \sim 1 \times 10^{-6}$ and
- iii) too small Brs of $K^*\eta, K\eta'$!

Impact of the subleading corrections



- ◆ If $\eta^{(\prime)}$ is emitted, we have the correction

$$a_3 + a_5 = (c_4 + c_6) \times 0.12 + a_3^{2\text{DAs}} + a_5^{2\text{DAs}}$$
- ◆ With (without) the subleading corrections, $K^-\eta' \gtrsim \bar{K}^0\eta' \approx 55$ (35), and $K^{*-}\eta \gtrsim \bar{K}^{*0}\eta \approx 24$ (20), in units of 10^{-6} , where X^{PP} is set to be 0.
- ◆ At $\gamma = 90^\circ$, we have $\bar{K}^0\rho^0, K^-\rho^+, \bar{K}^0\rho^-, K^-\rho^0 = 6, 7, 7, 2$ ($\times 10^{-6}$), and $K^{*-}\pi^+, \bar{K}^{*0}\pi^0, \rho^-\eta, \rho^-\eta' = 9, 2, 6, 4$ ($\times 10^{-6}$).
- ◆ QCDF without 3DA contributions (Du et al; Beneke, Neubert):

$$\rho^+K^{-,0} : \rho^0K^{-,0} : \omega K^{-,0} \approx 1 : (1/\sqrt{2})^2 : (1/\sqrt{2})^2$$
- ◆ PQCD (C.H. Chen): $\bar{K}^0\rho^0, K^-\rho^+, \bar{K}^0\rho^-, K^-\rho^0 = 2.5, 5.4, 3.0, 2.2$ ($\times 10^{-6}$)

Summary

- ◆ Including the $K_{\bar{q}sg}$ corrections we obtain $a_2(J/\psi K) \approx 0.27 + 0.05i$ which is well consistent with the data. The sign of $\text{Re}(a_2)$ turns out to be positive.
- ◆ With (without) 3-Fock state corrections,
 $a_3 = -55 + 26i$ ($26 + 26i$), $a_5 = 112 - 30i$ ($1.2 - 30i$)
- ◆ For $\gamma \approx (60 - 110)^\circ$, $\omega\pi^-, \omega K^-, \omega\bar{K}^0 \approx 6.0, (6 \sim 5), 5.1 \times 10^{-6}$
- ◆ With (without) the subleading corrections, $K^-\eta' \gtrsim \bar{K}^0\eta' \approx 55$ (35), and
 $K^{*-}\eta \gtrsim \bar{K}^{*0}\eta \approx 24$ (20), in units of 10^{-6} , where the annihilation contribution of the $K\eta'$ modes is chosen to be 0.
- ◆ At $\gamma = 90^\circ$, we have $K^{*-}\pi^+, \bar{K}^{*0}\pi^0, \rho^-\eta, \rho^-\eta' = 9, 2, 6, 4$ ($\times 10^{-6}$).
- ◆ **This work:** $\bar{K}^0\rho^0, K^-\rho^+, \bar{K}^0\rho^-, K^-\rho^0 = 6, 7, 7, 2 \times 10^{-6}$.
 $\bar{K}^0\rho^0/\omega K^- \gtrsim 1$, sensitive to γ .

- ◆ **PQCD** by C.H.Chen: $\bar{K}^0\rho^0, K^-\rho^+, \bar{K}^0\rho^-, K^-\rho^0 = 2.5, 5.4, 3.0, 2.2 \times 10^{-6}$.
- ◆ **QCDF** (Du et al; Beneke, Neubert):
 $\bar{K}^0\rho^0 : K^-\rho^+ : \bar{K}^0\rho^- : K^-\rho^0 \approx (1/\sqrt{2})^2 : 1 : 1 : (1/\sqrt{2})^2$

Brief review on PP, VP modes

Table 3: (in units of 10^{-6}), $\gamma = 70^\circ$. The first error: parameter variations, the second one: the uncertainty due to weak annihilation. The “default” refers to $m_s = 100$ MeV and $F_2 = 0$.

| Mode | Default | $m_s = 80$ MeV | $F_2 = 0.1$ | pQCD | Experiment |
|--|------------------------------|-------------------------------|------------------------------|--|----------------|
| $B^- \rightarrow K^- \eta'$ | 42^{+16+27}_{-12-11} | 59^{+22+41}_{-16-17} | 56^{+19+31}_{-14-13} | 41 | 77.6 ± 4.6 |
| $\bar{B}^0 \rightarrow \bar{K}^0 \eta'$ | 41^{+15+26}_{-11-11} | 57^{+21+39}_{-15-16} | 56^{+18+30}_{-13-13} | | 60.6 ± 7.0 |
| $B^- \rightarrow K^- \eta$ | $1.7^{+2.0+1.3}_{-1.5-0.5}$ | $2.2^{+2.7+1.9}_{-2.0-0.8}$ | $1.4^{+1.8+1.1}_{-1.2-0.5}$ | 7.0 | 3.1 ± 0.7 |
| $\bar{B}^0 \rightarrow \bar{K}^0 \eta$ | $1.0^{+1.7+1.1}_{-1.2-0.4}$ | $1.4^{+2.4+1.6}_{-1.7-0.6}$ | $0.7^{+1.5+0.9}_{-0.9-0.4}$ | | < 4.6 |
| $B^- \rightarrow K^- \pi^0$ | $9.4^{+3.2+5.6}_{-2.9-2.4}$ | $12.6^{+4.3+8.2}_{-3.8-3.5}$ | $9.4^{+3.2+5.6}_{-2.9-2.4}$ | 11 | 12.8 ± 1.1 |
| $\bar{B}^0 \rightarrow \bar{K}^0 \pi^0$ | $5.9^{+2.7+4.5}_{-2.3-1.9}$ | $8.5^{+3.7+6.8}_{-3.1-2.8}$ | $5.9^{+2.7+4.5}_{-2.3-1.9}$ | | 11.2 ± 1.4 |
| $B^- \rightarrow K^{*-} \eta'$ | $3.5^{+4.4+4.7}_{-3.7-1.7}$ | $7.7^{+7.6+8.0}_{-6.7-3.2}$ | $2.7^{+3.5+3.9}_{-6.7-3.2}$ | $2.8^{+1.6}_{-1.0} \pm 0.0$ $3.2^{+1.9+0.6}_{-1.2-0.2}$ | < 35 |
| $\bar{B}^0 \rightarrow \bar{K}^{*0} \eta'$ | $2.5^{+3.8+4.3}_{-3.1-1.5}$ | $6.3^{+6.8+7.4}_{-5.8-2.9}$ | $1.2^{+2.7+3.2}_{-1.8-0.9}$ | | < 13 |
| $B^- \rightarrow K^{*-} \eta$ | $8.6^{+3.0+14.0}_{-2.6-4.4}$ | $13.8^{+4.8+19.8}_{-4.2-6.7}$ | $9.1^{+3.1+14.3}_{-2.7-4.6}$ | | 25.4 ± 5.3 |
| $\bar{B}^0 \rightarrow \bar{K}^{*0} \eta$ | $8.7^{+2.9+14.0}_{-2.6-4.5}$ | $13.9^{+4.6+19.5}_{-4.1-6.7}$ | $9.2^{+3.0+14.2}_{-2.7-4.7}$ | | 16.4 ± 3.0 |
| $B^- \rightarrow K^{*-} \pi^0$ | $3.2^{+1.2+4.0}_{-1.1-1.3}$ | $3.3^{+1.3+4.8}_{-1.2-1.5}$ | $3.2^{+1.2+4.0}_{-1.1-1.3}$ | | < 31 |
| $\bar{B}^0 \rightarrow \bar{K}^{*0} \pi^0$ | $0.7^{+0.6+2.4}_{-0.5-0.6}$ | $0.7^{+0.6+3.0}_{-0.5-0.6}$ | $0.7^{+0.6+2.4}_{-0.5-0.6}$ | | < 3.5 |

Global fit with QCD factorization

By D.S.Du, J.F.Sun, D.S.Yang, G.H.Zhu using the **CKMFITTER** package

Table 4: “No chiral”: neglecting the chirally enhanced hard spectator contributions and the annihilation topology. The branching ratios are in units of 10^{-6} .

| | | | | | |
|-----------|-----------------|-----------------|-------------------|----------------|----------------|
| Mode | $\pi^+\pi^-$ | $\pi^+\pi^0$ | $K^+\pi^-$ | $K^+\pi^0$ | $K^0\pi^+$ |
| Expt. | 4.77 ± 0.54 | 5.78 ± 0.95 | 18.5 ± 1.0 | 12.7 ± 1.2 | 18.1 ± 1.7 |
| Best fit | 4.82 | 5.35 | 19.0 | 11.4 | 20.1 |
| No chiral | 5.68 | 3.25 | 18.8 | 12.6 | 20.2 |
| Mode | π^0K^0 | $\eta\pi^+$ | $\rho^\pm\pi^\mp$ | $\rho^0\pi^+$ | $\eta\rho^+$ |
| Expt. | 10.2 ± 1.5 | < 5.2 | 25.4 ± 4.3 | 8.6 ± 2.0 | < 6.2 |
| Best fit | 8.2 | 2.8 | 26.7 | 8.9 | 4.6 |
| No chiral | 7.3 | 1.8 | 29.5 | 8.5 | 3.8 |
| Mode | ϕK^+ | ϕK^0 | $K^+\rho^-$ | ωK^0 | |
| Expt. | 8.9 ± 1.0 | 8.6 ± 1.3 | 13.1 ± 4.7 | 5.9 ± 1.9 | |
| Best fit | 8.9 | 8.4 | 12.1 | 6.3 | |
| No chiral | 7.1 | 6.7 | 5.1 | 1.2 | |

Best fit values: $|V_{ub}| = 3.57 \times 10^{-3}$, $\gamma = 79^\circ$, $F^{B\pi} = 0.24$??, $A_0^{B\rho} = 0.31$, $m_s = 85$ MeV, $\mu = 2.5$ GeV, $f_B = 220$ MeV, $\rho_A^{PP} = 0.5$, $\phi_A^{PP} = 10^\circ$, $\rho_A^{PV} = 1$, $\phi_A^{PV} = -30^\circ$. As to F^{BK} , there is no strong constraint ?? and the range $[0.24, 0.30]$ is acceptable from the current global analysis.

$$\begin{aligned}
 M(B^- \rightarrow \pi^- \pi^0) &= -i \frac{G_F}{2} f_\pi F_0^{B \rightarrow \pi} (m_\pi^2) (m_B^2 - m_\pi^2) \\
 &\times \left\{ V_{ub} V_{ud}^* (a_1 + a_2) - V_{tb} V_{td}^* \times \frac{3}{2} [a_9 + a_{10} - a_7 + a_8 r_\chi^\pi] \right\}.
 \end{aligned}$$