Holography, Entropy and Extra Dimensions

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Deog Ki Hong (Pusan Nat'l U.) and S. D. H. Hsu (U. of Oregon)

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1. Introduction

Our Universe may have extra dimensions:

1. Kaluza 1919 and Klein 1926 to unify all forces.

$$\hat{g}_{\hat{\mu}\hat{\nu}} = \begin{pmatrix} g_{\mu\nu} - \varphi\, A_\mu A_\nu & \varphi\, A_\mu \\ \varphi\, A_\nu & \varphi \end{pmatrix} \tag{1}$$

   After dimensional reduction, the general coordinate transformation induces the gauge symmetry at 3+1 D.

2. Superstring theory as TOE is consistent only in 10 D: Vacuum configuration is $M^4 \times$ CY (1985).

3. Brane worlds

- Horava-Witten 1996:

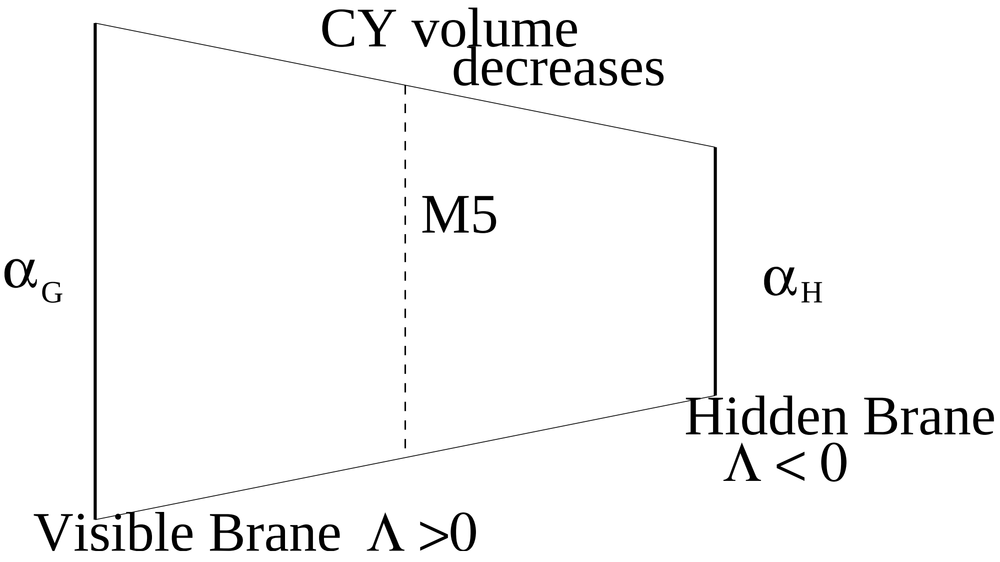

Figure 1: M theory on the orbifold $R^{10} \times S^1/Z_2$

- Low scale gravity as a solution to GHP: ADD 1998, RS 1999

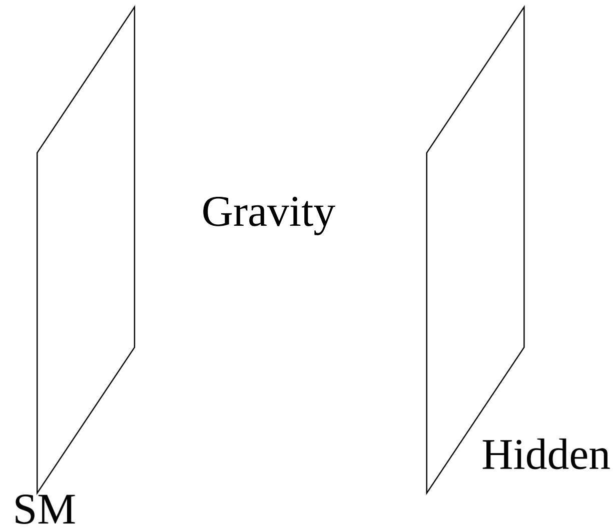

Figure 2: Only Gravity propagates in bulk.

  – In the brane world the Einstein action has the form

$$S = \int d^4x \; M_*^2 \mathcal{R} \; \left(d^{D-4}x \; M_*^{D-4} \right) \; \sqrt{-g} \; . \tag{2}$$

– The relation between the fundamental Planck scale M_* and the apparent one M_P is given by

$$V_w M_*^{D-2} = M_P^2 \qquad V_w \equiv \int d^{D-4}x \sqrt{-g_{(D-4)}}. \tag{3}$$

$\longrightarrow M_* = O(1)$ TeV solves the hierarchy problem.

Newton's law deviates at sub millimeter. Soon to be tested.

- We argue that Holography Bound requires $M_* > 10^{4-6}$ TeV.

2. What is Holography Bound?

- Bekenstein Bound: For a system of energy M in a radius R, its entropy is bounded from above

$$S < \frac{2\pi MR}{\hbar}. \tag{4}$$

- For weak gravity, R_s $(= 2GM) < R$,

$$S < \frac{2\pi MR}{\hbar} < \frac{A}{4G\hbar}, \qquad A = 4\pi R^2. \tag{5}$$

- Holography principle ('t Hooft 1993, Susskind 1995): The entropy in a spatial volume V enclosed by a surface area A cannot exceed $A/4$ in Planck units.

- States with $R_c < R_s$ are not accessible to outside observers.

- In D dimension, the Schwarzschild radius of a system with energy E

$$\Phi \sim \frac{E}{M_*^{D-2} R^{D-3}} \longrightarrow R_s \sim (M_*^{2-D} E)^{1/(D-3)} \tag{6}$$

- Maximum energy of a system in size R has a upper bound:

$$\begin{aligned} (M_*^{2-D} E_{\rm max})^{1/(D-3)} &< (M_*^{2-D} a^{-D} R^{D-1})^{1/(D-3)} < R \\ \rightarrow a &> M_*^{-1} (R\, M_*)^{2/D} . \end{aligned} \tag{7}$$

- Entropy of the system is given as ($k_B = 1$)

$$S = \ln 2^{(R/a)^{D-1}} = \left(\frac{R}{a}\right)^{D-1} \ln 2 < (RM_*)^{D-3+2/D} \ln 2 \tag{8}$$

- For $D = 4$, this bound gives $S < C\, A^{3/4}$.

- Violation of Bekenstein Bound

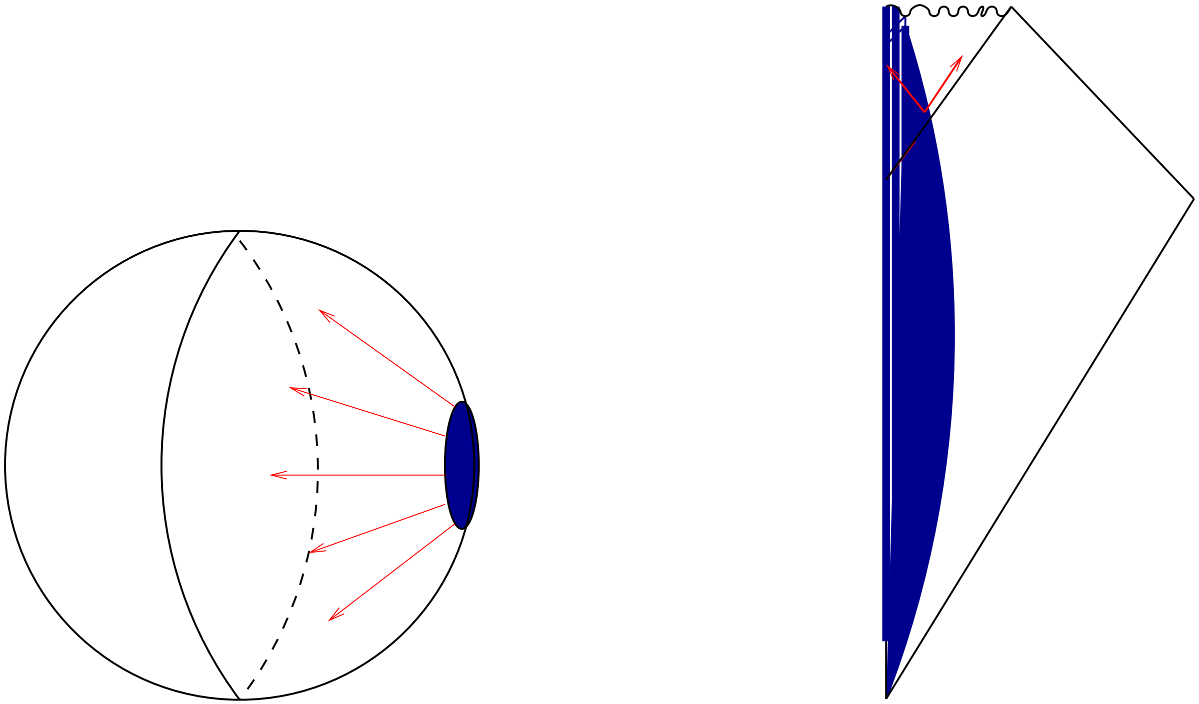

Figure 3: Closed universe and a collapsing star

- Covariant Entropy Bound (Bousso 1999)

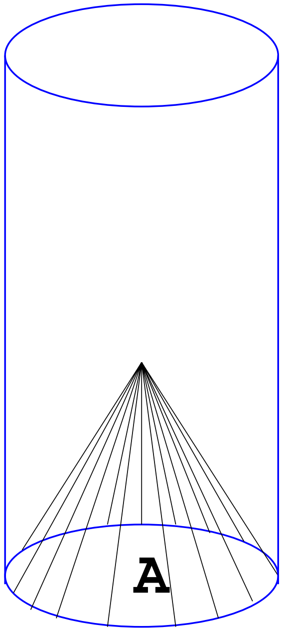

Figure 4: Entropy on Hyper Null Surface, L.

- All matter inside A (or V) will pass through the light-sheet L.

$$S_V < S_L < \frac{A}{4} \tag{9}$$

3. Black Hole Thermodynamics

- What is the number of states inside a black hole?
- Black holes radiate as black bodies with a certain temperature:

$$T_{\rm H} = \frac{1}{8\pi M}. \tag{10}$$

- Imagine an object with energy ΔE being dropped into a black hole with mass E. ($\Delta E \ll 1 \ll E$ in Planck units.)
- The absorption cross section is then

$$\sigma = \pi R^2, \qquad R \simeq 2E. \tag{11}$$

- From Hawking's result, the emission probability is

$$W \simeq \pi R^2 \rho_{\Delta E} e^{-\beta_{\rm H}\Delta E}, \tag{12}$$

where $\rho_{\Delta E}$ is the density of states for a particle with energy ΔE.

- Suppose the processes are described by a Hamiltonian acting in Hilbert space.

$$\begin{aligned} \sigma &= \left|\langle E+\Delta E\left|T\right|E,\Delta E\rangle\right|^2 \rho(E+\Delta E) \\ W &= \left|\langle E,\Delta E\left|T\right|E+\Delta E\rangle\right|^2 \rho(E)\rho_{\Delta E}. \end{aligned} \tag{13}$$

- By PCT invariance, the matrix elements are same.

$$\frac{\sigma}{W} = \frac{e^{\beta_{\rm H}\Delta E}}{\rho_{\Delta E}} = \frac{\rho(E+\Delta E)}{\rho(E)\rho_{\Delta E}} \tag{14}$$

- Therefore, the black hole entropy becomes

$$S = \ln\rho(E) = 4\pi E^2 + S_0 \quad \text{or} \quad S = \frac{A}{4} + S_0. \tag{15}$$

4. Holography Bounds on Brane World Scenario

1. The Holographic Bound (HB) is violated during the big bang.
   - Consider a spacelike region V of extent r_h on the 3-brane. If it saturates HB,

$$T^3 r_h^3 \sim M_*^2 r_h^2 \ , \quad \text{or} \quad r_h \sim T^{-1}\left(\frac{M_*}{T}\right)^2 \tag{16}$$

     (N.B. The entropy in the extra dimension factors out, assuming the thickness of the brane, $d \sim M_*^{-1}$.)

   - Cosmological horizon size, up to a degeneracy factor and a numerical constant

$$d_H \sim \begin{cases} \frac{M_P}{T^2} & \text{if } T > T_d \simeq 10 \text{ eV} \\ \left(\frac{M_P}{T_d^2}\right)\left(\frac{T_d}{T}\right)^{3/2} & \text{otherwise} \end{cases} \tag{17}$$

- The ratio between the horizon distance and the saturation distance becomes

$$\frac{d_H}{r_h} \sim \frac{T}{M_*}\frac{M_P}{M_*} \sim \frac{T}{T_*}, \quad T_* \simeq 10^{-2}\,\mathrm{eV}. \tag{18}$$

- For $T > T_*$ the causal horizon contains more degrees of freedom than allowed by HB of the fundamental theory.
- HB bound and the successful BBN requires $M_* > 10^4$ TeV.

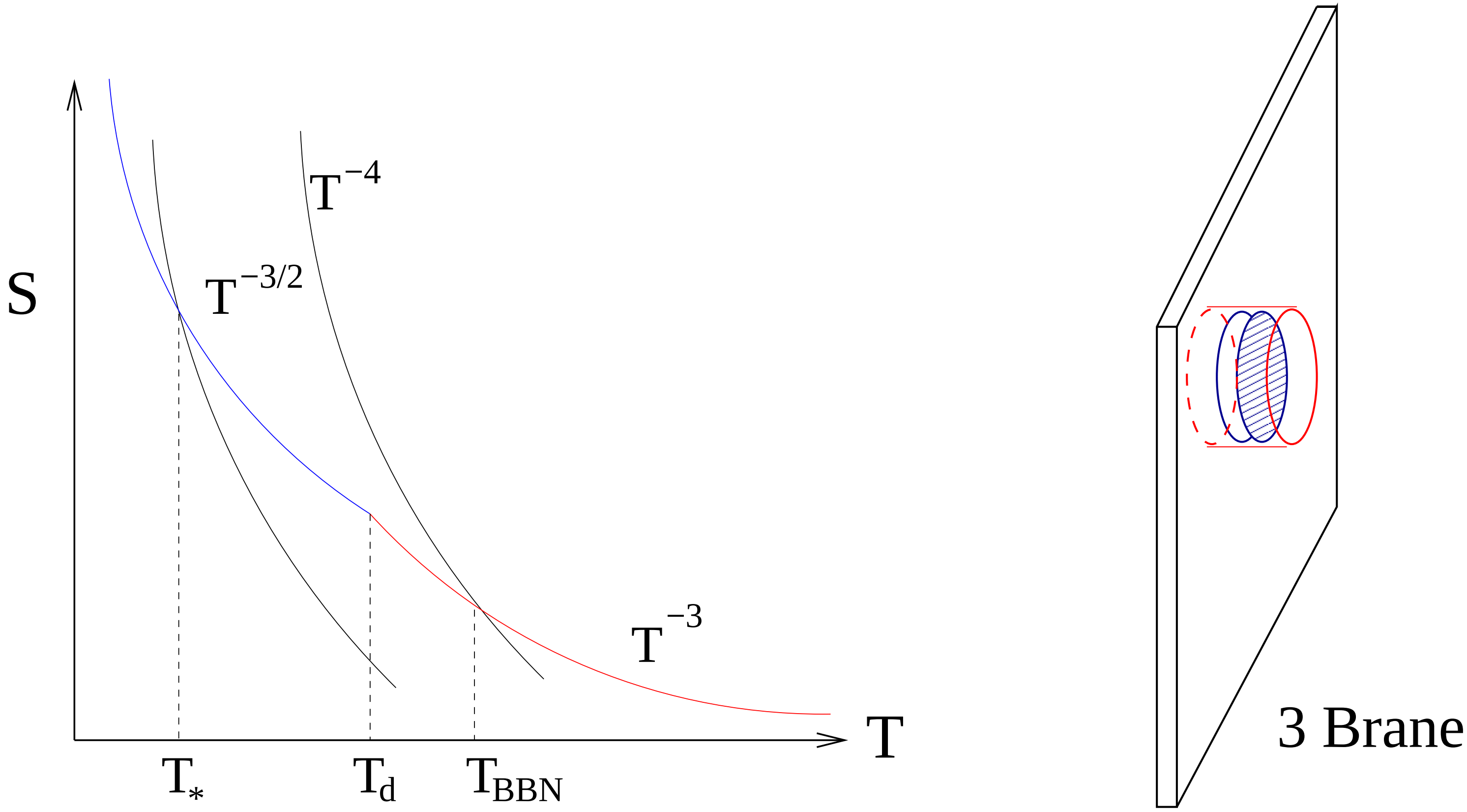

Figure 5: Entropy in the horizon

2. The black hole entropy bound is violated.

- The shape of black hole in brane world is of pancake

  (Giddings et. al 2000; Hawking et. al 2000; Casadio 2003)

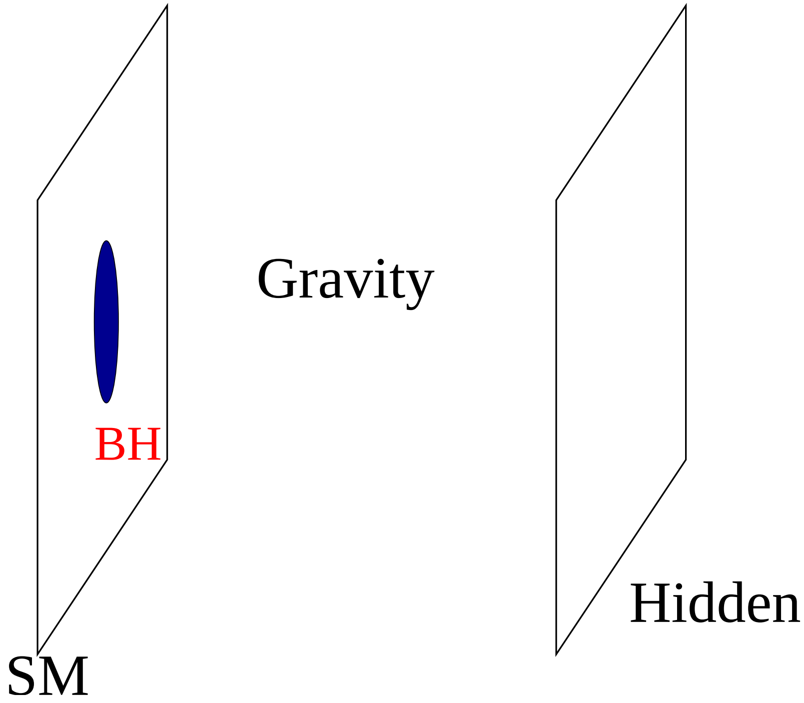

Figure 6: Black holes in a brane

- A black hole of energy density ρ has radius

$$R \sim \frac{1}{M_P^2} \rho R^3 \sim \left(\frac{V_w M_*^{D-2}}{\rho} \right)^{1/2} . \tag{19}$$

- To determine the size of the horizon off the brane, consider the potential energy per unit test mass given by

$$\Phi(u) \sim M_*^{2-D} \int^R \frac{d^3 r \, \rho}{(\sqrt{r^2 + u^2})^{D-3}} , \tag{20}$$

where ρ is an energy density of dimension 4.

- For $D > 6$ the integration is dominated by $u \sim r \sim 0$ and the horizon will be

$$u_h \sim M_*^{-1} \left(\rho M_*^{-4} \right)^{1/(D-6)} . \tag{21}$$

- For $D = 5, 6,$

$$\Phi(u << R) \sim \begin{cases} \frac{\rho}{M_*^4} \text{ (up to a log)}, & \text{if } D = 6 \\ \frac{\rho^{1/2}}{M_*^2} \frac{M_P}{M_*}, & \text{if } D = 5. \end{cases} \tag{22}$$

- Astrophysical black holes typically have $\rho \ll M_*^4$ and therefore they are of pancake-shape for $D > 5$.
- For $D = 5$, it has extension to the fifth dimension only if $\rho > (10\,\text{keV})^4$.
- In this process the entropy of the collapsed neutron star is roughly 10^{57}. The radius of the black hole is a few kilometers, so that its area (10^{11} cm^2) in M_* units is only 10^{45}.
- Supernovae of $M > 8M_\odot$ stars which lead to black hole formation violate the black hole entropy bound in brane world scenarios.
- $M_* > 10^6$ TeV in order to avoid a conflict between the usual thermodynamic description of typical supernova collapse and the

black hole entropy bound.

- Hawking temperature of pancake black holes

$$T = \frac{dM}{dS} \sim \frac{1}{R_s}(M_P/M_*)^2. \tag{23}$$

- For a solar mass black hole, if $M_* \sim$ TeV,

$$T \sim 10^{13}\ \mathrm{GeV} \tag{24}$$

- According to observation, astrophysical b.h. $T < O(1)$ keV:

$$M_* > 10^{-5}\, M_P \sim 10^{11}\ \mathrm{TeV}. \tag{25}$$

5. Discussions

- If the holography bound is correct as in the classical gravity (Wald et. al 2000), extra dimensional models are severely constrained.
- To reproduce the successful BBN and supernovae explosion,

$$M_* > 10^{4-6}\ \mathrm{TeV}.$$

- There might be highly non-local degrees of freedom in brane worlds.
- Alternatively, the brane world scenario poses a challenge on the holography principle.
- Maybe, the volume of the extra dimension, V_w, should be order one in the units of M_*^{D-4}, which then leaves the hierarchy problem unsolved and the possibility of tests in colliders closed.