Covariant Light-Front Approach for s-wave and p-wave Mesons

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Introduction

- Mesonic weak transition form factors and decay constants are two of the most important ingredients in the study of meson weak decays.
- Among many models, the light-front quark model (Terent'ev 76; Terent'ev,Berestetsky 76,77; Chung,Coester,Polyzou 88) is the only relativistic quark model. It has many advantages, for example,
  - applicable, in particular, at the maximum recoil point $q^2 = 0$ where the final-state meson could be highly relativistic.
  - hadron spin can be correctly constructed using the so-called Melosh rotation.
- It has been employed to obtain decay constants and weak form factors (Jaus 90,91,96; Ji,Chung,Cotanch 92, Cheng,Cheung,Hwang 97).
- There exist however, some ambiguities in extracting the physical quantities.
  - the usual recipe of taking only the plus component of the current matrix elements will miss the zero-mode contributions and renders the matrix element non-covariant. Well known examples are f_V, $F_0(q^2)$ in $P \to P$ pseudoscalar transition (Jaus 99).
- A covariant model has been constructed in (Cheng,Cheung,Hwang,Zhang 98) for heavy mesons within the framework of heavy quark effective theory.
- Without appealing to the heavy quark limit, a covariant approach of the light-front model for the usual pseudoscalar and vector mesons was put forward by Jaus (1999).

Introduction

- We wish to extend the covariant analysis of the light-front model in (Jaus99) to even-parity, p-wave mesons (light and heavy).
- The interest in even-parity mesons is revived by recent discoveries:
  - Two narrow resonances: $D^*_{s0}(2317)$ (BABAR 03):$^3P_0$, $D_{s1}(2460)$ (CLEO 03): $P_1^{1/2}$, and two broad resonances: $D^*_0(2308)$ and $D_1(2427)$ (Belle 03).
  - $B \to D^{**}\pi$ and $B \to D^{**}_s \overline{D}$ have been recently observed.
  - B factories observe many three-body modes. Resonance states.
- So far, the Isgur-Scora-Grinstein-Wise (ISGW) quark model (89) is the only model that provides a systematical estimate of the transition of a ground-state s-wave meson to a low-lying p-wave meson. It uses the non-relativistic constituent quark picture.
- Under heavy quark symmetry (HQS) the number of the independent form factors is reduced and they are related to some universal Isgur-Wise (IW) functions. In this work, we follow (Cheng,Cheung,Hwang,Zhang 98) to evaluate the form factors and decay constants in a covariant light-front formulism within the framework of heavy quark effective theory.
- It is found that the resultant decay constants do agree with those obtained from the covariant light-front approach and then extended to the heavy quark limit. The relevant IW functions, namely, ξ, $\tau_{1/2}$ and $\tau_{3/2}$ are obtained. One can then study some properties of these IW functions, including the slopes and sum rules (Bjorken 90;Bjorken,Dunietz,Taron 92; Uraltsev 01).

Decay constant

- Decay constants are defined in

$$\langle 0|A_\mu|P(P')\rangle = if_P P'_\mu, \qquad \langle 0|V_\mu|S(P')\rangle = f_S P'_\mu,$$
$$\langle 0|V_\mu|V(P',\varepsilon')\rangle = M'_V f_V \varepsilon'_\mu, \quad \langle 0|A_\mu|\,{}^{3(1)}A(P',\varepsilon') = M'_{{}^3A({}^1A)} f_{{}^3A({}^1A)} \varepsilon'_\mu,$$

where we denote the ${}^{2S+1}L_J = {}^1S_0$, ${}^3P_0$, ${}^3S_1$, ${}^3P_1$, ${}^1P_1$ and ${}^3P_2$ states of $q'_1\bar{q}_2$ mesons by P, S, V, ${}^3A$, ${}^1A$ and T, respectively.

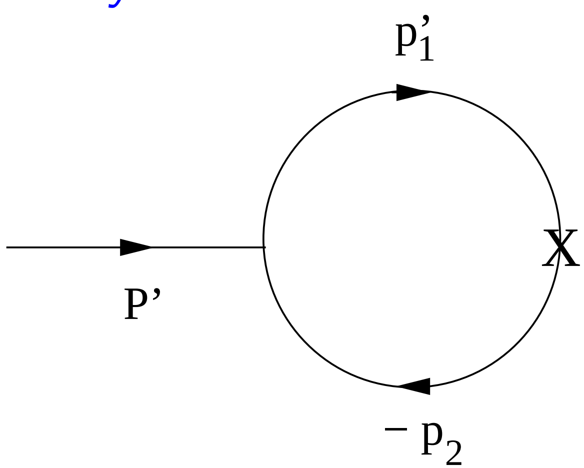
- There are some non-trivial constraint on p-wave meson decay constants:
  - $f_S = 0$ ($\because$ EOM) and $f_{{}^1A} = 0$ ($\because$ Charge conjugation) in the SU(3) limit (Suzuki 93).
  - $f_V = f_P$, $f_{A^{1/2}} = f_S$, $f_{A^{3/2}} = 0$ in the HQ limit.
    In the HQ limit, the HQ spin degree of freedom decouples and it is more convenient to use the $L_J^j = P_2^{3/2}$, $P_1^{3/2}$, $P_1^{1/2}$ and $P_0^{1/2}$ basis, where j is the total angular momentum of the light antiquark. $P^{3/2} = {}^3P_2$ and $P_0^{1/2} = {}^3P_0$, while (Isgur,Wise91)

$$\left|P_1^{3/2}\right\rangle = \sqrt{\frac{2}{3}}\,\left|{}^1P_1\right\rangle + \frac{1}{\sqrt{3}}\,\left|{}^3P_1\right\rangle, \qquad \left|P_1^{1/2}\right\rangle = \frac{1}{\sqrt{3}}\,\left|{}^1P_1\right\rangle - \sqrt{\frac{2}{3}}\,\left|{}^3P_1\right\rangle.$$

    One obtains the above rules on f_M by using some current algebra arguments (Le Yaouanc *et. al.* 96; Vseli,Dunietz 96).

- We follow (Jaus 99) by using a covariant approach of the light-front model for decay constant and FF calculations.

Decay constant



$M\ (^{2S+1}L_J)$	$i\Gamma'_M$
$P\ (^1S_0)$	$H'_P\gamma_5$
$V\ (^3S_1)$	$iH'_V[\gamma_\mu - \frac{1}{W'_V}(p'_1 - p_2)_\mu]$
$S\ (^3P_0)$	$-iH'_S$
$^3A\ (^3P_1)$	$-iH'_{^3A}[\gamma_\mu + \frac{1}{W'_{^3A}}(p'_1 - p_2)_\mu]\gamma_5$
$^1A\ (^1P_1)$	$-iH'_{^1A}[\frac{1}{W'_{^1A}}(p'_1 - p_2)_\mu]\gamma_5$
$T\ (^3P_2)$	$i\frac{1}{2}H'_T[\gamma_\mu - \frac{1}{W'_V}(p'_1 - p_2)_\mu](p'_1 - p_2)_\nu$

- **Step 1**: write down the Feynman amplitude (like any usual covariant calculation).
- **Step 2**: pass to LF formalism by performing the contour integration (Chang,Ma 69). Close the upper complex $p_1'^-(=p_1'^0 - p_1'^3)$ plane. Pick up a pole at $p_2^2 = m_2^2$. Contains spurious $\tilde{\omega} = (1,0,0,1)$ contribution.
- **Step 3**: Use the well-studied vertex function (in conventional LF approach)$H \to h$.
- **Step 4**: Inclusion of zero modes (Chang,Root,Yan 73;Yan 73). The above contour integration is not complete. Consider for example:

$$\int dp^- \frac{i}{p^2 - m^2 + i\epsilon} = \int dp^- \int_0^\infty d\alpha e^{i\alpha(p^2 - m^2 + i\epsilon)}$$
$$= 2\pi \int_0^\infty d\alpha e^{-i(m^2 + p_\perp^2)} \left(\frac{\delta(\alpha)}{p^+} + \frac{\delta(p^+)}{\alpha}\right).$$

The inclusion of zero mode contribution cancels the spurious term.

Decay constant

- We finally have (M': mass of meson; M_0: kinetic mass of meson, function of x_1, $p'_\perp$)

$$f_P = \frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{h'_P}{x_1 x_2 (M'^2 - M_0'^2)} 4(m'_1 x_2 + m_2 x_1).$$

- The decay constant of a scalar meson can be obtained in a similar manner.

$$f_S = \frac{N_c}{16\pi^3}\int dx_2 d^2p'_\perp \frac{h'_S}{x_1 x_2 (M'^2 - M_0'^2)} 4(m'_1 x_2 - m_2 x_1).$$

  – For $m'_1 = m_2$, the meson wave function is symmetric and hence $f_S = 0$.

- By similar procedure, one obtains

$$\begin{aligned} f_V &= \frac{N_c}{4\pi^3}\int \frac{dx_2 d^2p'_\perp\, h'_V}{x_1 x_2 (M'^2 - M_0'^2)}\left[x_1 M_0'^2 - m'_1(m'_1 - m_2) - p_\perp'^2 + \frac{m'_1 + m_2}{W'_V}\, p_\perp'^2\right], \\ f_{^3A} &= -\frac{N_c}{4\pi^3}\int \frac{dx_2 d^2p'_\perp\, h'_{^3A}}{x_1 x_2 (M'^2 - M_0'^2)}\left[x_1 M_0'^2 - m'_1(m'_1 + m_2) - p_\perp'^2 - \frac{m'_1 - m_2}{W'_{^3A}}\, p_\perp'^2\right], \\ f_{^1A} &= \frac{N_c}{4\pi^3}\int \frac{dx_2 d^2p'_\perp\, h'_{^1A}}{x_1 x_2 (M'^2 - M_0'^2)}\left(\frac{m'_1 - m_2}{W'_{^1A}}\, p_\perp'^2\right). \end{aligned}$$

  It is clear that $f_{^1A} = 0$ for $m'_1 = m_2$.

- The SU(N)-flavor constraints on f_S and $f_{^1A}$ are thus satisfied.

Decay Constant: numerical results

$^{2S+1}L_J$	$\beta_{u\bar d}$	$\beta_{s\bar u}$	$\beta_{c\bar u}$	$\beta_{c\bar s}$	$\beta_{b\bar u}$
$^1S_0$	0.3327	0.3788	0.4616	0.4923	0.5328
$^3P_0$	β_π	β_K	β_D	$\kappa\beta_D$	β_B
$^3S_1$	0.2668	0.27075	0.3739	0.3788	β_B
$^3P_1$	0.2972	0.3	β_{D^*}	$\beta_{D_s^*}$	β_B
$^1P_1$	β_{a_1}	$\beta_{K(^3P_1)}$	β_{D^*}	$\beta_{D_s^*}$	β_B

$^{2S+1}L_J$	$f_{u\bar d}$	$f_{s\bar u}$	$f_{c\bar u}$	$f_{c\bar s}$	$f_{b\bar u}$
$^1S_0$	(131)	(160)	(200)	(230)	(180)
$^3P_0$	0	39	139	53	159
$^3S_1$	(216)	(210)	(220)	(230)	214
$^3P_1$	(−203)	−191	−163	−152	−192
$^1P_1$	0	18	62	48	103
$P_1^{1/2}$	–	–	168	151	216
$P_1^{3/2}$	–	–	−43	−48	−26

- Vertex functions h are given by conventional LF approach.
- β: a parameter in the Gaussian-type wave function. $h \propto \exp[-(p_\perp^2 + p_z^2)/\beta^2]$.
- The decay constants in parentheses are used to determine β.
- β_V is smaller than previously obtained in literature (due to the new f_V formula).
- $f_S < f_P$, due to $(m_1 x_2 \mp m_2 x_1)$.

- $m(D_{s0}^*)(2317) \simeq m(D_0^*)(2308)$. Binding energy $\propto \beta$, $\beta_{D_{s0}^*} \sim \beta_{D_0^*} \frac{(m_{D_{s0}^*} - m_c - m_s)}{(m_{D_0^*} - m_c - m_u)} \sim 0.6\,\beta_{D_0^*}$.
  Belle $B \to \overline{D} D_{s0}^*$ rate hints at $f_{D_{s0}^*} \sim 60$ MeV.
- For $c\bar u$ and $b\bar u$ systems, we have $|f_{A^{3/2}}| \ll f_{A^{1/2}} \simeq f_S$, in accordance with the expectation from HQS. We do have $|f_{A^{3/2}}| \ll f_{A^{1/2}}$ in the $c\bar s$ system, but f_S is much smaller than $f_{A^{1/2}}$ due to the above mentioned subtlety.

Form Factors

- Form factors can be studied in a similar way (Step 1– Step 4).
- In Jaus 99, the calculation of the zero mode contribution is obtained in a frame where the momentum transfer $q^+ = q^0 + q^3 = 0$. Because of this $(q^+ = 0)$ condition, form factors are known only for spacelike momentum transfer $q^2 = -q_\perp^2 \leq 0$. One needs to analytically continue them to the timelike region Jaus96, where the physical decay processes is relevant. Recently, it has been shown that within a specific model, form factors obtained directly from the timelike region (with $q^+ > 0$) are identical to the ones obtained by the analytic continuation from the spacelike region (Bakker,Choi,Ji 03).
- To proceed we find that the form-factor momentum dependence in the spacelike region can be well parameterized and reproduced in the two-parameter form:

$$F(q^2) = \frac{F(0)}{1 - a(q^2/m_B^2) + b(q^2/m_B^2)^2},$$

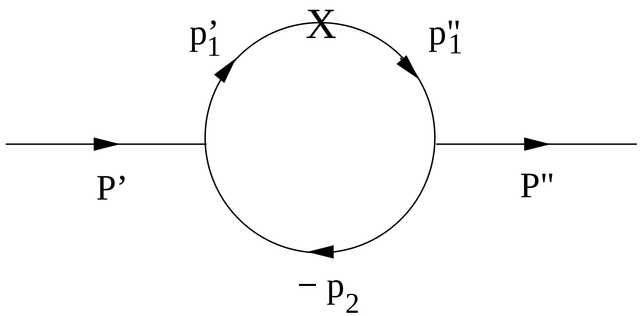
for $B \to M$ transitions. The parameters $F(0)$, a and b are first determined from the spacelike region. We then employ this parametrization to determine the physical form factors at $q^2 \geq 0$.

Form factors

- Form factors for $P \to P, V$ transitions are given by

$$\begin{aligned}\langle P(P'')|V_\mu|P(P')\rangle &= P_\mu f_+(q^2) + q_\mu f_-(q^2),\\ \langle S(P'')|A_\mu|P(P')\rangle &= i\Big[u_+(q^2)P_\mu + u_-(q^2)q_\mu\Big],\end{aligned}$$

where $P = P' + P''$, $q = P' - P''$ and the convention $\epsilon_{0123} = 1$ is adopted.



$$\mathcal{B}^{PP}_\mu = -i^3 \frac{N_c}{(2\pi)^4} \int d^4p'_1 \frac{H'_P H''_P}{N'_1 N''_1 N_2} S^{PP}_{V\mu},$$

$$\mathcal{B}^{PS}_\mu = -i^3 \frac{N_c}{(2\pi)^4} \int d^4p'_1 \frac{H'_P H''_S}{N'_1 N''_1 N_2} S^{PS}_{A\mu},$$

where ($N = p^2 - m^2 + i\epsilon$)

$$\begin{aligned}S^{PP}_{V\mu} &= \mathrm{Tr}[\gamma_5(\not p''_1 + m''_1)\gamma_\mu(\not p'_1 + m'_1)\gamma_5(-\not p_2 + m_2)]\\ S^{PS}_{A\mu} &= \mathrm{Tr}[(-i)(\not p''_1 + m''_1)\gamma_\mu\gamma_5(\not p'_1 + m'_1)\gamma_5(-\not p_2 + m_2)],\\ &= -i\, S^{PP}_{V\mu}(m''_1 \to -m''_1).\end{aligned}$$

- We obtain

$$u_\pm = -f_\pm(m''_1 \to -m''_1, h''_P \to h''_S).$$

Form factors

- For $P \to V, A, T$ transitions, we have

$$\begin{aligned}
\langle V(P'',\varepsilon'')|V_\mu|P(P')\rangle &= \epsilon_{\mu\nu\alpha\beta}\,\varepsilon''^{*\nu}P^\alpha q^\beta\, g(q^2),\\
\langle V(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -i\{\varepsilon''^*_\mu f(q^2)+\varepsilon^{*\prime\prime}\cdot P[P_\mu a_+(q^2)+q_\mu a_-(q^2)]\},\\
\langle A^{1/2}(P'',\varepsilon'')|V_\mu|P(P')\rangle &= i\left\{\ell_{1/2}(q^2)\varepsilon''^*_\mu+\varepsilon''^*\cdot P[P_\mu c_+^{1/2}(q^2)+q_\mu c_-^{1/2}(q^2)]\right\},\\
\langle A^{1/2}(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -q_{1/2}(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta,\\
\langle A^{3/2}(P'',\varepsilon'')|V_\mu|P(P')\rangle &= i\left\{\ell_{3/2}(q^2)\varepsilon''^*_\mu+\varepsilon''^*\cdot P[P_\mu c_+^{3/2}(q^2)+q_\mu c_-^{3/2}(q^2)]\right\},\\
\langle A^{3/2}(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -q_{3/2}(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta,\\
\langle T(P'',\varepsilon'')|V_\mu|P(P')\rangle &= h(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu\lambda}P_\lambda P^\alpha q^\beta,\\
\langle T(P'',\varepsilon'')|A_\mu|P(P')\rangle &= -i\Big\{k(q^2)\varepsilon''^*_{\mu\nu}P^\nu+\varepsilon''^*_{\alpha\beta}P^\alpha P^\beta[P_\mu b_+(q^2)+q_\mu b_-(q^2)]\Big\}.
\end{aligned}$$

- Similarly, we have

$$\begin{aligned}
\ell^{^3A,^1A}(q^2) &= f(q^2)\ \text{with}\ (m_1''\to -m_1'',\ h_V''\to h''_{^3A,^1A},\ W_V''\to W''_{^3A,^1A}),\\
q^{^3A,^1A}(q^2) &= g(q^2)\ \text{with}\ (m_1''\to -m_1'',\ h_V''\to h''_{^3A,^1A},\ W_V''\to W''_{^3A,^1A}),\\
c_+^{^3A,^1A}(q^2) &= a_+(q^2)\ \text{with}\ (m_1''\to -m_1'',\ h_V''\to h''_{^3A,^1A},\ W_V''\to W''_{^3A,^1A}),\\
c_-^{^3A,^1A}(q^2) &= a_-(q^2)\ \text{with}\ (m_1''\to -m_1'',\ h_V''\to h''_{^3A,^1A},\ W_V''\to W''_{^3A,^1A}),
\end{aligned}$$

while only the $1/W''$ terms for $P \to {}^1A$ form factors are kept.

Form factors

$B \to \pi, \rho, a_0(1450), a_1(1260), b_1(1235), a_2(1320)$,
$B \to K, K^*, K_0^*(1430), K_{^1P_1}, K_{^3P_1}, K_2^*(1430)$ transitions.

F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
$F_1^{B\pi}$	0.27	2.04	1.56	0.68	$F_0^{B\pi}$	0.27	0.69	0.68	0.04
$V^{B\rho}$	0.26	0.91	1.70	0.98	$A_0^{B\rho}$	0.29	0.92	1.63	0.96
$A_1^{B\rho}$	0.22	0.46	0.81	0.13	$A_2^{B\rho}$	0.20	0.61	1.51	0.80
$F_1^{Ba_0}$	0.28	0.71	1.41	0.47	$F_0^{Ba_0}$	0.28	0.36	0.42	0.00
A^{Ba_1}	0.37	1.08	1.40	0.50	$V_0^{Ba_1}$	0.16	0.41	1.59	0.96
$V_1^{Ba_1}$	0.23	0.26	0.28	0.10	$V_2^{Ba_1}$	0.27	0.52	1.05	0.38
A^{Bb_1}	0.16	0.40	1.79	1.32	$V_0^{Bb_1}$	0.35	1.00	1.35	0.42
$V_1^{Bb_1}$	0.11	0.20	0.89	0.23	$V_2^{Bb_1}$	-0.03	-0.01	3.24	9.04
h	0.007	0.015	2.08	1.92	k	0.034	0.012	-2.17	2.15
b_+	-0.005	-0.011	1.85	1.48	b_-	0.0014	0.0008	-0.52	1.22
F_1^{BK}	0.35	2.07	1.47	0.56	F_0^{BK}	0.35	0.73	0.65	0.01
V^{BK^*}	0.31	1.03	1.64	0.92	$A_0^{BK^*}$	0.33	1.02	1.57	0.87
$A_1^{BK^*}$	0.27	0.55	0.81	0.13	$A_2^{BK^*}$	0.25	0.75	1.50	0.77
$F_1^{BK_0^*}$	0.31	0.79	1.32	0.38	$F_0^{BK_0^*}$	0.31	0.35	0.23	0.04
$A^{BK_{^3P_1}}$	0.41	1.02	1.34	0.45	$V_0^{DK_{^3P_1}}$	0.17	0.37	1.50	0.91
$V_1^{BK_{^3P_1}}$	0.22	0.24	0.16	0.13	$V_2^{BK_{^3P_1}}$	0.26	0.41	0.86	0.35
$A^{BK_{^1P_1}}$	0.18	0.43	1.74	1.23	$V_0^{BK_{^1P_1}}$	0.38	0.99	1.34	0.44
$V_1^{BK_{^1P_1}}$	0.11	0.18	0.79	0.22	$V_2^{BK_{^1P_1}}$	-0.06	-0.06	2.79	5.31
h	0.008	0.017	2.03	1.83	k	0.036	0.013	-2.21	2.16
b_+	-0.006	-0.013	1.86	1.49	b_-	0.0027	0.0034	0.78	0.73

Form factors

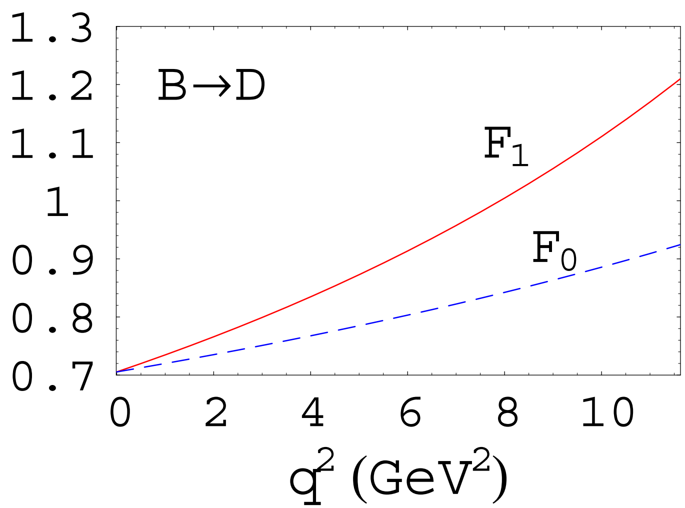
Table 1: $B \to D$, D^*, D_0^*, $D_1^{1/2}$, $D_1^{3/2}$, D_2^* transitions. For the purpose of comparing with heavy quark symmetry, the form factors $u_\pm, c_\pm, \ell, q$ are also shown.

F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
F_1^{BD}	0.71	1.21	1.12	0.29	F_0^{BD}	0.71	0.92	0.56	-0.01
V^{BD^*}	0.74	1.22	1.15	0.34	$A_0^{BD^*}$	0.72	1.22	1.17	0.23
$A_1^{BD^*}$	0.67	0.86	0.57	0.01	$A_2^{BD^*}$	0.62	0.92	1.03	0.42
$F_1^{BD_0^*}$	0.31	0.44	1.03	0.19	$F_0^{BD_0^*}$	0.31	0.24	-0.69	0.51
$A^{BD_1^{1/2}}$	-0.32	-0.41	0.74	0.09	$V_0^{BD_1^{1/2}}$	0.11	0.17	1.23	-0.34
$V_1^{BD_1^{1/2}}$	-0.06	-0.04	-1.51	1.17	$V_2^{BD_1^{1/2}}$	-0.33	-0.41	0.71	0.11
$A^{BD_1^{3/2}}$	0.64	0.91	1.07	0.27	$V_0^{BD_1^{3/2}}$	0.49	0.68	0.97	0.05
$V_1^{BD_1^{3/2}}$	0.19	0.17	-0.41	0.35	$V_2^{BD_1^{3/2}}$	-0.31	-0.61	2.37	2.37
u_+	-0.31	-0.44	1.03	0.19	u_-	0.48	0.69	1.02	0.15
$\ell_{1/2}$	0.46	0.30	-1.51	1.17	$q_{1/2}$	0.042	0.053	0.74	0.09
$c_+^{1/2}$	-0.043	-0.054	0.71	0.11	$c_-^{1/2}$	0.045	0.057	0.72	0.11
$\ell_{3/2}$	-1.48	-1.29	-0.41	0.35	$q_{3/2}$	-0.084	-0.117	1.07	0.27
$c_+^{3/2}$	-0.040	-0.079	2.37	2.37	$c_-^{3/2}$	-0.036	-0.039	0.35	0.25
h	0.016	0.025	1.53	0.93	k	0.42	0.47	0.43	0.16
b_+	-0.013	-0.020	1.46	0.79	b_-	0.014	0.022	1.45	0.79

- In the heavy quark limit, heavy quark symmetry requires that the form factors
  - u_-, $\ell_{1/2}$, $q_{1/2}$, $c_-^{1/2}$, h, k, $b_- > 0$, $\quad u_+$, $\ell_{3/2}$, $q_{3/2}$, $c_+^{1/2}$, $c_+^{3/2}$, $c_-^{3/2}$, $b_+ < 0$.
  - $c_+^{1/2} + c_-^{1/2} = 0$, $b_+ + b_- = 0$.

  Our results are in accordance with HQS.

$B \to D$, D_0^* transition form factors



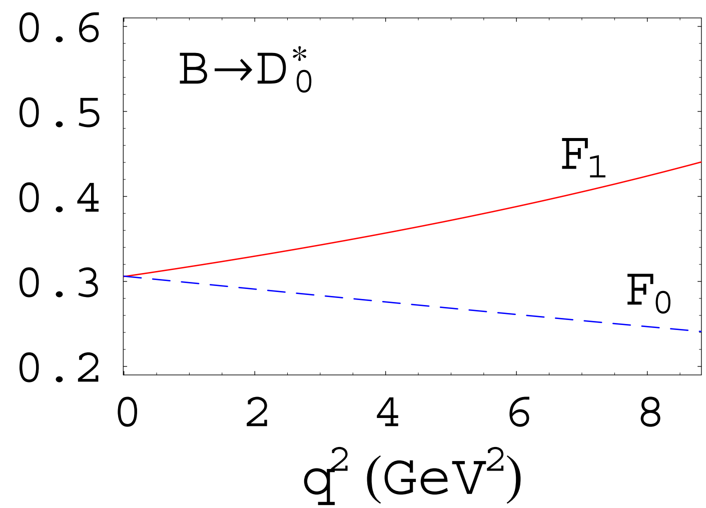
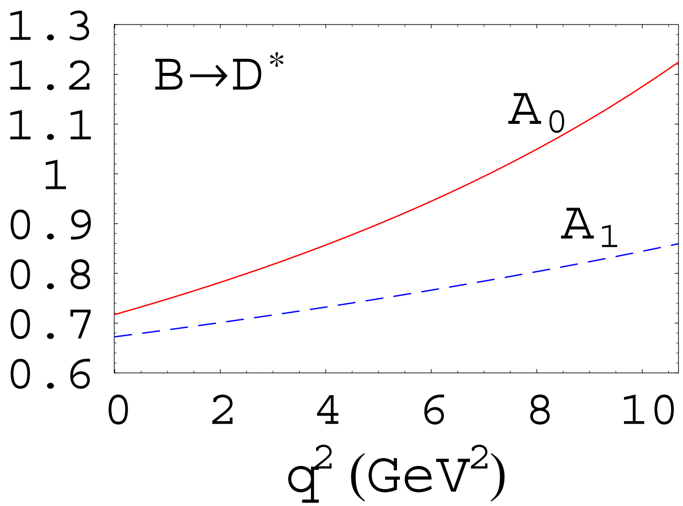


Figure 1: Form factors $F_1(q^2)$ and $F_0(q^2)$ for $B \to D$ and $B \to D_0^*$ transitions.

$$\langle P(P'')|V_\mu|P(P')\rangle = \left(P_\mu - \frac{M'^2 - M''^2}{q^2}\, q_\mu\right) F_1^{PP}(q^2) + \frac{M'^2 - M''^2}{q^2}\, q_\mu\, F_0^{PP}(q^2),$$

$$\langle S(P'')|A_\mu|P(P')\rangle = -i\left[\left(P_\mu - \frac{M'^2 - M''^2}{q^2}\, q_\mu\right) F_1^{PS}(q^2) + \frac{M'^2 - M''^2}{q^2}\, q_\mu\, F_0^{PS}(q^2)\right],$$

$B \to D^*$ transition form factors



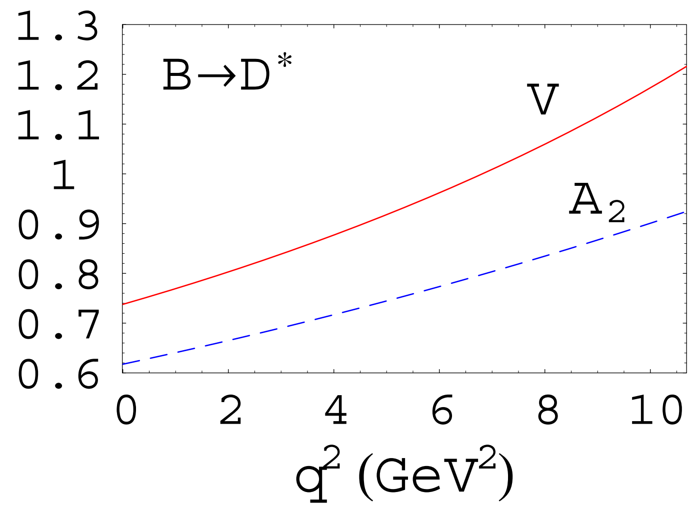


Figure 2: Form factors $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$ and $A_2(q^2)$ for $B \to D^*$ transitions.

$$\begin{aligned}
\langle V(P'',\varepsilon'')|V_\mu|P(P')\rangle &= -\frac{1}{M'+M''}\,\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu}P^\alpha q^\beta V^{PV}(q^2),\\
\langle V(P'',\varepsilon'')|A_\mu|P(P')\rangle &= i\Big\{(M'+M'')\varepsilon''^*_\mu A_1^{PV}(q^2) - \frac{\varepsilon''^*\cdot P}{M'+M''}\,P_\mu A_2^{PV}(q^2)\\
&\quad -2M''\,\frac{\varepsilon''^*\cdot P}{q^2}\,q_\mu[A_3^{PV}(q^2)-A_0^{PV}(q^2)]\Big\}.
\end{aligned}$$

$B \to D_1^{1/2}, D_1^{3/2}$ *transition form factors*

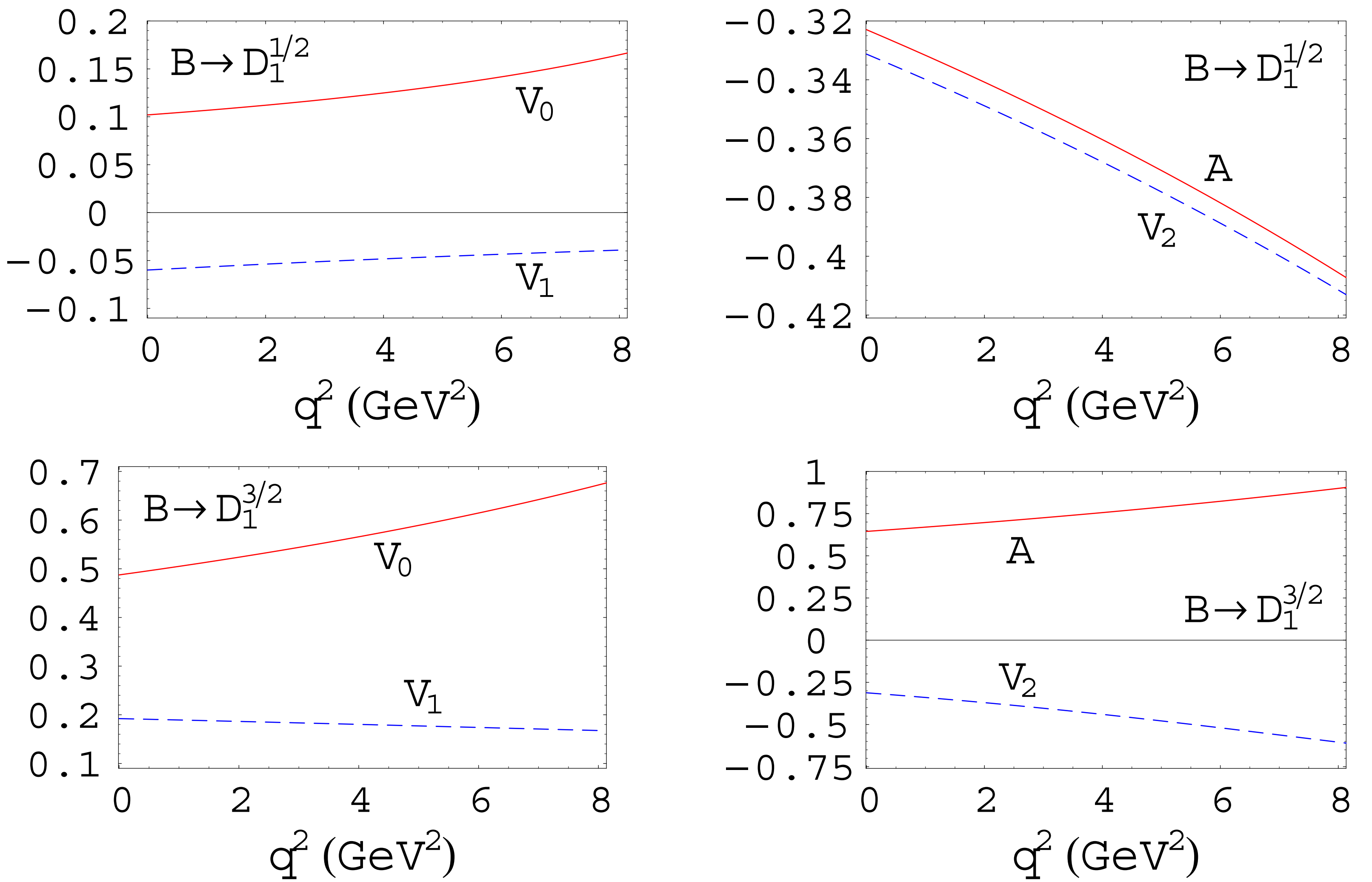


Figure 3: Form factors $A(q^2)$, $V_0(q^2)$, $V_1(q^2)$ and $V_2(q^2)$ for $B \to D_1^{1/2,3/2}$ transitions.

$B \to T$ transition form factors

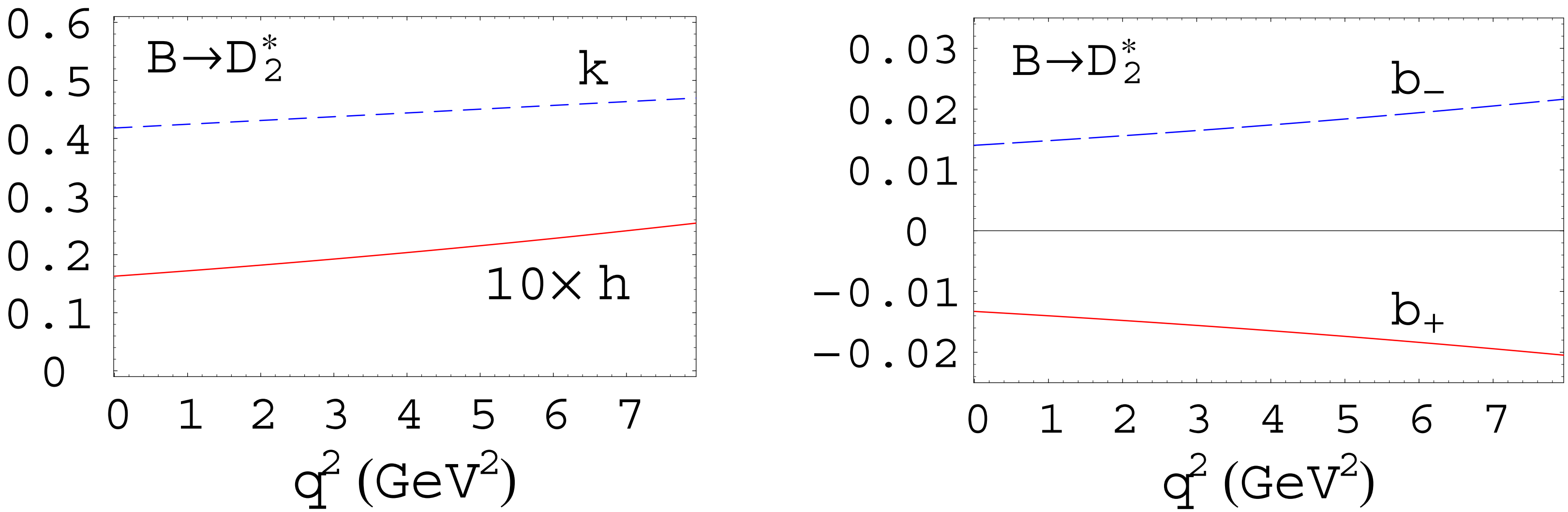


Figure 4: Form factors $k(q^2)$, $h(q^2)$, $b_+(q^2)$ and $b_-(q^2)$ for $B \to D_2^*$ transitions.

$$\langle T(P'', \varepsilon'')|V_\mu|P(P')\rangle = h(q^2)\epsilon_{\mu\nu\alpha\beta}\varepsilon''^{*\nu\lambda}P_\lambda P^\alpha q^\beta,$$

$$\langle T(P'', \varepsilon'')|A_\mu|P(P')\rangle = -i\Big\{k(q^2)\varepsilon''^{*}_{\mu\nu}P^\nu + \varepsilon''^{*}_{\alpha\beta}P^\alpha P^\beta[P_\mu b_+(q^2) + q_\mu b_-(q^2)]\Big\}.$$

Form factors

Table 2: Form factors of $B \to D^{**}$ transitions calculated in the ISGW2 model.

F	$F(0)$	$F(q^2_{\rm max})$	a	b	F	$F(0)$	$F(q^2_{\rm max})$	a	b
$F_1^{BD_0^*}$	0.18	0.24	0.28	0.25	$F_0^{BD_0^*}$	0.18	-0.008	–	–
$A^{BD_1^{1/2}}$	-0.44	-0.57	0.87	0.24	$V_0^{BD_1^{1/2}}$	0.18	0.23	0.89	0.25
$V_1^{BD_1^{1/2}}$	-0.07	0.002	–	–	$V_2^{BD_1^{1/2}}$	-0.49	-0.64	0.87	0.24
$A^{BD_1^{3/2}}$	0.44	0.51	0.46	0.065	$V_0^{BD_1^{3/2}}$	0.43	0.51	0.54	0.074
$V_1^{BD_1^{3/2}}$	0.15	0.12	-0.60	1.15	$V_2^{BD_1^{3/2}}$	-0.33	-0.51	1.45	0.83
u_+	-0.18	-0.24	0.88	0.25	u_-	0.46	0.62	0.87	0.25
$\ell_{1/2}$	0.54	-0.016	–	–	$q_{1/2}$	0.057	0.074	0.87	0.24
$c_+^{1/2}$	-0.064	-0.083	0.87	0.24	$c_-^{1/2}$	0.068	0.088	0.87	0.24
$\ell_{3/2}$	-1.15	-0.90	-0.60	1.15	$q_{3/2}$	-0.057	-0.066	0.46	0.065
$c_+^{3/2}$	-0.043	-0.066	1.45	0.83	$c_-^{3/2}$	-0.018	-0.013	0.23	5.38
h	0.011	0.014	0.86	0.23	k	0.60	0.68	0.40	0.68
b_+	-0.010	-0.013	0.86	0.23	b_-	0.010	0.013	0.86	0.23

(i) half of the fourteen form factors predicted by the covariant LF model and the ISGW2 model, namely, u_-, $q_{1/2}$, $\ell_{3/2}$, $c_+^{3/2}$, h, b_+ and b_- are consistent,

(ii) the values of the form factors u_+, $c_+^{1/2}$, $c_-^{1/2}$, $q_{3/2}$ and h differ by about a factor of 2,

(iii) $\ell_{1/2}$ decreases smoothly in the LF model as q^2 increases, while it decreases sharply with q^2 in the ISGW2 model, and

(iv) $|c_-^{3/2}(q^2)|$ has an opposite q^2 behavior in LF and ISGW2 models: it increases with q^2 in the former model, whereas it decreases with q^2 in the latter.

Form factors

- Chernyak (01) has estimated the $B \to a_0(1450)$ transition form factor based on the light-cone sum rules and obtained $F_{1,0}^{Ba_0(1450)}(0) = 0.46$ which is much larger than our result of 0.16 ; the latter is similar to the $B \to \pi$ form factor.
- For $B \to a_1(1260)$ form factors, there are two existing calculations: one in a quark-meson model (CQM) (Deandrea *et. al.* 99) and the other based on the QCD sum rule (QSR) (Aliev,Savci 99).
  - The results are quite different, for example, $V_0^{Ba_1}(0)$ computed in the quark-meson model, 1.20 , is larger than the sum-rule prediction, -0.23 ± 0.05 , by a factor of five. If $a_1(1260)$ behaves as the scalar partner of the ρ meson, it is expected that $V_0^{Ba_1}$ is similar to $A_0^{B\rho}(= 0.27)$. At $q^2 = 0$, relativistic effect become important.
  - In hadronic $B \to a_1 P$ decays, the relevant form factors are $V_0^{Ba_1}$ and F_1^{BP} under the factorization approach. In principle, the measurement of $\overline{B}^0 \to a_1^+ \pi^-$ will enable us to test the form factor $V_0^{Ba_1}$.

Table 3: $B \to a_1(1260)$ transition form factors at $q^2 = 0$ in various models.

Model	$A^{Ba_1}(0)$	$V_0^{Ba_1}(0)$	$V_1^{Ba_1}(0)$	$V_2^{Ba_1}(0)$
This Work	0.37	0.16	0.23	0.27
ISGW2	0.34	1.01	0.33	-0.08
CQM	0.14	1.20	0.81	0.56
QSR	-0.67 ± 0.10	-0.23 ± 0.05	-0.42 ± 0.05	-0.53 ± 0.05

Heavy Quark limit

- It can be shown that the decay constants satisfy the HQ relations:

$$f_V = f_P,\ f_{A^{1/2}} = f_S,\ f_{A^{3/2}} = 0$$

- Following (Cheng,Cheung,Hwang,Zhang98), one can study the Heavy Quark limit of FFs.
  The Isgur-Wise functions are defined in

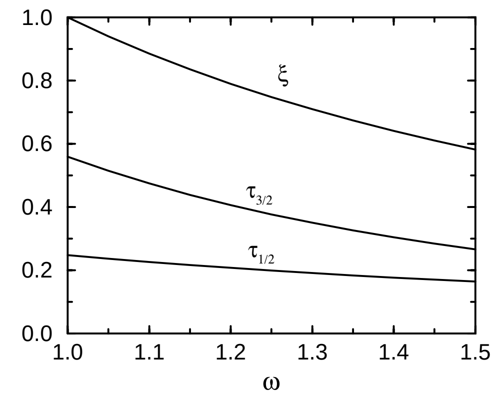
$$\begin{aligned}
\langle D_0^*(v')|A_\mu|B(v)\rangle &= i\,2\tau_{1/2}(\omega)(v-v')_\mu,\\
\langle D_1^{1/2}(v',\varepsilon)|V_\mu|B(v)\rangle &= -i\,2\tau_{1/2}(\omega)\Big[(1-\omega)\varepsilon^*_\mu + (\varepsilon^*\cdot v)v'_\mu\Big],\\
\langle D_1^{1/2}(v',\varepsilon)|A_\mu|B(v)\rangle &= -2\tau_{1/2}(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,\\
\langle D_1^{3/2}(v',\varepsilon)|V_\mu|B(v)\rangle &= \frac{1}{\sqrt{2}}\,\tau_{3/2}(\omega)i\Big\{(1-\omega^2)\varepsilon^*_\mu - (\varepsilon^*\cdot v)[3v_\mu + (2-\omega)v'_\mu]\Big\},\\
\langle D_1^{3/2}(v',\varepsilon)|A_\mu|B(v)\rangle &= \frac{1}{\sqrt{2}}\,\tau_{3/2}(\omega)(1+\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu}v'^\alpha v^\beta,\\
\langle D_2^*(v',\varepsilon)|V_\mu|B(v)\rangle &= \sqrt{3}\,\tau_{3/2}(\omega)\epsilon_{\mu\nu\alpha\beta}\varepsilon^{*\nu\gamma}v_\gamma v'^\alpha v^\beta,\\
\langle D_2^*(v',\varepsilon)|A_\mu|B(v)\rangle &= -i\sqrt{3}\,\tau_{3/2}(\omega)\Big\{(1+\omega)\varepsilon^*_{\mu\nu}v^\nu - \varepsilon^*_{\alpha\beta}v^\alpha v^\beta v'_\mu\Big\},
\end{aligned}$$

where $p_B = m_B v$, $p_{D^{**}} = m_{D^{**}}v'$ and $\omega = v\cdot v'$.

Heavy Quark Limit

We obtain

$$\xi(\omega) = 1 - 1.22(\omega-1) + 0.85(\omega-1)^2,$$
$$\tau_{1/2}(\omega) = 0.25 - 0.23(\omega-1) + 0.13(\omega-1)^2,$$
$$\tau_{3/2}(\omega) = 0.56 - 0.91(\omega-1) + 0.72(\omega-1)^2,$$
$$\tau(\omega) = \tau(1)[1 - \rho^2(\omega-1) + \ldots]$$



$\tau_{1/2}(1)$	$\rho^2_{1/2}$	$\tau_{3/2}(1)$	$\rho^2_{3/2}$	Ref.
0.25	0.92	0.56	1.63	This work
0.06	0.73	0.52	1.45	(Morenas ... 97,Cea ... 88))
0.09	1.1	0.28	0.9	(Deandrea ... 98)
0.13	0.57	0.43	1.39	(Morenas...97 ,Veseli,Dunietz 96)
0.22	0.83	0.54	1.50	(Morenas ...97,Godfrey,Isgur 85)
0.34	1.08	0.59	1.76	(Morenas ...97,ISGW 89)
0.35 ± 0.08	2.5 ± 1.0	–	–	(Colangelo ... 98)
0.41 ± 0.04	1.30 ± 0.23	0.66 ± 0.02	1.93 ± 0.16	(Wambach 95)
–	–	0.74 ± 0.15	0.90 ± 0.05	(Huang,Dai 99)

$$\sum_n |\tau^{(n)}_{3/2}(1)|^2 - \sum_n |\tau^{(n)}_{1/2}(1)|^2 = \frac{1}{4}, \quad \text{Uraltsev Sum Rule}$$

$$\rho^2 = \frac{1}{4} + \sum_n |\tau^{(n)}_{1/2}(1)|^2 + 2\sum_n |\tau^{(n)}_{3/2}(1)|^2, \quad \text{Bjorken Sum Rule}$$

Conclusion

In this work we have studied the decay constants and form factors of the ground-state s-wave and low-lying p-wave mesons within a covariant light-front approach. Our main results are in the follows:

- The main ingredients of the covariant light-front model, namely, the vertex functions, are worked out for both s-wave and p-wave mesons.
- The decay constant of light scalar mesons is suppressed relative to that of the pseudoscalar mesons and this suppression becomes less effective for heavy scalar resonances. We argue that the smallness of the decay constant of the newly observed $D^*_{s0}(2317)$ implied by a recent Belle's measurement on $B \to \overline{D} D^*_{s0}$ might be intimately related to the mass degeneracy of $D^*_{s0}(2317)$ and $D^*_0(2308)$.
- In the limit of SU(N)-flavor symmetry, the decay constants of the scalar meson and the $^1P_1$ axial-vector meson vanish, as it should be.
- The analytic expressions for $P \to S, A$ transitions can be obtained from that of $P \to P, V$ transitions by some simple replacements.
- The momentum dependence of the physical form factors is determined by first fitting the calculated form factors in the spacelike region to a 2-parameter function in q^2 and then analytically continuing them to the timelike region.
- Numerical results of the form factors for $B \to \pi,\ \rho,\ a_0(1450)$, $a_1(1260),\ b_1(1235),\ a_2(1320)$, $B \to K,\ K^*,\ K^{**}$ and $B \to D,\ D^*,\ D^{**}$ transitions are presented in detail. q^2 dependence of FFs are given.

- Comparison with the ISGW2 model based on the nonrelativistic constituent quark picture is made for $B \to D^{**}$ transition form factors. Some noticeable differences are pointed out, for example, the $B \to D_0^*$ form factor $F_0(0) = -u_+(0)$ is equal to 0.31 in this work and 0.18 in the ISGW2 model. This can be tested in the measurements of $B \to D_0^* \pi$ decays.
- The heavy quark limit behavior of decay constants and form factors is examined and it is found that the requirement of heavy quark symmetry is satisfied.
- Decay constants and form factors are also evaluated independently in a covariant light-front formulism within the framework of heavy quark effective theory. The resultant decay constants agree with those obtained from the covariant light-front model and then extended to the heavy quark limit. The universal Isgur-Wise functions $\xi(\omega), \tau_{1/2}(\omega)$ and $\tau_{3/2}(\omega)$ are obtained. In the infinite quark mass limit, all the form factors are related to the Isgur-Wise functions. At zero recoil $\omega = 1$, $\tau_{3/2}(1) = 0.25$, $\tau_{1/2}(1) = 0.56$ and $\xi(1) = 1$. The sum rules for the Isgur-Wise functions derived by Bjorken and by Uraltsev can be checked.