Weak Phases from Topological-Amplitude Parametrization

- Introduction
  - Theoretically Clean Cases
  - Cases Which Need Theoretical Inputs
- Amplitude Parametrization
  - $B \to K\pi$ case
  - $B \to \pi\pi$ case
- Conclusions

I. Introduction

One of the major missions in B physics is to determine the weak phases in the CKM matrix for CP violation

Unitarity Test:Search for deviaton from SM

- Using direct measurements of angles check:

$$\alpha + \gamma + \beta = \pi$$

- Check if B decays are consistent with the range of angles from the CKM fit?

To determine the weak phase angles, one has two different methods:

- Theoretically Clean Cases

- Cases Which Need Theoretical Inputs

Theoretically Clean Cases

Tree dominated process $\rightarrow$ Penguins Free or Small Penguins contributions

Problem: Some of the cases have very small branching ratio $\rightarrow$ difficult to measure.

Cases Which Need Theoretical Inputs

Have both Tree(T) and Penguins(P) contribution. The interference between T and P causes the uncertainties up to $\sim 30\%$

Problem: One needs theoretical inputs to constrain the uncertainties. For example, impose Isosping Symmetry, SU(3), U-Spin....

II. Amplitude Parametrization

We impose counting rules for the various amplitudes in terms of power of Wolfenstein parameter $\lambda \sim 0.22$.

- Assign an explicit power of λ to each topology according to PQCD.

- Drop the topologies with higher power of λ until the number of free parameters are equal to the number of measurements.
  - To $O(\lambda^2)$, the error is $\sim O(\lambda^3) \sim 1\%$
  - To $O(\lambda)$, the error is $\sim O(\lambda^2) \sim 5\%$

- Solve the simulanious equations to get weak phase and amplitudes.

- Check the solved amplitudes see if they satisfy the power counting rules from PQCD.

The Branching ratio:

$$B(B \to M_1 M_2) = \frac{\tau_B}{16\pi m_B} |A(B \to M_1 M_2)|^2$$

where

$$m_B = 5.28\text{Gev}, \quad \tau_{B^\pm} = 1.674 \times 10^{-12}\text{s},$$

$$\tau_{B^0} = 1.542 \times 10^{-12}\text{s}$$

The effective Hamiltonian:

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_{qs}^* V_{qb} \left[C_1(\mu) O_1^{(q)}(\mu) + C_2(\mu) O_2^{(q)}(\mu) \right.$$

$$\left. + \sum_{i=3}^{10} C_i(\mu) O_i(\mu) \right]$$

Where V_{qq} is the elements of CKM matrix.

The operators

$$
\begin{aligned}
O_1^{(q)} &= (\bar{s}_i q_j)_{V-A}(\bar{q}_j b_i)_{V-A} \,, \\
O_2^{(q)} &= (\bar{s}_i q_i)_{V-A}(\bar{q}_j b_j)_{V-A} \,, \\
O_3 &= (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V-A} \,, \\
O_4 &= (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V-A} \,, \\
O_5 &= (\bar{s}_i b_i)_{V-A}\sum_q (\bar{q}_j q_j)_{V+A} \,, \\
O_6 &= (\bar{s}_i b_j)_{V-A}\sum_q (\bar{q}_j q_i)_{V+A} \,, \\
O_7 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q (\bar{q}_j q_j)_{V+A} \,, \\
O_8 &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q (\bar{q}_j q_i)_{V+A} \,, \\
O_9 &= \frac{3}{2}(\bar{s}_i b_i)_{V-A}\sum_q e_q (\bar{q}_j q_j)_{V-A} \,, \\
O_{10} &= \frac{3}{2}(\bar{s}_i b_j)_{V-A}\sum_q e_q (\bar{q}_j q_i)_{V-A} \,,
\end{aligned}
$$

The i, j is the color indices.

The caracteristic scale is

$$\mu \sim \sqrt{m_b \bar{\Lambda}} \sim 1.5\text{Gev}$$

and $\bar{\Lambda} = m_B - m_b$.

The Wilson coefficients are

$$\begin{aligned}
C_1 &= -0.510\ , \qquad C_2 = 1.268\ , \\
C_3 &= 2.7\times 10^{-2}\ , \quad C_4 = -5.0\times 10^{-2}\ , \\
C_5 &= 1.3\times 10^{-2}\ , \quad C_6 = -7.4\times 10^{-2}\ , \\
C_7 &= 2.6\times 10^{-4}\ , \quad C_8 = 6.6\times 10^{-4}\ , \\
C_9 &= -1.0\times 10^{-2}\ , C_{10} = 4.0\times 10^{-3}\ .
\end{aligned}$$

The Wolfenstein parametrization for the CKM matrix is

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$$= \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}\ ,$$

where $\lambda = 0.2196 \pm 0.0023$, $A = 0.819 \pm 0.035$, and $R_b \equiv \sqrt{\rho^2+\eta^2} = 0.41 \pm 0.07$.

The phases ϕ_1 and ϕ_3 are defined via $V_{td} = |V_{td}|\exp(-i\phi_1)$ and $V_{ub} = |V_{ub}|\exp(-i\phi_3)$, respectively.

Case study $B \to K\pi$

The most general parametrization of the $B \to K\pi$ decay amplitudes are given by

$$
\begin{aligned}
A(B^+ \to K^0\pi^+) &= P\left(1 - \frac{P^c_{ew}}{P} + \frac{T^a}{P}e^{i\phi_3}\right) , \\
A(B^0_d \to K^+\pi^-) &= -P\left(1 - \frac{P^a_{ew}}{P} + \frac{T}{P}e^{i\phi_3}\right) , \\
\sqrt{2}A(B^+ \to K^+\pi^0) &= -P\left[1 + \frac{P_{ew}}{P}\right. \\
&\quad \left. + \left(\frac{T}{P} + \frac{C}{P} + \frac{T^a}{P}\right) e^{i\phi_3}\right] , \\
\sqrt{2}A(B^0_d \to K^0\pi^0) &= P\left(1 - \frac{P_{ew}}{P} - \frac{P^c_{ew}}{P}\right. \\
&\quad \left. - \frac{P^a_{ew}}{P} - \frac{C}{P}e^{i\phi_3}\right) ,
\end{aligned}
$$

where $P = P_{QCD} + e_u P^c_{ew} + e_u P^a_{ew}$

We have 14 unknows(7 amplitudes(13 unknowns)+ϕ_3) with 9 experimental inputs.

From PQCD, one have

$$
\frac{M^{nf}}{F^e} \sim \left[\ln \frac{m_B}{\Lambda_{\mathrm{QCD}}}\right]^{-1} \sim \lambda ,
$$

$$\frac{F^a_{(V-A)}}{F^e} \sim \frac{\Lambda_{\rm QCD}}{m_B} \sim \lambda^2 ,$$
$$\frac{F^a_{(V+A)}}{F^e} \sim \frac{2m_0}{m_B} \sim \lambda^0 ,$$

Assign the power counting rule to the Wilson coefficients

$$\begin{aligned} O(1) &: a_1 , \\ O(\lambda) &: a_2 , \\ O(\lambda^2) &: C_4 , C_6 , \\ O(\lambda^3) &: C_3 , C_5 , C_9 , \\ O(\lambda^4) &: C_{10} , \\ O(\lambda^5) &: C_7 , C_8 , \end{aligned}$$

with $a_1 = C_2 + C_1/N_c$ and $a_2 = C_1 + C_2/N_c$.

Combine all above, one have

$$\frac{T}{P} \sim \frac{V_{us}V^*_{ub}}{V_{ts}V^*_{tb}} \frac{a_1}{C_{4,6}} \sim \lambda ,$$
$$\frac{P_{ew}}{P} \sim \frac{C_9}{C_{4,6}} \sim \lambda ,$$
$$\frac{C}{T} \sim \frac{a_2}{a_1} \sim \lambda ,$$

$$
\begin{aligned}
\frac{T^a}{T} &\sim \frac{F^a_{(V-A)}}{F^e} \sim \frac{M^{nf}}{F^e}\frac{C_1}{a_1 N_c} \sim \lambda^2 \, , \\
\frac{P^c_{ew}}{P} &\sim \frac{C_{8,10}+C_{7,9}/N_c}{C_{4,6}} \sim \frac{M^{nf}}{F^e}\frac{C_9}{C_{4,6}N_c} \sim \lambda^3 \, , \\
\frac{P^a_{ew}}{P} &\sim \frac{F^a_{(V+A)}}{F_e}\frac{C_{8,10}+C_{7,9}/N_c}{C_{4,6}} \sim \frac{M^{nf}}{F^e}\frac{C_9}{C_{4,6}N_c} \sim \lambda^3 \, .
\end{aligned}
$$

The power of $P^c_{ew}/P \sim \lambda^3$, is different from the parametrization from Gronau and Lendon(λ^2). Drop $O(\lambda^3)$ term(T^a/P, P^c_{ew}/P, P^a_{ew}/P),and $A_{CP}(B^\pm \to K^0\pi^\pm)$,
$\Rightarrow$ 8 equations with 8 unknows.
$\Rightarrow$ Error is $O(\lambda^3) \sim 1\%$

Since the time-dependent asymmetry in the $B^0_d \to K_S\pi^0$ decay still has big uncertainty, drop futher $O(\lambda^2)$ term. We have

$$
\begin{aligned}
A(B^+ \to K^0\pi^+) &= P \, , \\
A(B^0_d \to K^+\pi^-) &= -P\left(1+\frac{|T|}{P}e^{i\phi_3}e^{i\delta_T}\right) \, , \\
\sqrt{2}A(B^+ \to K^+\pi^0) &= -P\left(1+\frac{|P_{ew}|}{P}e^{i\delta_{ew}}\right. \\
&\qquad \left. +\frac{|T|}{P}e^{i\phi_3}e^{i\delta_T}\right) \, , \\
\sqrt{2}A(B^0_d \to K^0\pi^0) &= P\left(1-\frac{|P_{ew}|}{P}e^{i\delta_{ew}}\right) \, ,
\end{aligned}
$$

and 6 experimental data

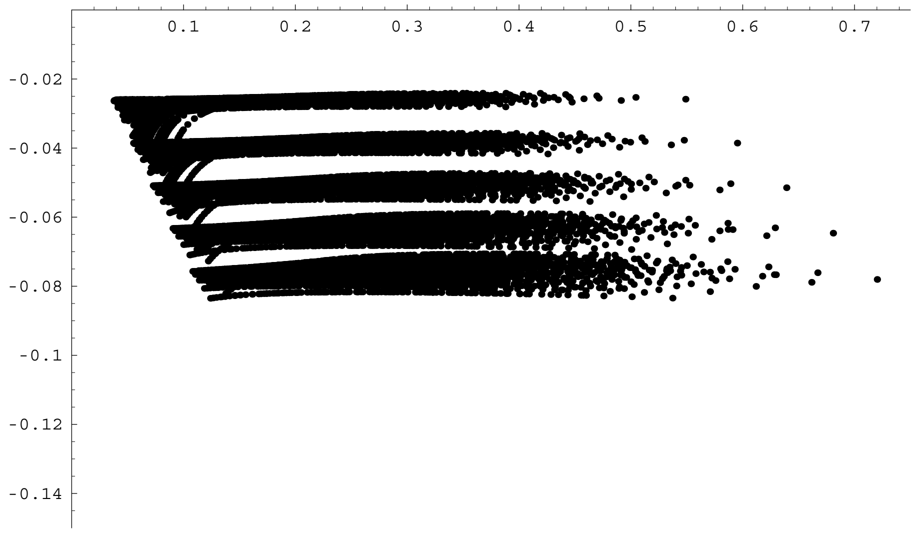
$$\begin{aligned}
&\mathrm{Br}(B^{\pm} \to K^0\pi^{\pm}) = (20.6 \pm 1.4) \times 10^{-6} \, , \\
&\mathrm{Br}(B_d^0 \to K^{\pm}\pi^{\mp}) = (18.2 \pm 0.8) \times 10^{-6} \, , \\
&\mathrm{Br}(B^{\pm} \to K^{\pm}\pi^0) = (12.8 \pm 1.1 \times 10^{-6} \, , \\
&\mathrm{Br}(B_d^0 \to K^0\pi^0) = (11.5 \pm 1.7) \times 10^{-6} \, , \\
&\mathcal{A}(B_d^0 \to K^{\pm}\pi^{\pm}) = -(10.2 \pm 5.0)\% \, , \\
&\mathcal{A}(B^{\pm} \to K^{\pm}\pi^0) = -(9.0 \pm 9.0)\% \, .
\end{aligned}$$

Solve the simutanious equations, We have center values

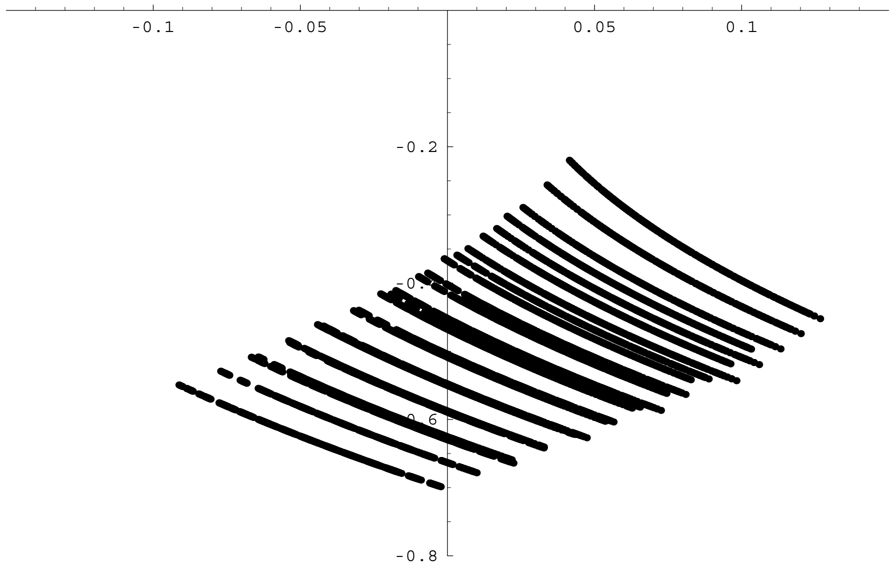
$$\begin{aligned}
&\frac{|T|}{P} = 0.23 \, , \quad \delta_T = -13^o \, , \;\; 0.06 < \frac{|T|}{P} < 0.72 \\
&\frac{|P_{ew}|}{P} = 0.50 \, , \quad \delta_{ew} = -88^o \, , \;\; 0.22 < \frac{P_{ew}}{P} < 0.70 \\
&\phi_3 = 102^o \, , \quad 26^o < \phi_3 < 151^o \, ,
\end{aligned}$$

Agree with the results from PQCD, QCDF and Rosner and Gronau(hep-ph/0307095)

T/P



P_{ew}/P



$B \to \pi\pi$ Case

The general parametrizations are:

$$
\begin{aligned}
\sqrt{2}A(B^+ \to \pi^+\pi^0) &= -T\left[1 + \frac{C}{T} + \left(\frac{P_{ew}}{T} \right.\right. \\
&\quad \left.\left. + \frac{P^c_{ew}}{T} + \frac{P^a_{ew}}{T}\right) e^{i\phi_2}\right] , \\
A(B^0_d \to \pi^+\pi^-) &= -T\left(1 + \frac{T^a}{T} + \frac{P}{T}e^{i\phi_2}\right) , \\
\sqrt{2}A(B^0_d \to \pi^0\pi^0) &= T\left[\left(\frac{P}{T} - \frac{P_{ew}}{T} \right.\right. \\
&\quad \left.\left. - \frac{P^c_{ew}}{T} - \frac{P^c_{ew}}{T}\right) e^{i\phi_2} - \frac{C}{T} + \frac{T^a}{T}\right] ,
\end{aligned}
$$

The power counting rules are

$$
\begin{aligned}
&\frac{P}{T} \sim \frac{V_{td}V^*_{tb}}{V_{ud}V^*_{ub}}\frac{C_{4,6}}{a_1} \sim \lambda , \\
&\frac{C}{T} \sim \lambda , \\
&\frac{P_{ew}}{T} \sim \lambda^2 , \\
&\frac{T^a}{T} \sim \frac{M_{nf}}{M_e}\frac{C_2}{a_1 N_c} \sim \lambda^2 , \\
&\frac{P^c_{ew}}{T} \sim \frac{P^a_{ew}}{T} \sim \lambda^4 .
\end{aligned}
$$

Drop the $O(\lambda^2)$ and higher terms, we have

$$\begin{aligned}
\sqrt{2}A(B^+ \to \pi^+\pi^0) &= -T\left(1 + \frac{|C|}{T}e^{i\delta_C}\right) , \\
A(B_d^0 \to \pi^+\pi^-) &= -T\left(1 + \frac{|P|}{T}e^{i\phi_2}e^{i\delta_P}\right) , \\
\sqrt{2}A(B_d^0 \to \pi^0\pi^0) &= T\left(\frac{|P|}{T}e^{i\phi_2}e^{i\delta_P} - \frac{|C|}{T}e^{i\delta_C}\right) ,
\end{aligned}$$

and the time dependent CP asymmetry

$$\begin{aligned}
\mathcal{A}(B_d^0(t) &\to \pi^+\pi^-) \\
&\equiv \frac{B(\bar{B}_d^0(t) \to \pi^+\pi^-) - B(B_d^0(t) \to \pi^+\pi^-)}{B(\bar{B}_d^0(t) \to \pi^+\pi^-) + B(B_d^0(t) \to \pi^+\pi^-)} \\
&= -C_{\pi\pi}\cos(\Delta M_d t) + S_{\pi\pi}\sin(\Delta M_d t) ,
\end{aligned}$$

where

$$C_{\pi\pi} = \frac{1 - |\lambda_{\pi\pi}|^2}{1 + |\lambda_{\pi\pi}|^2} , \qquad S_{\pi\pi} = \frac{2\,Im(\lambda_{\pi\pi})}{1 + |\lambda_{\pi\pi}|^2} ,$$

and

$$\lambda_{\pi\pi} = e^{2i\phi_2}\frac{1 + e^{-i\phi_2}P/T}{1 + e^{i\phi_2}P/T} .$$

The 5 experimental data are

$$\mathrm{Br}(B^{\pm} \to \pi^{\pm}\pi^{0}) = (5.2 \pm 0.8) \times 10^{-6} ,$$
$$\mathrm{Br}(B_d^{0} \to \pi^{\pm}\pi^{\mp}) = (4.6 \pm 0.4) \times 10^{-6} ,$$
$$\mathrm{Br}(B_d^{0} \to \pi^{0}\pi^{0}) = (1.97 \pm 0.47) \times 10^{-6} ,$$
$$C_{\pi\pi} = -(38 \pm 16)\% ,$$
$$S_{\pi\pi} = -(58 \pm 20)\% .$$

and assume

$$\mathcal{A}(B_d^{0} \to \pi^{0}\pi^{0}) = (0 \pm 50)\% .$$

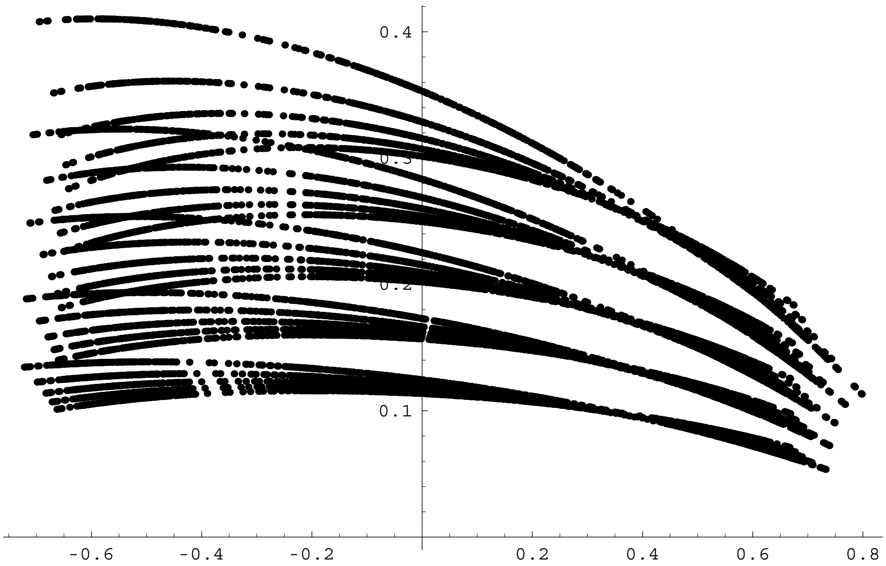
We have four avaliable solutions

$$\frac{P}{T} = 0.21 e^{96^{o} i} , \quad \frac{C}{T} = 0.99 e^{-84^{o} i} , \quad \phi_2 = 107^{o} ,$$
$$\frac{P}{T} = 0.21 e^{76^{o} i} , \quad \frac{C}{T} = 0.83 e^{76^{o} i} , \quad \phi_2 = 111^{o} ,$$
$$\frac{P}{T} = 0.67 e^{162^{o} i} , \quad \frac{C}{T} = 0.49 e^{-18^{o} i} , \quad \phi_2 = 72^{o} ,$$
$$\frac{P}{T} = 0.67 e^{10^{o} i} , \quad \frac{C}{T} = 0.16 e^{170^{o} i} , \quad \phi_2 = 147^{o} .$$

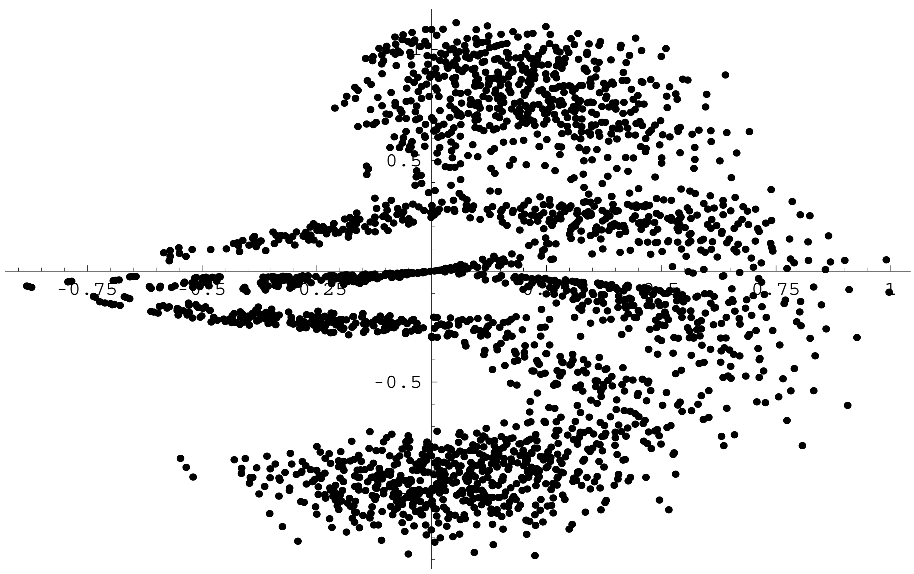
with the range of ϕ_2 is

$$51^{o} < \phi_2 < 176^{o}$$

P/T



C/T



III. Conclusions

- From $B \to K\pi$ case, we get $\phi_3 = 102^o$. The results satisfy the power counting and agree with PQCD.

- In $B \to K\pi$ case, the uncertainty can be as small as $O(\lambda^3) \sim 1\%$, $\Rightarrow$ No evidence for new physics!

- from $B \to \pi\pi$ case, the P_{ew} might not be samll($O(\lambda^2)$). One need to re-exame the calculation from PQCD,as well as QCDF, analysis of $B \to \pi\pi$.

- Need to put more theoretical efforts to extract ϕ_2 from $B \to \pi\pi$ data.