Inclusive Quarkonium Production and the NRQCD Factorization Approach

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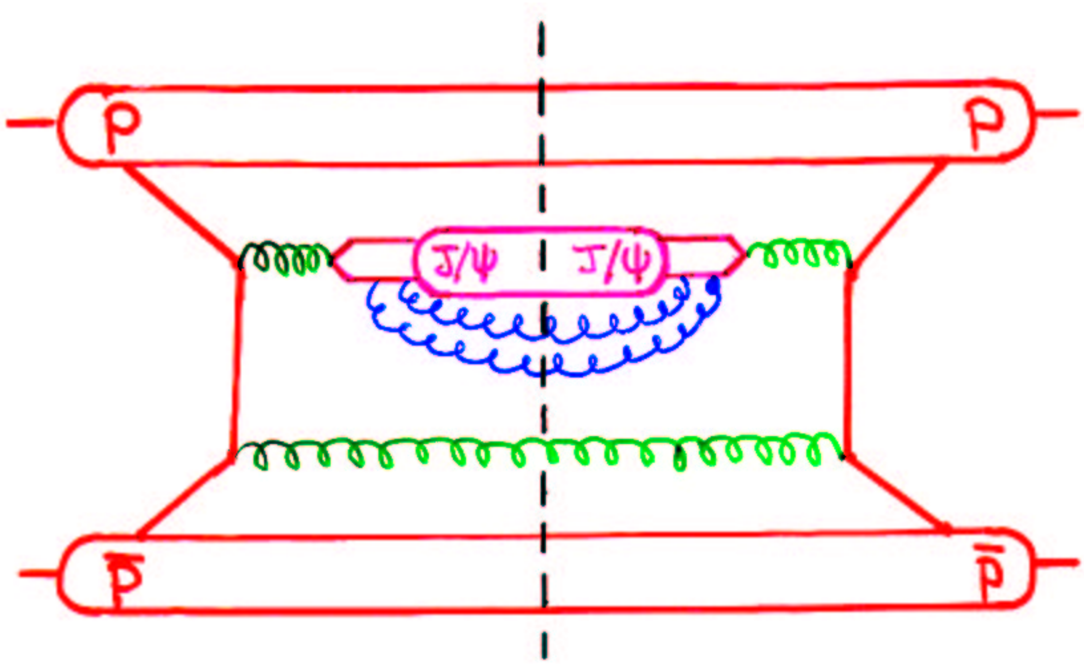
Factorization of the Inclusive Cross Section

The NRQCD Factorization Formula

(GTB, E. Braaten, G. P. Lepage)

- The effective field theory Nonrelativistic QCD (NRQCD) separates long-distance quarkonium dynamics ($p \lesssim mv$) from short-distance processes ($p \gtrsim m$).
- At large p_T (or p^*), the inclusive quarkonium production cross section can be written as a sum of products of NRQCD matrix elements and "short-distance" coefficients:

$$\sigma(H) = \sum_n \frac{F_n(\Lambda)}{m^{d_n-4}} \langle 0|\mathcal{O}_n^H(\Lambda)|0\rangle.$$



- The $F_n(\Lambda)$ are short-distance coefficients.
  - Partonic cross sections to make a $Q\bar{Q}$ pair convolved with parton distributions.
  - Calculate as an expansion in α_s.
- Four-fermion operators:

$$\mathcal{O}_n^H = \chi^\dagger \kappa_n \psi \left(\sum_X |H + X\rangle\langle H + X| \right) \psi^\dagger \kappa_n' \chi.$$

  - ψ is the Pauli spinor field that annihilates a heavy quark.
  - χ is the Pauli spinor field that creates a heavy antiquark.
  - κ contains Pauli matrices, color matrices, and covariant derivatives.
- The operator matrix elements contain all of the long-distance (nonperturbative physics).
  - Probabilities for a $Q\bar{Q}$ pair to evolve into a heavy-quarkonium.
  - They are universal (process independent).
- NRQCD predicts v-scaling rules for matrix elements.
  - $v^2 \approx 0.3$ for charmonium.
  - $v^2 \approx 0.1$ for bottomonium.
  - The sum over operator matrix elements is an expansion in powers of v.

- A similar factorization formula applies to inclusive quarkonium decays:

$$\Gamma(H \to \mathrm{LH}) = \sum_n \frac{2\,\mathrm{Im}\, f_n(\Lambda)}{m_Q^{d_n-4}} \langle H|\mathcal{O}_n(\Lambda)|H\rangle,$$

$$\mathcal{O}_n = \psi^\dagger \kappa_n \chi \chi^\dagger \kappa'_n \psi.$$

- The production matrix elements are the crossed versions of quarkonium decay matrix elements.
  - Only the color-singlet production and decay matrix elements are simply related.
- An important feature of NRQCD factorization: Quarkonium decay and production occur through color-octet, as well as color-singlet, $Q\overline{Q}$ states.
- If we drop all of the color-octet contributions, then we have the color-singlet model (CSM).
- In contrast, NRQCD factorization is not a model.
  - Sometimes erroneously called “the color-octet model.”
  - A rigorous consequence of QCD in the limit m, $p_T \gg \Lambda_{\mathrm{QCD}}$.

- NRQCD factorization relies on
  - NRQCD,
  - Hard-scattering factorization machinery. (Qiu, Sterman)
- Errors of order
  - $\Lambda_{\mathrm{QCD}}^2/p_T^2$ for unpolarized cross sections,
  - $\Lambda_{\mathrm{QCD}}/p_T$ for polarized cross sections.

Some Successes of the NRQCD Factorization Formalism

Inclusive P-Wave Quarkonium Decays

- IR finite predictions.
- Early NRQCD prediction for $\Gamma(\chi_{c2}) \to \gamma\gamma)$ borne out by experiment.
  - Old PDG value: $(11 \pm 6) \times 10^{-4}$ MeV.
  - NRQCD prediction: $(4.1 \pm 1.8) \times 10^{-4}$ MeV.
  - New PDG value: $(4.6 \pm 1.7) \times 10^{-4}$ MeV.
- NRQCD P-wave charmonium matrix elements:
  Global fit to data and lattice results are in agreement;
  Matrix elements are consistent with velocity-scaling rules.

– Global fit (Maltoni):

  * χ^2/d.o.f. $= 15.0/10$,
  * $\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle = (7.2 \pm 0.9) \times 10^{-2}$ GeV^5,
  * $\langle\chi_{cJ}|\mathcal{O}_8(^3S_1)|\chi_{cJ}\rangle = (4.3 \pm 0.9) \times 10^{-3}$ GeV^3.

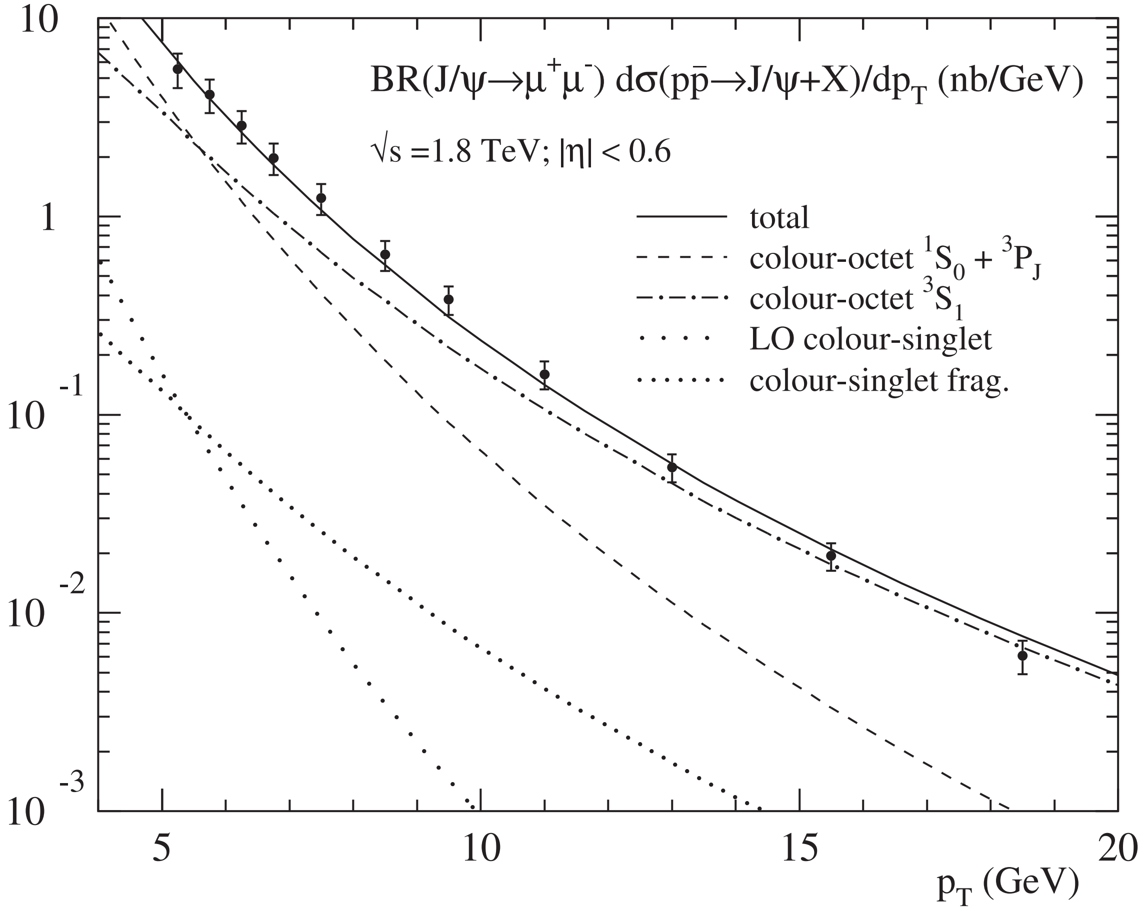
– Lattice (GTB, D.K. Sinclair, S. Kim):

  * $\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle = (8.0 \pm 1.7) \times 10^{-2}$ GeV^5,
  * $\langle\chi_{cJ}|\mathcal{O}_8(^3S_1)|\chi_{cJ}\rangle = (4.6 \pm 2.5) \times 10^{-3}$ GeV^3.

– Velocity scaling (Petrelli, Cacciari, Greco, Maltoni, Mangano):

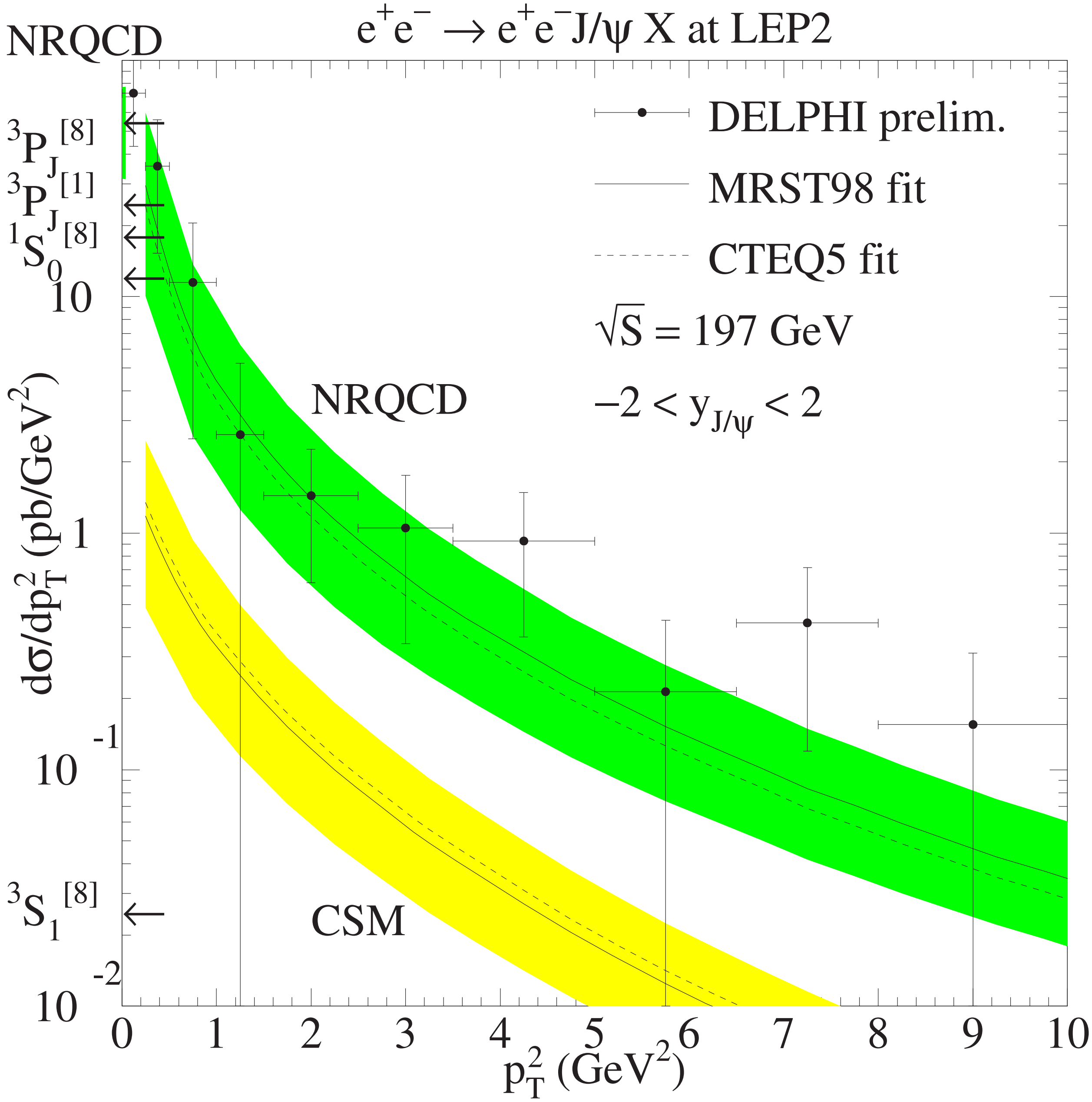
$$\langle\chi_{cJ}|\mathcal{O}_8(^3S_1)|\chi_{cJ}\rangle/\langle\chi_{cJ}|\mathcal{O}_1(^3P_J)|\chi_{cJ}\rangle \sim 1/(2N_c m_c^2) \approx 0.07 \text{ GeV}^{-2}.$$

- Explanation (color-octet mechanism) of Tevatron data for J/ψ, ψ', Υ production.
  - Matrix elements are determined from a fit to the data.
  - Shape consistent with NRQCD, but not with the color-singlet model.



$\gamma\gamma \to J/\psi + X$ at LEP

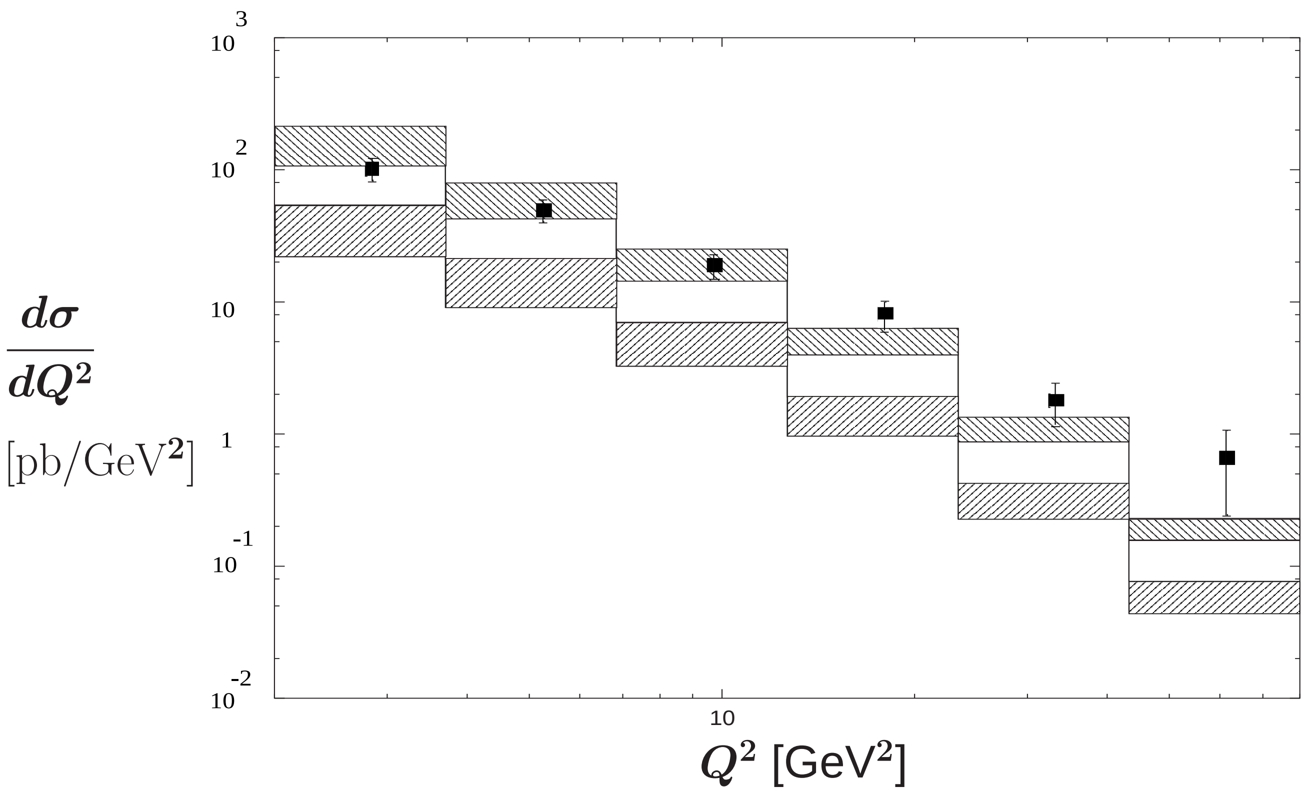
- Comparison of theory (Klasen, Kniehl, Mihaila, Steinhauser) with Delphi data clearly favors NRQCD over the color-singlet model.

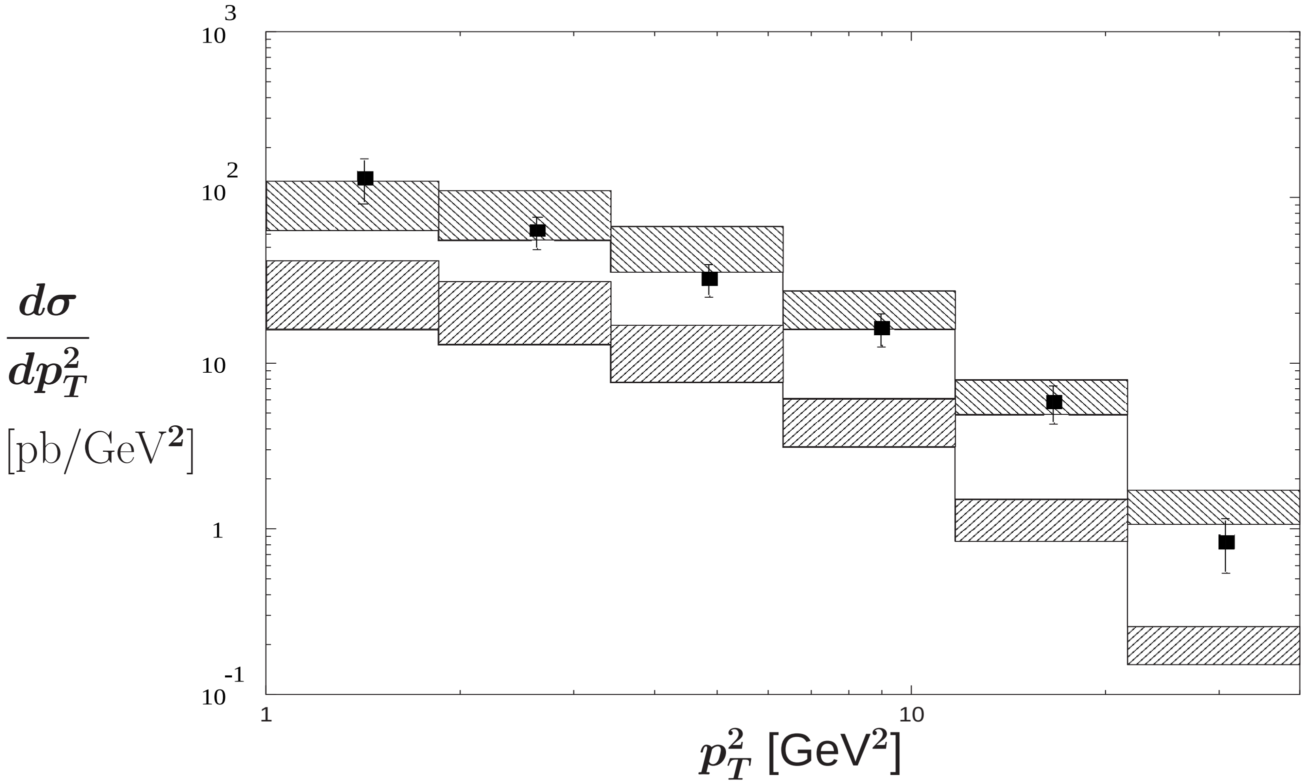


- Theory uses Braaten-Kniehl-Lee matrix elements and MRST98LO (solid) and CTEQ5L (dashed) PDF's.
- Theoretical uncertainties from
  - Renormalization and factorization scales (varied by a factor 2),
  - Color-octet matrix elements.
    - Different linear combination of matrix elements than in Tevatron cross sections.

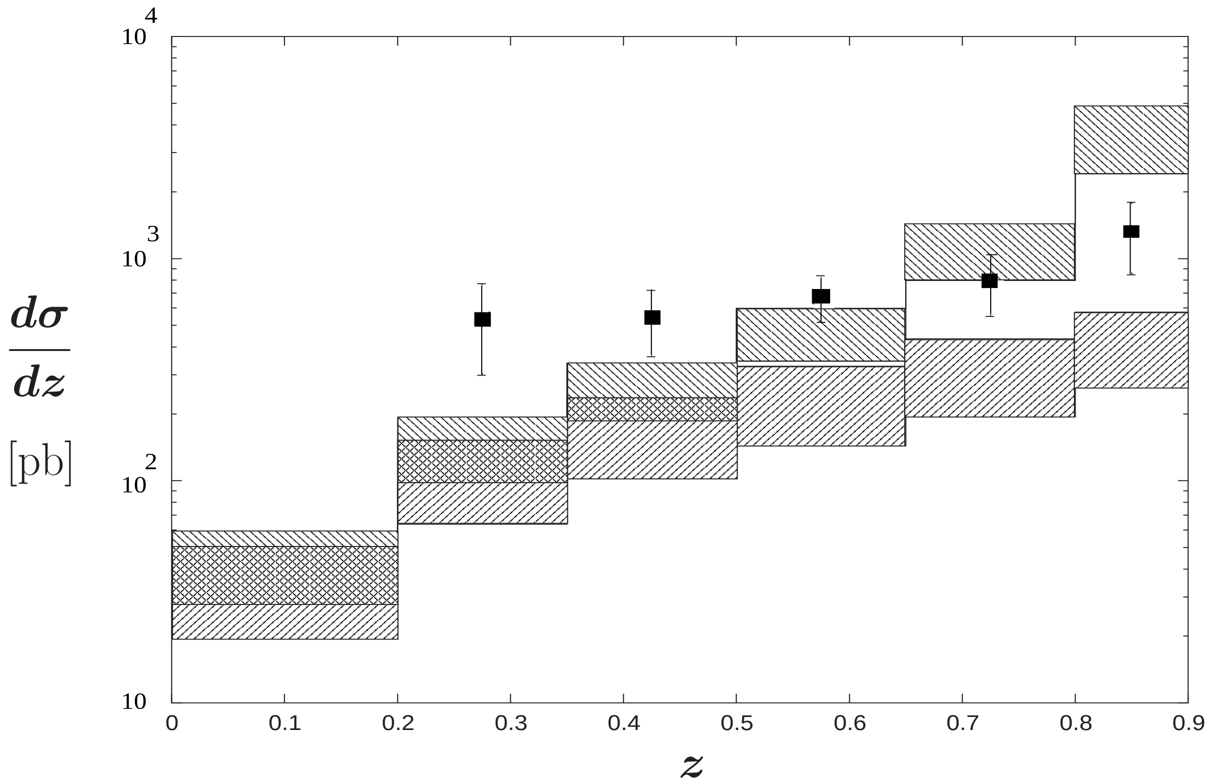
Quarkonium Production in DIS at HERA

- H1 data vs. leading-order NRQCD (upper) and Color-Singlet Model (lower).
- The data favor the NRQCD result when plotted vs. Q^2 and p_T^2, but not z.





$\frac{d\sigma}{dp_T^2}$
[pb/GeV^2]
10^3
10^2
10
1
10^{-1}
1
10
p_T^2 [GeV^2]



$\frac{d\sigma}{dz}$
[pb]
10^4
10^3
10^2
10
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
z

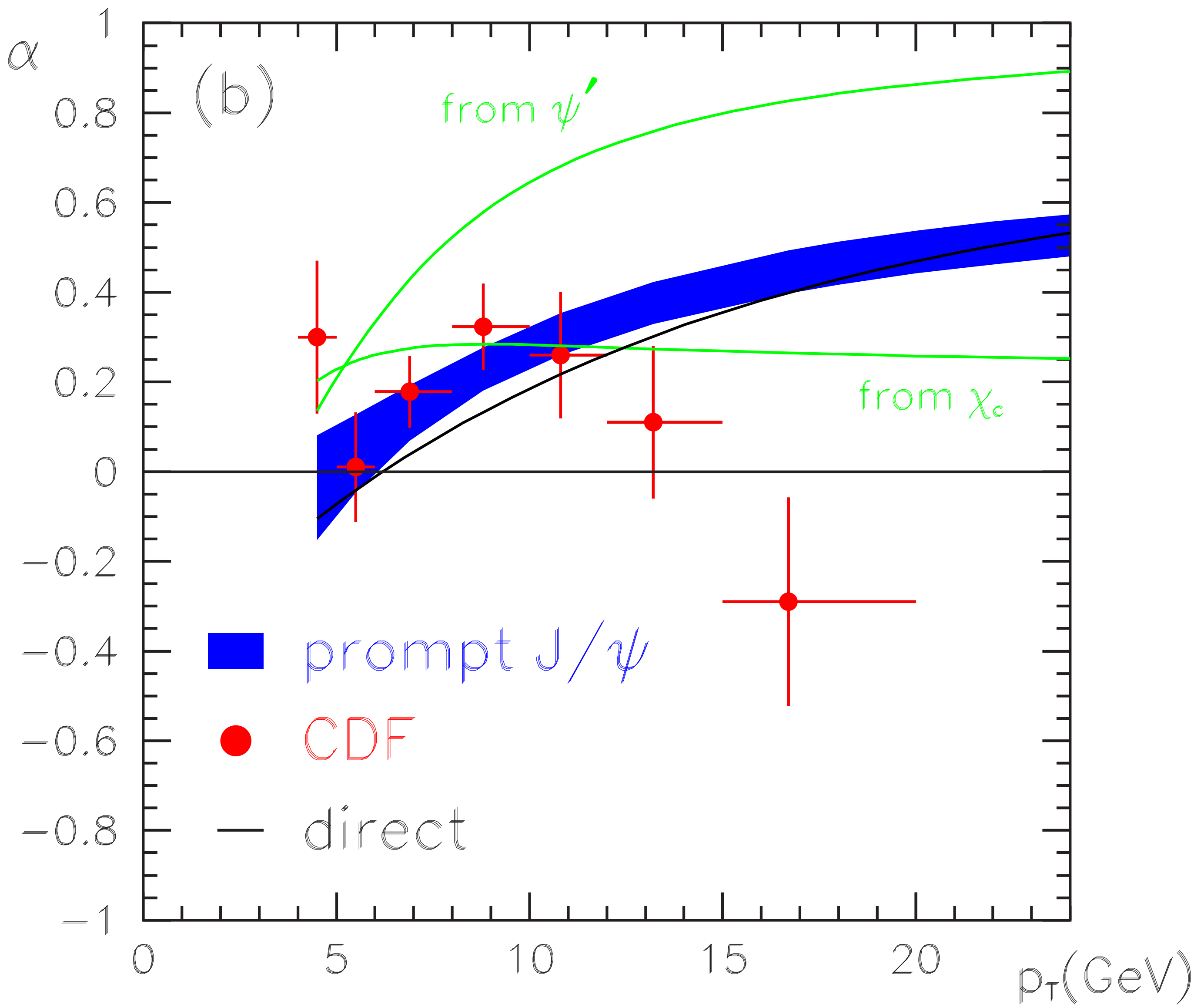
- Theory (Kniehl, Zwirner) uses Braaten-Kniehl-Lee matrix elements and MRST98LO and CTEQ5L PDF's.
- Theoretical uncertainties from
  - PDF's
  - Renormalization and factorization scales (varied by a factor 2),
  - Color-octet matrix elements.
    - Different linear combination of matrix elements than in Tevatron cross sections.
- The calculation of Kniehl and Zwirner disagrees with a number of previous results.

  These disagreements have not yet been resolved fully.

Some Problematic Comparisons with Experiment

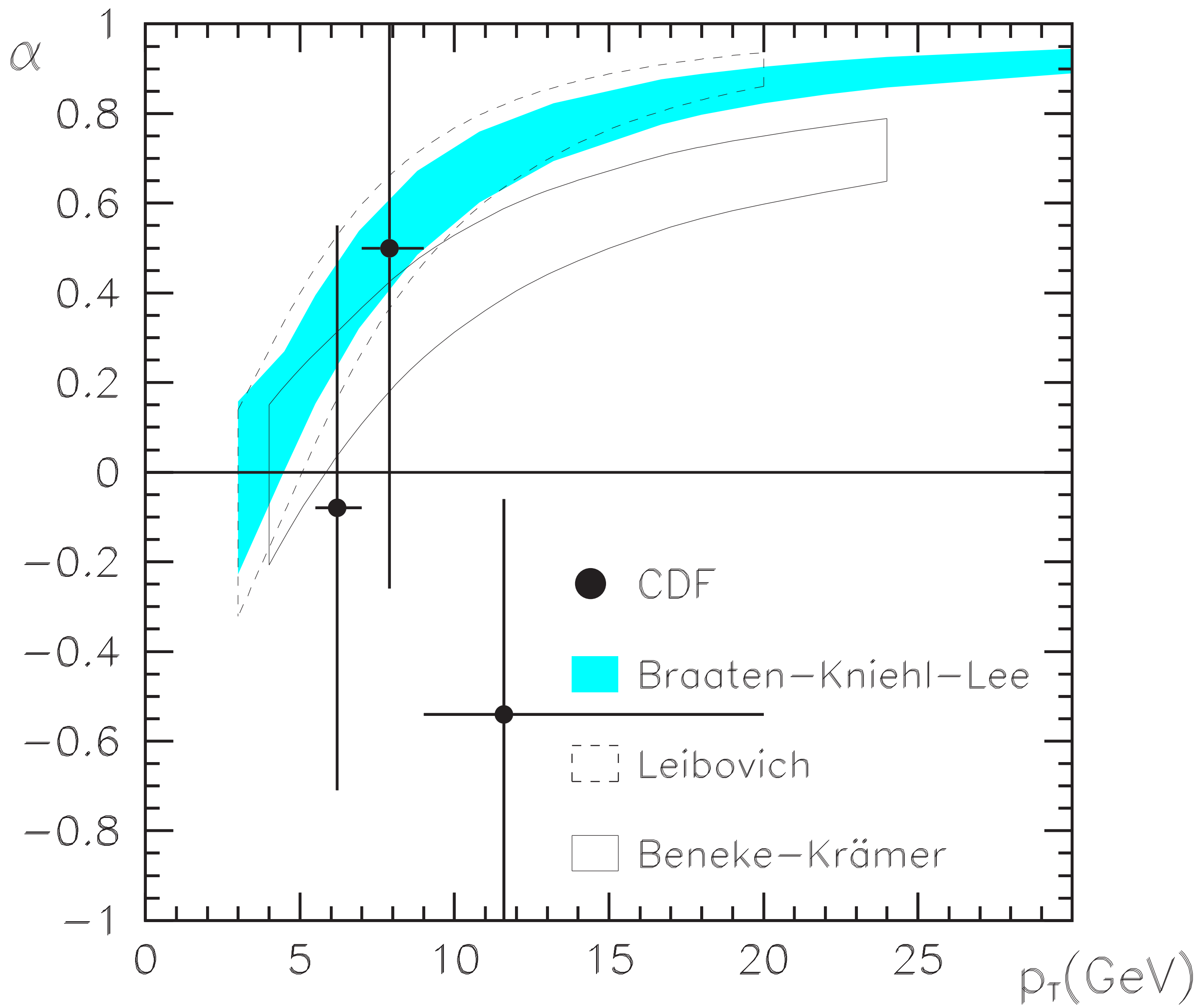
Polarization of Quarkonium at the Tevatron

- Potentially a "smoking gun" for the Color-Octet Mechanism.
- For large-p_T quarkonium production ($p_T \gtrsim 4m_c$ for J/ψ), gluon fragmentation via the color-octet mechanism dominates ($\langle \mathcal{O}_8(^3S_1)\rangle$).
- At large p_T, the gluon is nearly on mass shell, and, so, is transversely polarized.
- In color-octet gluon fragmentation, most of the gluon's polarization is transferred to the J/ψ. (Cho, Wise)
- Radiative corrections, color-singlet production dilute this. (Beneke, Rothstein; Beneke, Krämer)
- In the J/ψ case, feeddown is important, but has now been taken into account. (Braaten, Lee)
  - Feeddown from χ_c states is about 30% of the J/ψ sample and dilutes the polarization.
  - Feeddown from ψ' is about 10% of the J/ψ sample and is largely transversely polarized.



- $d\sigma/d(\cos\theta) \propto 1 + \alpha \cos^2\theta$.
  - $\alpha = 1$ is completely transverse;
  - $\alpha = -1$ is completely longitudinal.

- In the ψ' case, feeddown is not important, but statistics are not as good.



- The observed J/ψ and ψ' polarizations are much smaller than the prediction and seem to decrease with p_T.
- Polarization depends on a ratio of matrix elements.
  - It probably is not strongly affected by multiple soft-gluon emission or k-factors.

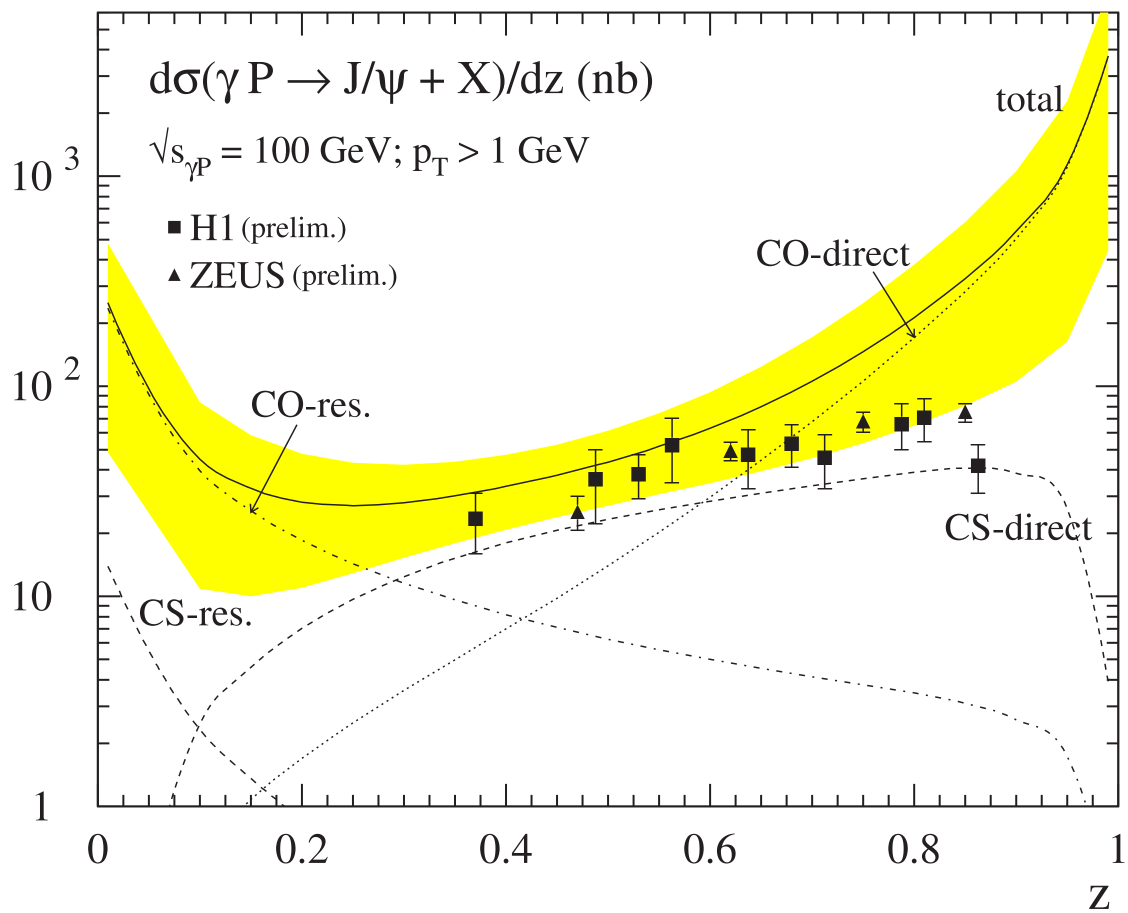
There are large theoretical uncertainties

- Uncertainties in matrix elements (shown in plots)
- Contributions of higher order in α_s
  - Calculated for ${}^3S_1$ color-octet fragmentation (Braaten, Lee), which gives the bulk of the polarization.
  - Corrections to the non-fragmentation process could conceivably increase the unpolarized contribution by a factor 2.
- Large order-v^2 corrections to gluon fragmentation to quarkonium. (GTB, Lee)
  - $+51\%$ for the color-singlet part.
    Yields a small correction to total the rate.
  - -39% for the color-octet part.
    Changes the normalization of the fitted matrix element, but not the rate.
  - Does the v expansion converge?

- Existing calculations assume that 100% of the $Q\bar{Q}$ polarization is transferred to the quarkonium.
  - Spin-flip corrections are suppressed only by v^2, not v^4, relative to the non-flip part. (GTB, Braaten, Lepage)
  - It could happen that the spin-flip corrections are anomalously large.
  - Do the velocity-scaling rules need to be modified? (Brambilla, Pineda, Soto, Vairo; Fleming, Rothstein, Leibovich)
  - These issues should be resolved by a lattice calculation that is in progress. (GTB, Lee, Sinclair)

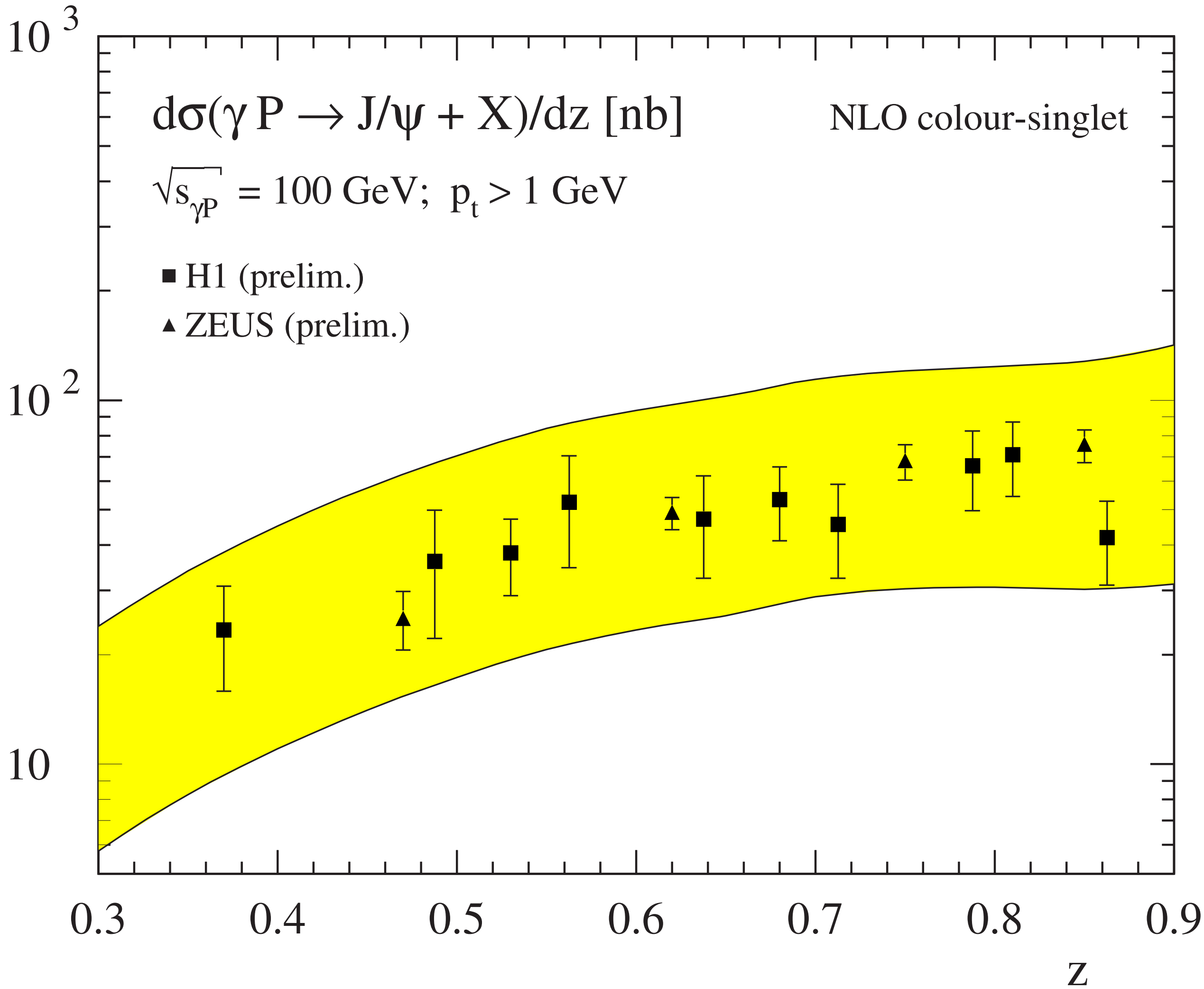
Inelastic Quarkonium Photoproduction at HERA

- Calculations by Cacciari, Krämer; Amundson, Fleming, Maksymyk; Ko, Lee, Song; Kniehl, Krämer.
- There seems to be no room for the color-octet contribution in the photoproduction data.

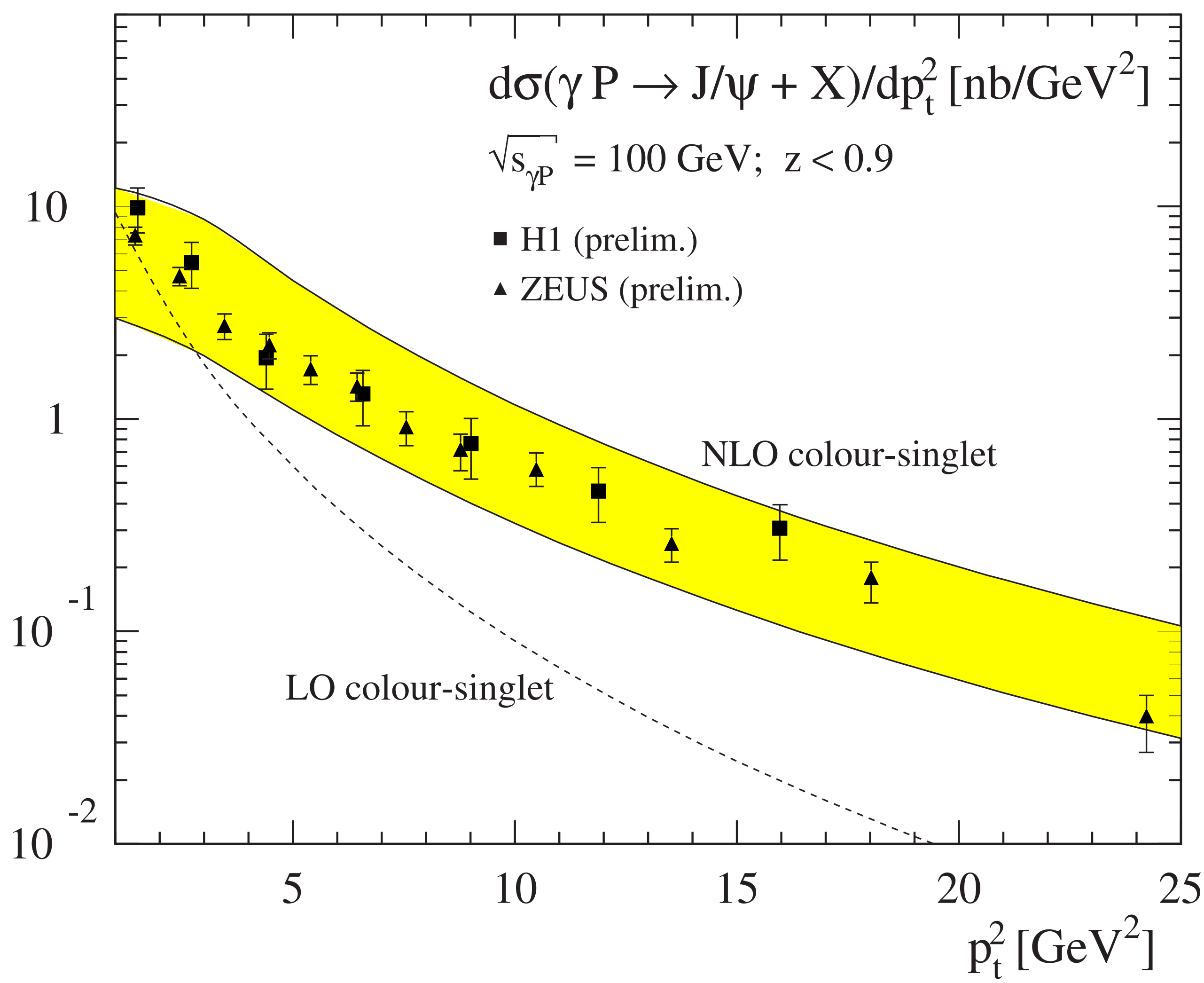


- Uncertainty is from color-octet matrix elements.

- NLO corrections (Krämer) increase the color-singlet piece by about a factor of 2.



- $p_T > 1$GeV cut.
  Can question whether factorization is OK at such small p_T.
- However, the data differential in p_T are compatible with color-singlet production alone at large p_T.



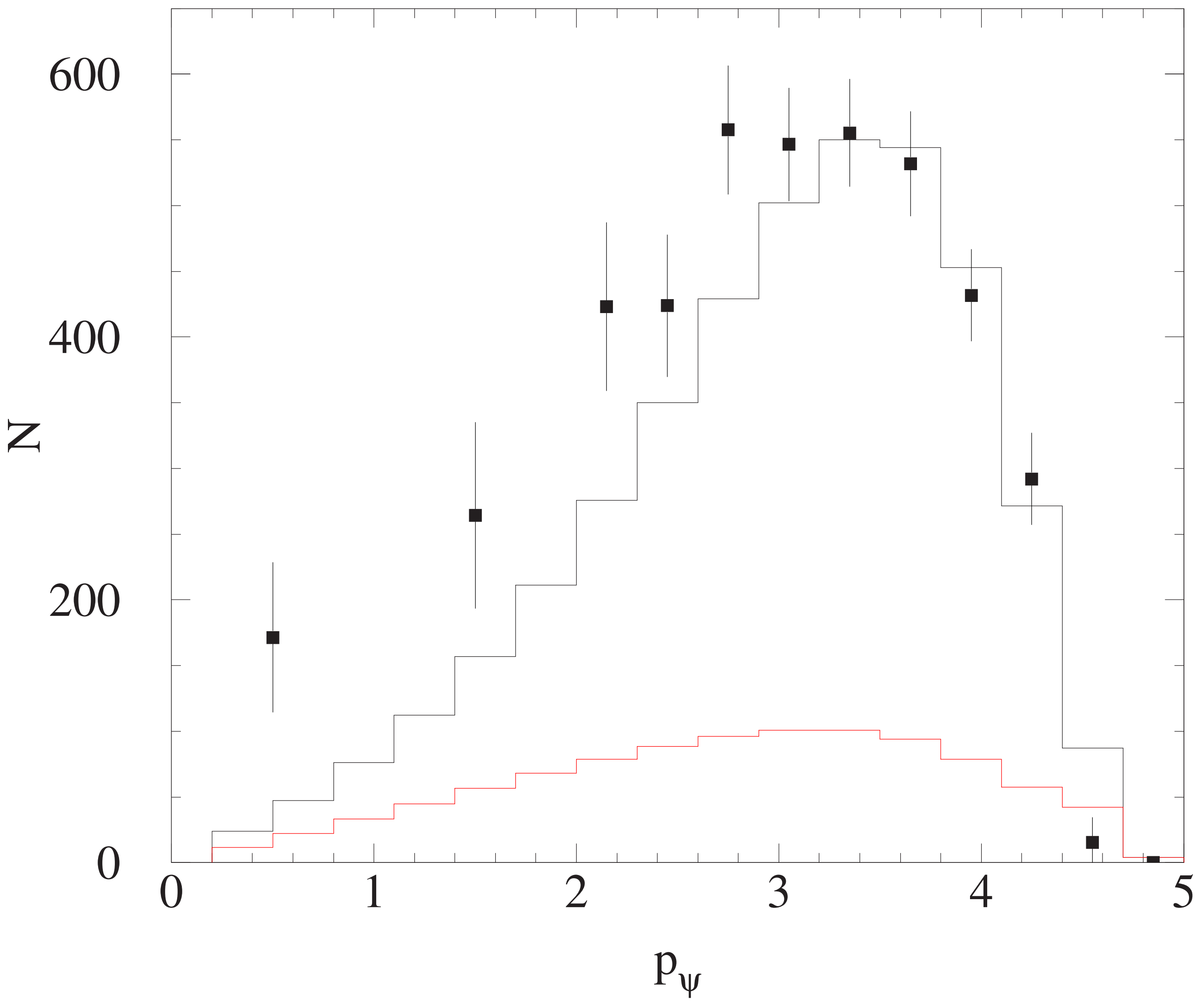
- Data seem to be incompatible with color-octet matrix elements extracted at the Tevatron.

But. . .

- Uncertainties in m_c could lower the color-singlet contribution by about a factor of 2, leaving more room for a color-octet contribution.
- There are large uncertainties in the color-octet matrix elements
  - Different linear combinations appear in photoproduction than appear in hadroproduction at the Tevatron.
  - Soft-gluon resummation should decrease the sizes of the matrix elements extracted from the Tevatron data.
- The color-octet contribution is calculated only at leading order in α_s for photoproduction.
  - Soft-gluon resummation is needed near the endpoint. (Beneke, Schuler, Wolf)
- The v expansion breaks down near $z = 1$.
  - Resummation of the v expansion leads to a nonperturbative shape function. (Beneke, Rothstein, Wise)

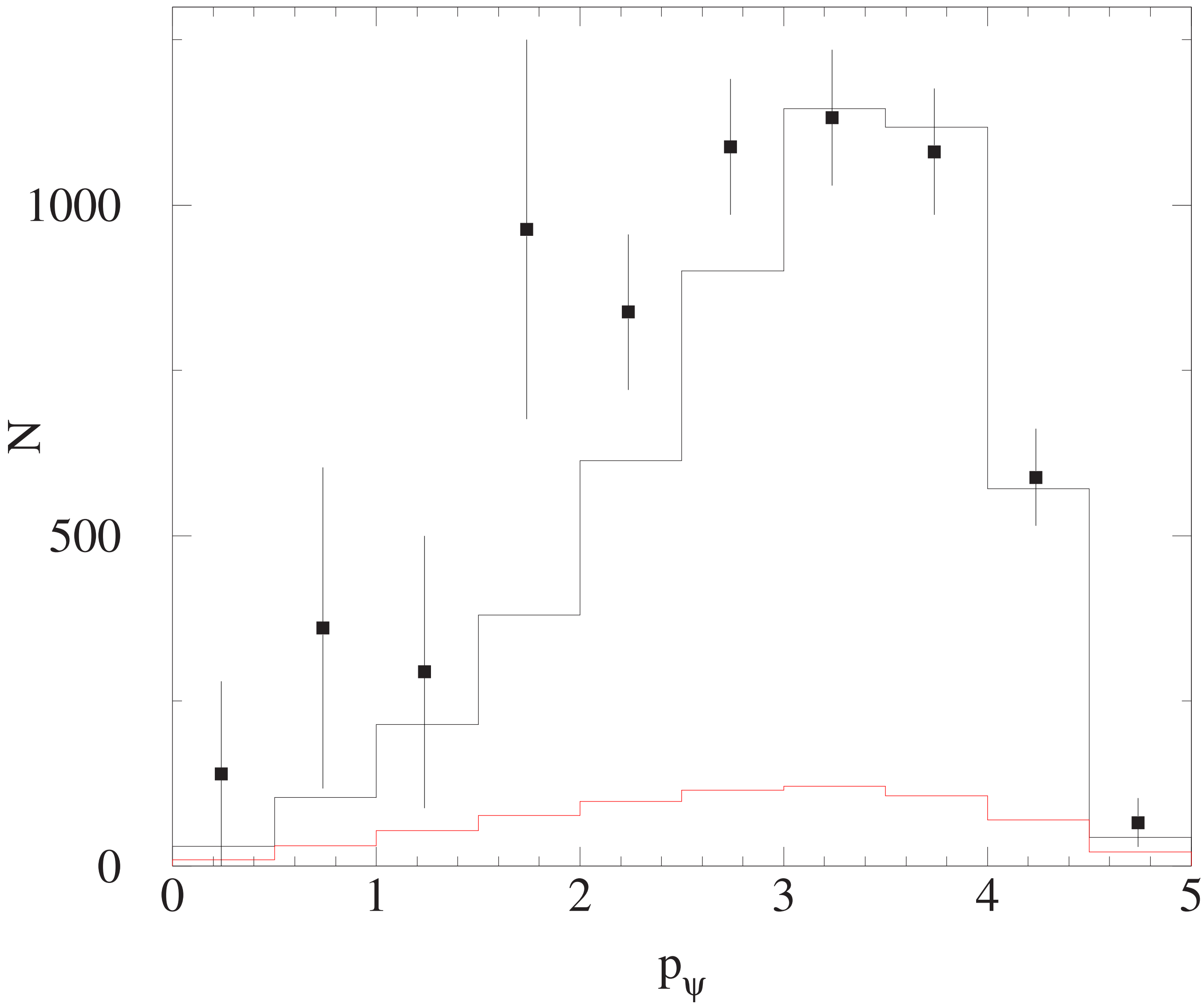
- Soft-gluon-resummation and shape-function effects have been calculated for $e^+e^- \longrightarrow J/\psi + X$ by Fleming, Leibovich, and Mehen.

Belle data:

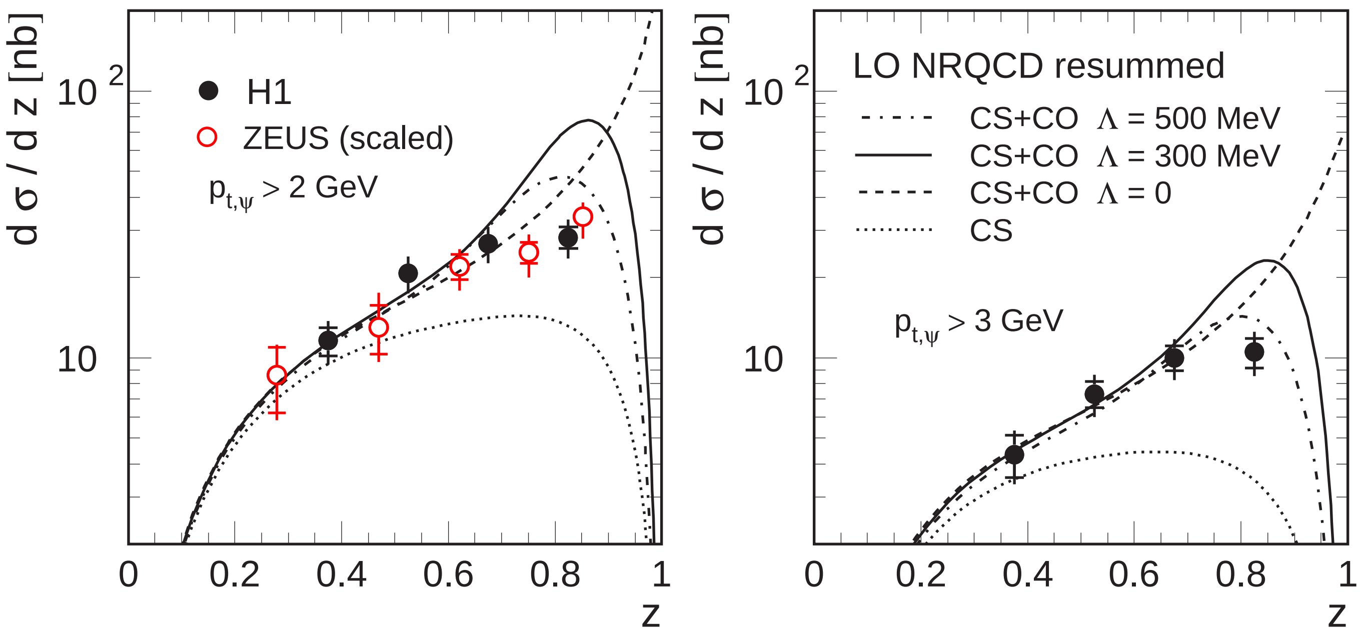


- Red is color singlet; black is color-octet plus color singlet.

BaBar data:



- Inclusion of a shape function with reasonable choices of parameters leads to an improved fit.



- New higher-p_T data are more compatible with a color-octet contribution.
- Strategy for future calculations:
  Use a shape function fitted to e^+e^- data plus soft-gluon resummation to make a firm prediction.

Double $c\bar{c}$ Production at Belle

$e^+e^- \to J/\psi + \eta_c$

- Belle obtains

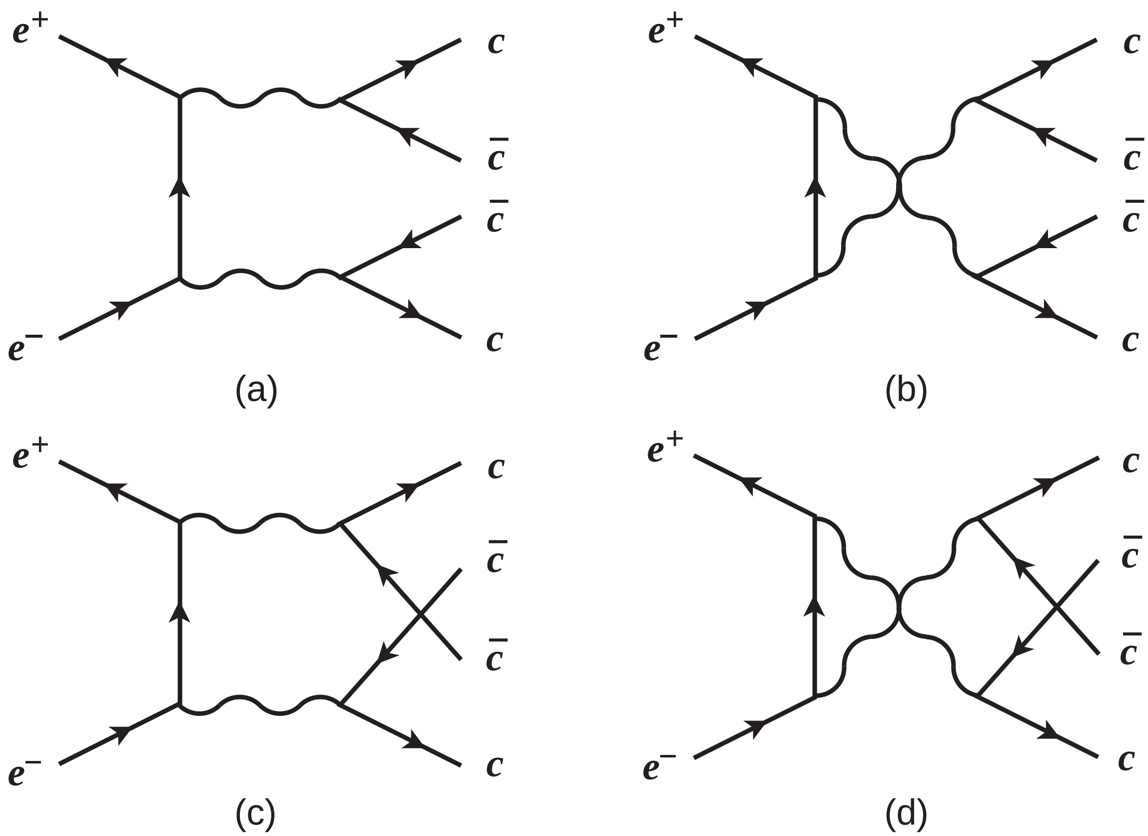
$$\sigma(e^+e^- \to J/\psi + \eta_c)B[\geq 4] = 33^{+7}_{-6} \pm 9 \text{ fb.}$$

- NRQCD predicts

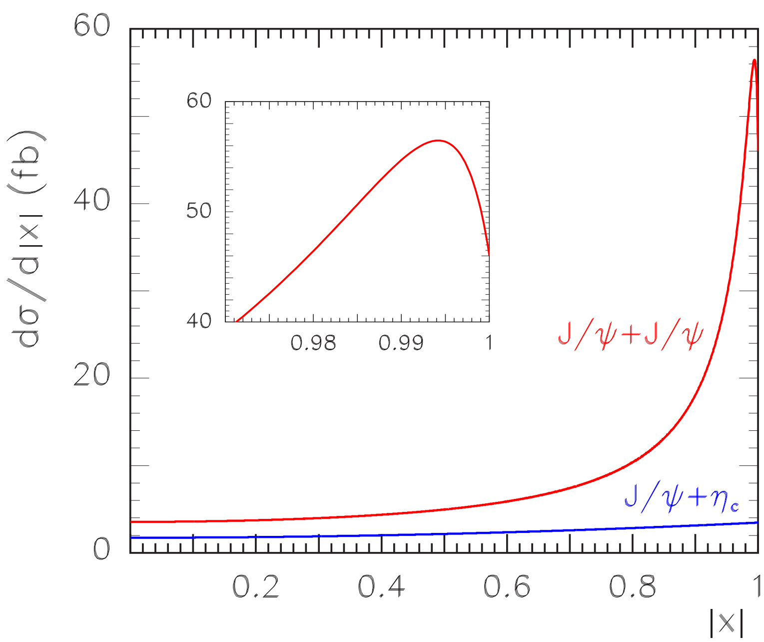
$$\sigma(e^+e^- \to J/\psi + \eta_c) = 2.31 \pm 1.09 \text{ fb.}$$

  – First calculation by Braaten, Lee.
  – Confirmed by Liu, He, Chao (NRQCD) and Brodsky, Ji, and Lee (light-front QCD).
  – Includes -21% QED interference correction.
  – Uncertainties from higher orders in α_s, v, matrix elements.
  – Exclusive process: color-singlet only.
  – Matrix elements are fairly well determined from $J/\psi \to e^+e^-$ and $\eta_c \to \gamma\gamma$.

- Some of the $J/\psi + \eta_c$ data sample may consist of $J/\psi + J/\psi$ events. (GTB, Braaten, Lee)
  - The Belle resolution is 110 MeV, but $M_{J/\psi} - M_{\eta_c} = 120$ MeV.
  - $J/\psi + J/\psi$ is $C = +1$, so that state is produced in a two-photon process.
  - Suppressed by relative to $J/\psi + \eta_c$ by $(\alpha/\alpha_s)^2$
  - But fragmentation diagrams are enhanced by
    * $(E_{\rm beam}/2m_c)^4$ from gluon propagators,
    * $\log[8(E_{\rm beam}/2m_c)^4]$ from a would-be collinear divergence.



e+
e−
c
c̄
(a)
(b)
(c)
(d)



dσ/d|x| (fb)
|x|
J/ψ+J/ψ
J/ψ+η_c
60
40
20
0
0.2
0.4
0.6
0.8
1
60
50
40
0.98
0.99
1

- Prediction:
  $\sigma(e^+e^- \to J/\psi + J/\psi) = 8.70 \pm 2.94$ fb.
  - Corrections of higher order in α and v may reduce this by a factor 3.
  - Comparable with the prediction

  $\sigma(e^+e^- \to J/\psi + \eta_c) = 2.31 \pm 1.09$ fb.
- New Belle result:
  There is no significant $J/\psi + J/\psi$ signal observed.

$$\sigma(e^+e^- \to J/\psi + J/\psi) < 7 \text{ fb.}$$

$e^+e^- \to J/\psi + c\bar{c}$

- New Belle result:

$$\begin{aligned}&\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X)\\ &\quad = 0.82 \pm 0.15 \pm 0.14\\ &\quad > 0.48 \text{ (90\% confidence level)}\end{aligned}$$

- pQCD plus color-singlet model (Cho, Leibovich; Baek, Ko, Lee, Song; Yuan, Qiao, Chao):

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X) \approx 0.1.$$

- The experimental and theoretical double-$c\bar{c}$ cross sections also disagree.
  - Belle: $\sigma(e^+e^- \to J/\psi + c\bar{c}) \approx 0.9$ pb.
  - Theory: $\sigma(e^+e^- \to J/\psi + c\bar{c}) =$ 0.10–0.15 pb.
- Corrections of higher order in α_s and v are not expected to be large.

The discrepancies in the double $c\bar{c}$ cross sections are among the largest in the standard model.

- Theory and experiment differ by almost an order of magnitude—larger than any known "k-factor."
- This is a problem not just for NRQCD factorization, but for pQCD in general.
- For $e^+e^- \to J/\psi + \eta_c$, one obtains exactly the same result in the NRQCD and light-cone formalisms.
- It is difficult to see how any perturbative calculation of

$$\sigma(e^+e^- \to J/\psi + c\bar{c})/\sigma(e^+e^- \to J/\psi + X)$$

  could give a value as large as 80%.
  - The color-evaporation model should give a result that is close to that of NRQCD factorization.

- It is very important for BaBar to check the Belle double $c\bar{c}$ results.
- Other possibilities:
  - New production mechanisms
  - Perturbative QCD is inapplicable
  - New physics

Summary

- The NRQCD factorization approach provides a systematic method for calculating quarkonium decay production rates as a double expansion in powers of α_s and v.
- Calculation of production rates also relies upon hard-scattering factorization (corrections suppressed by powers of $\Lambda_{\rm QCD}/p_T$).
- NRQCD factorization has enjoyed a number of successes:
  - Inclusive P-wave quarkonium decays,
  - Quarkonium production at the Tevatron,
  - $\gamma\gamma \to J/\psi + X$ at LEP,
  - Quarkonium production at DIS at HERA.
- Other processes are (so far) more problematic:
  - Quarkonium polarization at the Tevatron,
  - Inelastic quarkonium photoproduction at HERA,
  - Double $c\bar{c}$ production at Belle.
- The Belle double $c\bar{c}$ production results are a severe challenge to pQCD and should be checked by BaBar.

- In other cases, inclusion of corrections of higher order in α_s and v and soft-gluon resummation should help.
- More precise theoretical predictions are hampered by uncertainties in the NRQCD matrix elements.
  - Lattice calculations can help to pin down the decay matrix elements.
  - It is not yet known how to formulate the calculation of production matrix elements on the lattice.
- This is an exciting time for heavy-quarkonium physics, with a great deal of experimental and theoretical activity in production, decay, and spectroscopy.
- There are still many interesting and challenging problems in heavy-quarkonium physics that remain to be solved.