$D_s(2317)$ and $D_s(2460)$ from HQET Sum Rules

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I. Introduction

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III. Sum rule calculations

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I. Introduction

The end of April, BaBar :

$D_s(2317)$, narrow

surprisingly low compared to quark models.

Later, CLEO : confirmed , & $D_s(2460)$

Belle : confirmed

	$D_s(2317)$	$D_s(2460)$
J^P	0^+	1^+
Decays	$D_s\pi^0$	$D_s^*\pi^0$

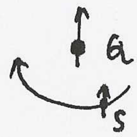
We take them as $L=1$ excited D_s.

narrowness : $D_s(2317) \to D_s(1969) + \eta' \hookrightarrow \pi^0$

masses : use QCD sum rules to calculate.

II. HQET description

picture :



$J=0,\ J_\ell=\frac{1}{2}$

$J=1,\ J_\ell=\frac{3}{2}$

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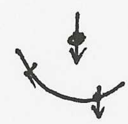


$J=2,\ J_\ell=\frac{3}{2}$

$$D_s(2317) = D_s\,(J=0,\ J_\ell=\tfrac{1}{2})$$

$$D_s(2460) = D_s\,(J=1,\ J_\ell=\tfrac{1}{2})$$

$m_Q/\Lambda_{QCD} \to \infty$:

$$\begin{pmatrix} D_s(2317) \\ D_s(2460) \end{pmatrix}, \quad \begin{pmatrix} D_s(J=1,\ J_\ell=\frac{3}{2}) \\ D_s(J=2,\ J_\ell=\frac{3}{2}) \end{pmatrix}$$

$$\Delta M \sim \frac{1}{m_Q}, \qquad \text{mixing} \sim \frac{1}{m_Q}$$

$$M = m_Q + \bar{\Lambda}_s + \mathcal{O}\left(\frac{1}{m_Q}\right)$$

Currents :

$$J = \frac{1}{\sqrt{2}}\, \bar{h}_v\, \Gamma^\rho D_\rho\, q$$

$$\Gamma^\rho = -\gamma_t^\rho \qquad \text{for } D_s(2317)$$

$$\Gamma^\rho = \gamma_5\, \gamma_t^\mu\, \gamma_t^\rho \qquad \text{for } D_s(2460)$$

$$\Gamma^\rho = -\sqrt{\tfrac{3}{2}}\, \gamma_5 \left(g_t^{\mu\rho} - \tfrac{1}{3}\gamma_t^\mu \gamma_t^\rho \right) \qquad \text{for } D_s(1, \tfrac{3}{2})$$

$$\Gamma^\rho = \tfrac{1}{2}\left(\gamma_t^\mu g_t^{\nu\rho} + \gamma_t^\nu g_t^{\mu\rho} \right) - \tfrac{1}{3} g_t^{\mu\nu} \gamma_t^\rho \qquad \text{for } D_s(2, \tfrac{3}{2})$$

where

$$\gamma_t^\mu \equiv \gamma^\mu - v^\mu \not{v}, \qquad g_t^{\mu\nu} \equiv g^{\mu\nu} - v^\mu v^\nu .$$

Decay constants :

$$\langle 0 | J^\dagger | D_s^{**}(\eta) \rangle \equiv f \eta$$

polarization vector

III. QCD sum rules

	Q^2	
high		perturbation
1 GeV^2		perturbation $+ \dfrac{\langle \bar{q} q \rangle}{(Q^2)^{3/2}} + \cdots$ (QSR)
low		chiral Lagrangian

construct a Green's function

hadron language $\qquad$ quark-gluon with VACUUM CONDENSATES

sum rule

procedure :

duality assumption

Borel transformation

Sum rule window

This method is more close to QCD itself

than quark models.

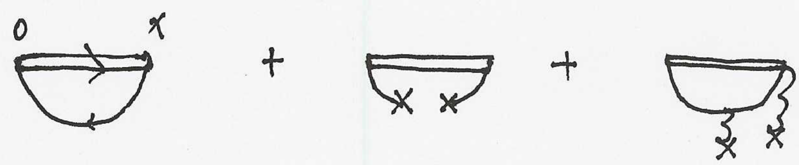
1. Green's function:

$$\Gamma(\omega) = i \int d^4x \, e^{ik\cdot x} \langle 0 | T \, J^\dagger(x) \, J(0) | 0 \rangle$$

with $\omega = 2k\cdot v$

hadron: $\Gamma(\omega) = \dfrac{2f^2}{2\bar\Lambda_s - \omega}$ + resonance.

quark-gluon:



$m_s \neq 0$

$\langle \bar{s} s \rangle = 0.8 \langle \bar{q} q \rangle$ $\hookrightarrow$ u, d

2. sum rules:

$(0^+, 1^+)$:

$$f_{+,\frac{1}{2}}'^{2}\, e^{-2\bar{\Lambda}_s/T} = \frac{3}{64\pi^2}\int_{2m_s}^{w_c}\left(w^4 + 2m_s w^3 - 6m_s^2 w^2 - 12 m_s^3 w\right)e^{-w/T}dw$$

$$- \frac{1}{16} m_0^2 \langle \bar{s} s\rangle + \frac{3}{8} m_s^2 \langle \bar{s} s\rangle - \frac{m_s}{16\pi} \langle \alpha_s GG\rangle .$$

$(1^+, 2^+)$:

$$f_{+,\frac{3}{2}}^{2}\, e^{-2\bar{\Lambda}_s/T} = \frac{1}{64\pi^2}\int_{2m_s}^{w_c}\left(w^4 + 2m_s w^3 - 6m_s^2 w^2 - 12 m_s^3 w\right)e^{-w/T}dw$$

$$- \frac{1}{12} m_0^2 \langle \bar{s} s\rangle - \frac{1}{32}\langle \frac{\alpha_s}{\pi} GG\rangle T + \frac{1}{8} m_s^2 \langle \bar{s} s\rangle - \frac{m_s}{48\pi}\langle \frac{\alpha_s}{\pi} GG\rangle .$$

3. numerical analysis: $m_s = 150$ MeV

S. R. window:
$$\begin{cases} T_{min}: & \text{high power condensates} < 30\% \text{ pert.} \\ T_{max}: & \text{pole term} > 60\% \text{ pert.} \end{cases}$$

$(0^+, 1^+)$: $0.38\ \text{GeV} < T < 0.58\ \text{GeV}$,

$\bar{\Lambda}_s(\frac{1}{2}^+) = 0.86 \pm 0.10\ \text{GeV}$.

$(1^+, 2^+)$: $0.55\ \text{GeV} < T < 0.65\ \text{GeV}$,

$\bar{\Lambda}_s(\frac{3}{2}^+) = 0.83 \pm 0.10\ \text{GeV}$.

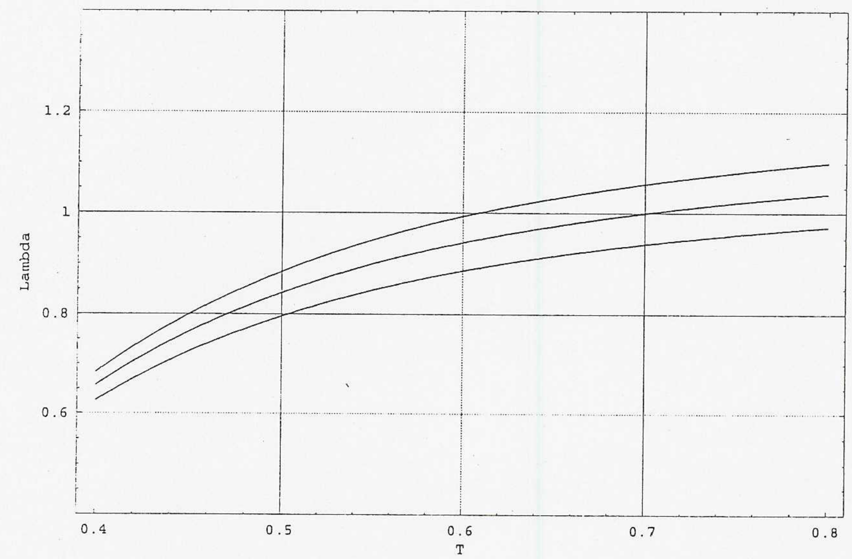


Figure 1: The variation of $\bar{\Lambda}(\frac{1}{2}^+)$ (in unit of GeV) of the $(0^+, 1^+)$ doublet with T and ω_c for the derivative currents. The vertical and horizontal axes correspond to $\bar{\Lambda}$ and T. From top to bottom, the curves correspond to ω_c being $3.1, 2.9, 2.7$ GeV respectively.

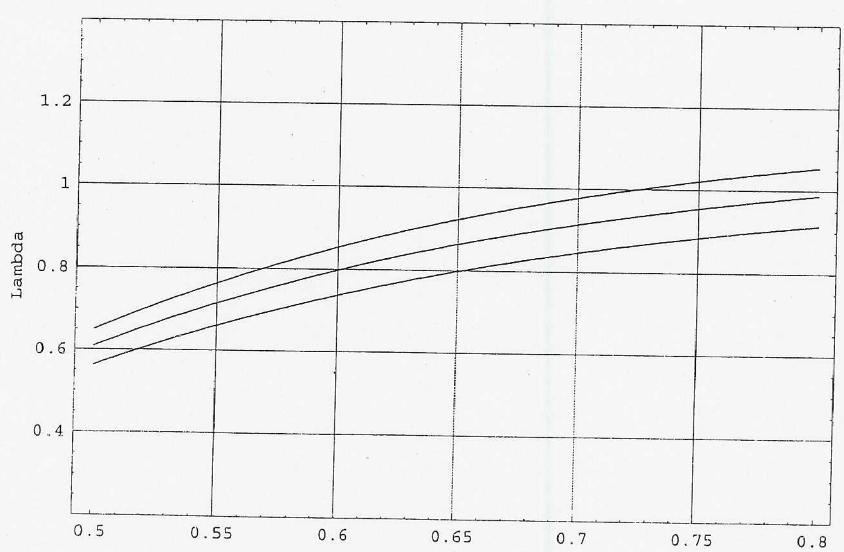


Figure 2: The variation of $\bar{\Lambda}(\frac{3}{2}^+)$ (in unit of GeV) of the $(1^+, 2^+)$ doublet with T and ω_c. From top to bottom, $\omega_c = 3.2, 3.0, 2.8$ GeV respectively.

5. final results :

$$(0^+\ 1^+)\begin{cases}\frac{1}{4}\left(m_{D_{s0}}+3m_{D'_{s1}}\right)=m_c+(0.86\pm 0.10)\,\mathrm{GeV}+\frac{1}{m_c}(0.40\pm 0.08)\,\mathrm{GeV}^2\\ m_{D'_{s1}}-m_{D_{s0}}=\frac{1}{m_c}(0.28\pm 0.05)\,\mathrm{GeV}^2\end{cases}$$

$$(1^+\ 2^+)\begin{cases}\frac{1}{8}\left(3m_{D_{s1}}+5m_{D^*_{s2}}\right)=m_c+(0.83\pm 0.10)\,\mathrm{GeV}+\frac{1}{m_c}(0.41\pm 0.10)\,\mathrm{GeV}^2\\ m_{D^*_{s2}}-m_{D_{s1}}=\frac{1}{m_c}(0.116\pm 0.06)\,\mathrm{GeV}^2\end{cases}$$

$m_c = 1.44$ GeV consistent with ground States

$$\Rightarrow\begin{cases}m_{D^*_{s2}}-m_{D_{s1}}=0.080\pm 0.042\ \mathrm{GeV} & (\text{Exp.: } 37\ \mathrm{MeV})\\ \frac{1}{4}\left(m_{D_{s0}}+3m_{D'_{s1}}\right)=2.57\pm 0.12\ \mathrm{GeV} & (\text{Exp: } 2.42\ \mathrm{GeV})\\ m_{D'_{s1}}-m_{D_{s0}}=0.19\pm 0.04\ \mathrm{GeV} & (\text{Exp: } 143\ \mathrm{MeV})\end{cases}$$

And $m_{D_{s0}} = 2.42 \pm 0.13$ GeV (Exp. 2.32 GeV)

IV Discussions

HQET, $1/m_c$, QCD sum rules

D_{s0} has been calculated.

Consistency within uncertainties of Q.S.R.

But the central value is still 100 MeV higher.

uncertainties:

- Q.S.R. stability is not as good as in ground states
- $1/m_c^2$ corrections

Compare to non-strange case:

$$m_{D_0} = 2.35 \quad (2.29) \quad \text{GeV}$$

$$m_{D_1'} = 2.54 \quad (2.40)$$

$$m_{D_1} = 2.40 \quad (2.42)$$

$$m_{D_2^*} = 2.48 \quad (2.46)$$

4. $1/m_c$ corrections

$$\mathcal{L} = \bar{h}_v\, v\cdot A\, h_v + \frac{\mathcal{K}}{2m_Q} + \frac{\mathcal{S}}{2m_Q} + O(1/m_Q^2)$$

with

$$\mathcal{K} = \bar{h}_v\,(iD_t)^2 h_v\ , \quad \mathcal{S} = C\,\frac{g}{2}\,\bar{h}_v\,\sigma\cdot G\, h_v$$

$$M = m_Q + \bar{\Lambda}_s - \frac{K}{4m_Q} - d\,\frac{\Sigma}{4m_Q} + O\left(\frac{1}{m_Q^2}\right)$$

For K and Σ, three-point Green's functions.

m_s corrections omitted.

results:

$(0^+\ 1^+)$: $K(\tfrac{1}{2}) = -1.60 \pm 0.30\ \mathrm{GeV}^2$,

$\Sigma(\tfrac{1}{2}) = 0.28 \pm 0.05\ \mathrm{GeV}^2$.

$(1^+\ 2^+)$: $K(\tfrac{3}{2}) = -1.64 \pm 0.40\ \mathrm{GeV}^2$

$\Sigma(\tfrac{3}{2}) = 0.058 \pm 0.01\ \mathrm{GeV}^2$