Factorization of B decays in soft-collinear effective theory

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Abstract

I consider factorization properties in B decays in soft-collinear effective theory. The idea of two-step matching is used to prove factorization in SCET.

Outline

- Soft-collinear effective theory
- Description of B decays in SCET
- Factorization
- Factorized results
- Perspective and conclusion

Soft-collinear effective theory

Bauer et al., Phys. Rev. D **63**, 114020, (2001), Phys. Lett. B **516**, 134 (2001); M. Beneke et al., Nucl. Phys. B **643**, 431 (2002).

- Three scales exist for an energetic particle. The momentum can be decomposed as

$$\begin{aligned} p^\mu &= \frac{\overline{n}\cdot p}{2}n^\mu + p_\perp^\mu + \frac{n\cdot p}{2}\overline{n}^\mu \\ &= \mathcal{O}(Q) + \mathcal{O}(Q\lambda) + \mathcal{O}(Q\lambda^2). \\ &\quad \left(n^2 = \overline{n}^2 = 0,\ n\cdot\overline{n} = 2,\ \lambda \sim \frac{p_\perp}{Q}\right) \end{aligned}$$

- Construct effective theories $(\mathrm{SCET_I}, \mathrm{SCET_{II}})$ by integrating out the degrees of freedom, not excitable at the scale.

- Matching: Physics is the same at the boundary.

1. $\mathrm{SCET_I}$ ($\sqrt{Q\Lambda} < \mu < Q$): $\lambda' \sim \sqrt{\Lambda/Q}$,

   collinear fields with $(p^+, p^-, p^\perp) \sim Q(\lambda'^2, 1, \lambda')$ and ultrasoft fields with $p^\mu_{us} \sim Q(\lambda')^2$.

2. $\mathrm{SCET_{II}}$ ($\mu < \sqrt{Q\Lambda}$): $\lambda \sim \Lambda/E$.

   collinear fields with $(p^+, p^-, p^\perp) \sim Q(\lambda^2, 1, \lambda)$ and soft fields with $p^\mu_s \sim Q\lambda^2$.

   $\longrightarrow$ Power counting of the operators can be made consistent.

full QCD ψ, A^μ

$p^2 \sim Q^2$

collinear, ultrasoft, soft particles

$\mathrm{SCET_I}$ ξ, q_{us}, A^μ_n, A^μ_{us}, $\cdots$

$p^2 \sim Q\Lambda$

$\mathrm{SCET_{II}}$ collinear, soft particles

ξ, q_s, A^μ_n, A^μ_s, $\cdots$

Factorization

- matrix elements of four-quark operators $\stackrel{?}{=}$ product of current matrix elements

$$\langle \pi\pi|\overline{q}_1\Gamma_1 q_2 \cdot \overline{q}_3\Gamma_2 b|B\rangle \stackrel{?}{=} \langle\pi|\overline{q}_1\Gamma_1 q_2|0\rangle\langle\pi|\overline{q}_3\Gamma_2 b|B\rangle.$$

1. Naive factorization has been assumed with the argument of color transparency.
2. It can be proved at leading order in SCET.

- long-distance and short-distance physics separable?

In SCET, the decay amplitudes can be written as

$$A = \int d\omega d\eta dk_+ T(\omega) J(\omega,\eta,k_+)\mathcal{O}(\eta,k_+),$$

where T (J) is the Wilson coefficient in $\mathrm{SCET_I}$ ($\mathrm{SCET_{II}}$), and $\mathcal{O}$ is the four-quark operator.

1. This property is important in B decays.
2. We can consider higher-order corrections based on this.

Effective Lagrangian in $\mathrm{SCET_I}$

Collinear Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\xi\xi}^{(0)} &= \bar{\xi}\Big(in\cdot D + i\not{D}_{c\perp} W\frac{1}{\bar{n}\cdot\mathcal{P}}W^\dagger i\not{D}_{c\perp}\Big)\frac{\not{\bar{n}}}{2}\xi, \\
\mathcal{L}_{\xi\xi}^{(1)} &= (\bar{\xi}W)i\not{D}_{us}^{\perp}\frac{1}{\overline{\mathcal{P}}}W^\dagger i\not{D}_{c\perp}\frac{\not{\bar{n}}}{2}\xi \\
&+ (\bar{\xi}i\not{D}_{c\perp}W)\frac{1}{\overline{\mathcal{P}}}i\not{D}_{us}^{\perp}W^\dagger\frac{\not{\bar{n}}}{2}\xi, \\
\mathcal{L}_{\xi\xi}^{(2)} &= (\bar{\xi}W)i\not{D}_{us}^{\perp}\frac{1}{\overline{\mathcal{P}}}i\not{D}_{us}^{\perp}\frac{\not{\bar{n}}}{2}(W^\dagger\xi) \\
&\quad +(\bar{\xi}i\not{D}_{c\perp}W)\frac{1}{\overline{\mathcal{P}}}i\bar{n}\cdot D_{us}\frac{1}{\overline{\mathcal{P}}}\frac{\not{\bar{n}}}{2}W^\dagger i\not{D}_{c\perp}\xi.
\end{aligned}
$$

Ultrasoft-collinear Lagrangian

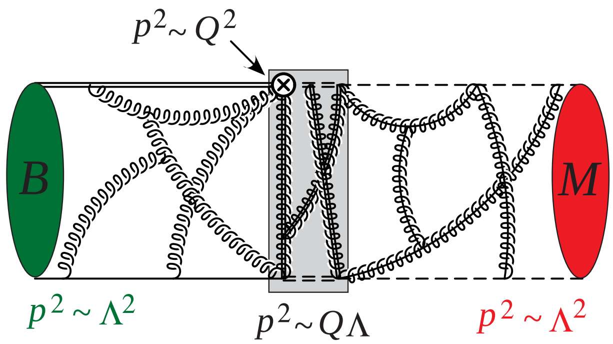
$$\mathcal{L}_{\xi q}^{(1)} = \overline{\xi}\Big(g\not{A}_{n\perp} - i\not{D}_{c\perp}\frac{1}{i\overline{n}\cdot D_c}g\overline{n}\cdot A_n\Big)q_{us} + \text{h.c.},$$

$$\mathcal{L}_{\xi q}^{(2)} = \overline{\xi}\frac{\not{\overline{n}}}{2}\Big(gn\cdot A_n + i\not{D}_{c\perp}g\not{A}_{n\perp}\Big)q_{us} - \overline{\xi}i\not{D}_{us}^{\perp}\frac{1}{i\overline{n}\cdot D_c}g\overline{n}\cdot A_n q_{us} + \text{h.c.}.$$

$$W = \sum_{\text{perm}} \exp\Big(-\frac{g}{\overline{\mathcal{P}}}\overline{n}\cdot A_n\Big).$$

Description of B decays in SCET

C. W. Bauer, D. Pirjol, I. W. Stewart, hep-ph/0303156.



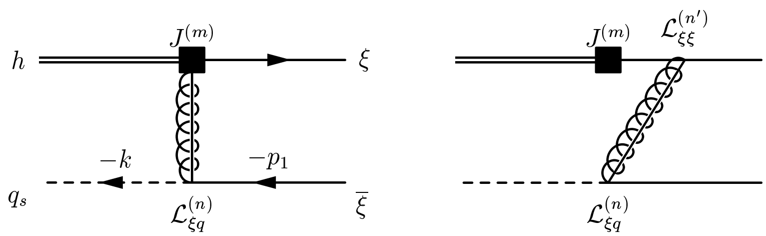
1. $\mathrm{SCET_I}$

- Construct heavy-to-light current operators in $\mathrm{SCET_I}$.

$$\overline{q}\Gamma b \quad \rightarrow \quad C_1 \overline{\xi} W \Gamma h + C_{2i}(\overline{\xi} W)(W^\dagger i D^\alpha_{n\perp} W)\frac{\Gamma_{\alpha i}}{\overline{\mathcal{P}}^\dagger} h$$

$$+ \quad C_{3i}(\overline{\xi} W)(W^\dagger i D^\alpha_{n\perp} W)\frac{\Gamma_{\alpha i}}{m_b} h + \cdots$$

- Compute the time-ordered products and integrate out the modes of order $p^2 \sim Q\Lambda$.



- Decouple the ultrasoft modes by the redefinition

$$\begin{aligned} \xi_{\rm II} &= Y^\dagger \xi, \ \ A_{n,{\rm II}} = Y^\dagger A_n Y, \\ Y(x) &= P\exp\Big(ig\int_{-\infty}^0 dsn\cdot A_{us}(ns+x)\Big), \end{aligned}$$

and evolve the operators in $\mathrm{SCET_{II}}$.

2. $\mathrm{SCET_{II}}$

- Construct gauge-invariant heavy-to-light current operators.

$$B\to D\pi : (\overline{h}_c\Gamma_h h_b)(\overline{\xi}W\Gamma_l W^\dagger\xi),$$

C. W. Bauer, D. Pirjol, I. W. Stewart, Phys. Rev. Lett. **87**, 201806 (2001); S. Mantry, D. Pirjol, I. W. Stewart, hep-ph/0306254.

$$B\to\pi\pi : \Big[(\overline{\xi}^u W)_\alpha (S^\dagger h)_\alpha\Big]_{V-A}\Big[(\overline{\chi}^d\overline{W})_\beta(\overline{W}^\dagger\chi^u)_\beta\Big]_{V-A}$$

J. Chay, C. Kim, hep-ph/0301055, hep-ph/0301262.

$$B\to K^*\gamma : \frac{em_b^2}{8\pi^2}\Big(\overline{\xi}W\Big)\not{\overline{n}}\not{A}(1+\gamma_5)S^\dagger h$$

J. Chay, C. Kim, Phys. Rev. D **68**, 034013 (2003).

- Collect all the terms at leading order in SCET.

(a) leading-operator contributions without spectator quarks
(b) nonfactorizable spectator contributions
(c) spectator contributions to the form factor

Factorized results

The decay amplitudes can be written as

$$A_i[\mathcal{O}] = T_i + N_i + F_i.$$

For example, in nonleptonic decays, they are given as

J. Chay, C. Kim, hep-ph/0301262.

$$\begin{aligned} T_i &= \int d\eta\ C^T_{\mathrm{eff},i}(\eta,\mu_0,\mu) \\ &\times \langle \overline{\xi} W \gamma_\mu (1-\gamma_5) S^\dagger h \cdot \overline{\chi}\overline{W}\delta(\eta - \mathcal{Q}_+)\gamma^\mu (1\mp\gamma_5)\overline{W}^\dagger \chi\rangle \\ &= \pm i f_{M2} 2E \int_0^1 du C^T_{\mathrm{eff},i}(u,\mu_0,\mu)\phi_{M2}(u,\mu) \\ &\times \langle M_1 | \overline{\xi} W \frac{\overline{\not n}}{2}(1-\gamma_5) S^\dagger h | \overline{B}\rangle \\ &= \pm i m_B^2 f_{M2} \int_0^1 du \zeta(\mu_0,\mu) C^T_{\mathrm{eff},i}(u,\mu_0,\mu)\phi_{M2}(u,\mu), \end{aligned}$$

$$
\begin{aligned}
N_i &= \int d^4x C^N_{\mathrm{eff},i} T[O_i^{(1a)}(x) + O_i^{(1b)}(x), i\mathcal{L}^{(1)}_{\xi q}(0)] \\
&= \int dudvdr_+ C^N_{\mathrm{eff},i}(\mu_0,\mu) J_i^N(u,v,r_+,\mu_0,\mu) \\
&\times \mathcal{N}_i f_B f_{M1} f_{M_2} \phi_{M1}(u,\mu)\phi_{M2}(v,\mu)\phi_B^+(r_+,\mu), \\
F_i &= \int dudvdr_+ C^F_{\mathrm{eff},i}(\mu_0,\mu) J_i^F(u,v,r_+,\mu_0,\mu) \\
&\times \mathcal{N}_i f_B f_{M1} f_{M2} \phi_{M1}(u,\mu)\phi_{M2}(v,\mu)\phi_B^+(r_+,\mu).
\end{aligned}
$$

- All the contributions can be written in a factorized form.

- The convolution integrals are finite at leading order.

Perspectives and conclusion

- Many types of B meson decays are proved to be factorized at leading order in SCET and to all orders in α_s.

- We may need subleading contributions.
  1. Operators at subleading order,
  2. Meson distribution amplitudes at subleading order.

- Argument for the need of new modes called "messenger modes" ($p^\mu \sim (\lambda^2, \lambda, \lambda^{3/2})$). T. Becher et al., hep-ph/0309227.
  1. Can we treat the modes with $p^2 \sim E\lambda^3$?
  2. We have some evidence that they should not be included.
  3. It can be the difference in the formalism in SCET.

- Power counting of various physical quantities in $\mathrm{SCET}_{\mathrm{II}}$.

- A new way of considering B decays is wide open from first principles.