A truly minimal left-right model of quark and lepton masses

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1. The most natural extension of the $G_{std} \equiv SU(3)_c \times SU(2)_L \times U(1)$ invariant standard model (SM) is to include left-right symmetry.

2. Left-right symmetry means that we have to include right handed gauge bosons W_R. The gauge symmetry then becomes $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$

3. Right handed symmetry spontaneously breaks to SM by the following symmetry breaking chain.
$$\begin{array}{rl} & G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \\ {}_{M_R}\!\!\longrightarrow & G_{std} \equiv SU(3)_c \times SU(2)_L \times U(1)_Y \\ {}_{M_z}\!\!\longrightarrow & G_{obs} \equiv SU(3)_c \times U(1)_{EM} \end{array}$$

4. Note that in low energy we can lebel all scalars and fermions by their QCD quantum number and their electric charges.

5. $\boxed{Q = T_L^3 + T_R^3 + \frac{B-L}{2}}$. Charge and color must be conserved.

6. Therefore, the standard Higgs choice is the following,
$$\begin{aligned} \Delta_L &= (3,1,2) \\ \Delta_R &= (1,3,-2) \\ \phi &= (2,2,0) \end{aligned}$$

7. Δ_R is color singlet and has a component with Q=0 namely
$$Q = 0 = 0 - 1 + 1$$
This will break G_{LR}, but keep G_{std} as well as G_{obs} intact

8. Similarly ϕ has two components with Q=0, namely,
$$Q = 0 = 1/2 - 1/2 + 0$$
$$Q = 0 = -1//2 + 1/2 + 0$$
This will break G_{LR} straight to G_{obs}. Note that ϕ has no color.

9. Therefore we must have $< \Delta_R > \quad >> \quad < \phi >$ to recover $M_R >> M_Z$. This will explain why the right handed gauge bosons W_R are not found by experiments.

1. It is much more tight a situation when we work with the fermions. This is because elementary scalars are still 'hypothetical' particles yet to be observed. Therefore we have some freedom in the choice of scalars. On the contrary all fermions, which are to be included, have already been observed in experiments.

2. Quarks and leptons transform under $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ as

$$
\begin{aligned}
q_L &\equiv \begin{pmatrix} u \\ d \end{pmatrix}_L \longrightarrow (3,2,1,1/3) \\
q_R &\equiv \begin{pmatrix} u \\ d \end{pmatrix}_R \longrightarrow (3,1,2,1/3) \\
l_L &\equiv \begin{pmatrix} \nu \\ e \end{pmatrix}_L \longrightarrow (1,2,1,-1) \\
l_R &\equiv \begin{pmatrix} N \\ e \end{pmatrix}_R \longrightarrow (1,1,2,-1)
\end{aligned}
\tag{1}
$$

3. Because $G_{LR} \subset SO(10)$ we can embed all fermions in a simple SO(10) spinor. This spinor is a 16-plet of SO(10).
$16 \supset \underbrace{(3,2,1,1/3)}_{q_L} + \underbrace{(3,1,2,1/3)}_{q_R} + \underbrace{(1,2,1,-1)}_{l_L} + \underbrace{(1,1,2,-1)}_{l_R}$
Note that we have not normalized $U(1)_{B-L}$ as a generator of SO(10)

4. Similarly Δ_L and Δ_R representations can be embedded in 126 and $\overline{126}$ representations of SO(10). And (2,2,0) scalar can be embedded in a 10-plet of SO(10). Therefore in this case where we include a 10-plet Higgs, the fermion masses are generated from the Yukawa coupling
$$h^{ij}\ 16_F^i \times 16_F^j \times 10_H$$

5. This means that in the conventional left-right symmetric case the fermions get Dirac masses at the tree level. Neutrino's however, get both Dirac type as well as Majorana type masses. The question is to what extent $\phi \equiv (2,2,0)$ is necessary ?

1. It was pointed out by Weinberg about 20 years ago that there exists a unique dimension-5 operator in standard model which can give a Majorana mass to the neutrino even if we do not have a right handed neutrino.

$$\mathcal{L} = \frac{f_{ij}}{\Lambda}(\nu_i \phi^0 - e_i \phi^+)(\nu_j \phi^0 - e_j \phi^+)$$

where the Majorana mass is generated by the VEV of the neutral component of ϕ. The mass is given by,

$$\mathcal{M}_{ij} = f_{ij}\frac{v^2}{\Lambda} \quad (2)$$

This also means that whatever be the underlying mechanism to generate the light Majorana neutrino mass, the mass has to be see-saw in character. The usual see-saw formula for the neutrino mass is well-known

$$\mathcal{M}_{light} = \frac{m_D^2}{M_R} \quad (3)$$

The formula in Equation(3) is more model dependent than that of Equation(2) in the sense that we have to have a mechanism to generate M_R, which is the Majorana mass of a right handed neutrino. In general it is obtained from a triplet Higgs by a coupling like,

$$M_R^{ij} \rightarrow f_M^{ij} \underbrace{l_R^i}_{2} \underbrace{l_R^j}_{2} \underbrace{\Delta_R}_{3}$$

We have given the $SU(2)_R$ quantum numbers in underbrace.

2. We will refer to Equation(2) as a SINGLE SEE-SAW.

3. Now suppose that the Dirac type masses are also generated via higher dimensional operators, then we can write a see-saw formula for the Dirac mass in a left-right symmetric model as,

$$m_D^{ij} \approx f_D^{ij}\frac{v_L v_R}{\Lambda_D} \quad (4)$$

This type of Dirac mass leads to DOUBLE SEE-SAW. We will explain all relevant operators in a left-right symmetric model next. Note that we will exclude the usual bi-doublet scalar.

1. The following higher dimensional operators exist if we exclude the bidoublet from a left-right symmetric model.

$$\begin{aligned} Majorana\ left &\rightarrow \frac{f_L^{ij}}{\Lambda_M} \ (l_{iL}\phi_L) \ \ (l_{jL}\phi_L) \\ Majorana\ right &\rightarrow \frac{f_R^{ij}}{\Lambda_M} \ (l_{iR}\phi_R) \ \ (l_{jR}\phi_R) \\ Dirac &\rightarrow \frac{f_D^{ij}}{\Lambda_D} \ (\overline{l_{iL}}\phi_L^*) \ \ (l_{jR}\phi_R) \end{aligned}$$

2. In this case we get left and right handed Majorana masses as,

$$\begin{aligned} M_L &= f_L^{ij}\frac{v_L^2}{\Lambda_M} \\ M_R &= f_R^{ij}\frac{v_R^2}{\Lambda_M} \end{aligned}$$

Therefore after diagonalizing the 6×6 mass matrix of the neutrinos, the light neutrino mass matrix of neutrinos is

$$M_{light}^{\nu} = M_L + m_D^T \frac{1}{M_R} m_D \qquad (5)$$

Using the expressions of M_L, M_R, m_D matrices,

$$M_{light}^{\nu} = f_L \frac{v_L^2}{\Lambda_M} + [\frac{v_L^2 v_R^2}{v_R^2}\frac{\Lambda_M}{\Lambda_D^2}] \ f_D^T \frac{1}{f_R} f_D \qquad (6)$$

Setting aside the Yukawa matrices, the order of magnitude of the neutrino mass is

$$M_{light}^{\nu} = \frac{v_L^2}{\Lambda_M} + \frac{v_L^2 \ \Lambda_M}{\Lambda_D^2} \qquad (7)$$

Note that even-if here we are ignoring the Yukawa matrices, they will in general give very interesting Physics of the neutrino mixing angles.

3. Taking $v_L \approx 100$ GeV, $\Lambda_M \approx M_P$, $\Lambda_D \approx M_{GUT} \approx 10^{16}$ GeV, we get $M_L \approx 10^{-15}$ GeV $\approx 10^{-6}$ eV. So we drop the first term in Equation(5).

1. Let us first get some order of magnitude estimates of the neutrino mass predicted in this scenario where,

$$M^{\nu}_{light} \approx \frac{v_L^2\ \Lambda_M}{\Lambda_D^2}$$

v_L/GeV	Λ_M/GeV	Λ_D/GeV	m_ν/eV
100	10^{19}	10^{16}	1
100	10^{18}	10^{16}	0.1
100	10^{18}	2×10^{16}	0.025
91	10^{18}	2×10^{16}	0.0207

2. Quark masses are given by (See: Eqn. (1)) the higher dimensional operator,

$$\begin{aligned} Quark\ mass &\rightarrow \frac{f_q^{ij}}{\Lambda_D}\ \ (\overline{q_{iL}}\ \phi_L^*)\ \ (q_{jR}\ \phi_R) \\ m^{ij}_{quark} &\approx f_q^{ij}\frac{v_L v_R}{\Lambda_D} \end{aligned} \tag{8}$$

Therefore because $v_L \approx 100$ GeV, $v_R \approx \Lambda_D \approx 10^{16}$ GeV. This means that $SU(2)_R \times U(1)_{B-L}$ is broken at a very high scale. At low energy we have just the standard model.

3. We do however have a right handed singlet neutrino whose mass is

$$M_N \approx \frac{v_R^2}{\Lambda_M} \approx 10^{13}\ \ \text{GeV} \tag{9}$$

This mass scale is very useful from the point of view of leptogenesis. This is because leptogenesis must occur after the inflation stops and the universe starts to re-heat.

4. Because $m_{top} \sim 174$ GeV, we must have $v_R \sim \Lambda_D$ one may wonder whether one can write $\frac{f_q^{ij}}{\Lambda_D}\ \ (\overline{q_{iL}}\ \phi_L^*)\ \ (q_{jR}\ \phi_R)$ as an effective operator? The answer is yes. We are giving an example.

1. Consider extra singlets U_L and U_R with an invariant mass $M_U\ \overline{U_L} U_R$ near the GUT scale. The quantum numbers are,

$$U_L, U_R \quad \rightarrow \quad (3, 1, 1, 4/3) \tag{10}$$

Then we can write a 2×2 mass matrix of the top quark linking $(\overline{t_L}, \overline{U_L})$ with (t_R, U_R) as

$$\begin{pmatrix} 0 & f_L\ v_L \\ f_R\ v_R & M_U \end{pmatrix}$$

This matrix has an eigenvalue

$$\begin{aligned} m_{top} &= \frac{f_L\ f_R\ v_L\ v_R}{M_U} [1 + \frac{(f_R v_R)^2}{M_U^2}]^{-1/2} \\ &\approx \frac{f_L\ f_R\ v_L\ v_R}{\sqrt{2}\ M_U} \\ &\approx \frac{f_L\ f_R\ v_L\ v_R}{\sqrt{2}\ \Lambda_D} \end{aligned}$$

2. **GAUGE COUPLING UNIFICATION**

Let us have supersymmetry and also add two new superfields, $\phi_L^c \rightarrow (1, 2, 1, -1)$ $\phi_R^c \rightarrow (1, 1, 2, -1)$. $SU(2)_R$ breaking happens at 10^{16} GeV, and at low energy we have simply MSSM with two HIggs doublets. Therefore it has a well known property that gauge couplings unify at 2×10^{16} GeV.