<u>**GEOMETRIC ANALYSIS AND PDE SEMINAR**</u>

September 2012 – August 2013   KIAS

Year 2013

1. On the Boltzmann equation in cosmology

Ho Lee (Kyung Hee Univ.)

[2013. 08. 22. 11:00-12:00 Room 1423]

Abstract. In this talk, we consider the Boltzmann equation in some cosmological settings. The Boltzmann equation describes the time evolution of matter distribution in the universe, where matter is treated as a collection of particles, for instance each galaxy in the universe can be considered as a particle. In the nonrelativistic case the Poisson equation is coupled to the Boltzmann equation in order to describe the gravitational field, and we obtain the Vlasov-Poisson-Boltzmann system. In the relativistic case we consider the Robertson-Walker spacetime and obtain the relativistic Boltzmann equation in the Robertson-Walker spacetime. In both cases we study the global existence and asymptotic behaviours of solutions.

2. Translating Monsters versus CMC surfaces

Hojoo Lee (KIAS)

[2013. 08. 02. 13:30-14:30 Room 1423]

Abstract. The recent decades admit intensive research devoted to the study of solitons [6] to the mean curvature flow ($\mathcal{H}$-flow for short). As known in [1, 7, 14, 15], there exist fascinating geometric dualities between the $\mathcal{H}$-flow solitons and minimal submanifolds. We say that a surface is the $\mathcal{H}$-flow *translator* [15] when its mean curvature vector field agrees with the normal component of a constant Killing vector field. Translators arise as Hamilton's convex eternal solutions and Huisken-Sinestrari's Type II singularities for the $\mathcal{H}$-flow. In our survey talk, we will sketch the construction of various translators, and explicitly describe that translators become natural generalization of classical objects: minimal surfaces and constant mean curvature surfaces.

References

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3. Self-Shrinker Solitons in Mean Curvature Flow

Niels Martin Møller (Princeton Univ.)

[2013. 08. 01. 13:30-15:30 Room 1424]

[2013. 08. 02. 14:30-16:30 Room 1423]

Abstract. The first lecture (1) will cover geometric preliminaries on Mean Curvature Flow (a geometric nonlinear heat equation quite analogous to the Ricci flow), and self-similar solutions to the flow: Maximum principles, monotonicity, parabolic blow-up and regularity. We will then describe some of the basic features of the soliton PDEs, with focus mostly on self-shrinkers and their role in modeling the finite time singularities of the flow (a good reference for preliminaries is Ecker's book [2]. See also [1] for an introduction, and one may consult some original sources [3], [4]), and relation to the theory of CMC and minimal surfaces. For graphs $(p, u(p))$, where $p \in \mathbb{R}^n$,

the nonlinear PDE for a self-shrinker is

$$(\dagger) \quad \operatorname{div}\left(\frac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = \frac{1}{2}\frac{p\cdot\nabla u - u}{\sqrt{1+|\nabla u|^2}},$$

for which the local theory is classical. The global geometry and analysis of complete, embedded hypersurfaces $\Sigma^n \subseteq \mathbb{R}^{n+1}$ (both compact and non-compact with ends), which locally solve (†), is thus the main theme of the lectures.

In the second lecture (2), an overview of the known examples of such hypersurfaces and some of the classification results will be given, and in particular some details from [9], which will serve to further emphasize the basic geometric properties of the self-shrinkers.

The two final lectures (3)-(4), will be a work-through of the gluing construction in [8] (subsidiarily [10]), which yields new complete, embedded, self-shrinkers Σ_g^2 of high genus g, in $\mathbb{R}^3$ (as conjectured from numerics by Tom Ilmanen, and others, in the early 90's), by fusing known low-genus examples. We will first cover some basic geometric and analytical generalities of such desingularization constructions (as in [5]– [7]), then concentrate on some specific features of self-shrinkers. For example, the analysis in the situation with non-compact ends is complicated by the unbounded geometry. Thus, Schrödinger operators with fast growth of the coefficients need to be understood well via geometric Liouville-type results, which amounts to a certain decomposition into a compact model space plus decaying remainders. This in turn enables the construction of the resolvent of the stability operator, and controlling the higher order terms in the nonlinear PDE, in appropriate weighted Hölder spaces.

References

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4. Solitons in geometric evolution problems

Kwangho Choi (Seoul Nat'l Univ.)

[2013. 07. 26. 13:30-15:30 Room 1423]

Abstract. Geometric flows as a solution of geometric evolution equations may have singularity at finite time or infinity. It is known that solitons are all possible singularity models near singularities of geometric flows. In this talk, we study the notion of soliton solutions in mean curvature/Ricci flows of surfaces.

5. Introduction to mean curvature flow

Hyunsuk Kang (KIAS)

[2013. 07. 26. 10:00-12:00 Room 1423]

Abstract. We review the basic theory and computation for the mean curvature flow on convex hypersurfaces in Euclidean spaces. Should time permit, those in Riemannian manifolds will be discussed.

6. On a multi-marginal optimal transport problem

Young-Heon Kim (Univ. of British Columbia)

[2013. 06. 20. 11:00-12:00 Room 1423]

Abstract. I will discuss an optimal transport problem arising when there are many mass distributions to match together in a cost efficient way, explaining a joint work with Brendan Pass. This talk will be introductory.

7. Generalized Calabi's correspondence

Hojoo Lee (KIAS)

[2013. 04. 16. 17:00-18:00 Room 7323]

Abstract. We introduce various generalizations of Calabi's correspondence between the minimal surfaces in the Euclidean three space and the maximal surfaces in the Lorentz three space.

8. Degeneration of hyper-elliptic curve

Jihye Seo (McGill university & CRM)

[2013. 04. 16. 16:00-17:00 Room 7323]

Abstract. At a generic place in moduli space, hyper-elliptic curves are regular and smooth. A 1-cycle may degenerate at complex codimension-1 locus, giving non-trivial monodromy around it. When two or more 1-cycles degenerate at the same time, the hyper-elliptic curve forms a cusp-like singularity. This is a great interest to theoretical physics community because it corresponds to electron and magnetic monopole becoming massless at the same time in Seiberg-Witten theory.

9. The limit behavior of Dirichlet problems defined on half-spaces

Minha Yoo (Seoul Nat'l Univ.)

[2013. 03. 28. 11:00-13:00 Room 1424]

Abstract. In this talk, we are going to investigate the limit behavior of the following Dirichlet problem,

$$\begin{cases} \Delta u(y) = 0 & \text{in } H(\nu, y_0), \\ u(y) = g(y) & \text{on } \partial H(\nu, y_0). \end{cases} \tag{9.1}$$

Here, $H(\nu, y_0)$ is a half-plane defined as $\{y \in \mathbb{R}^n : y \cdot \nu = y_0 \cdot \nu\}$ and $g(y)$ is a smooth function periodic in the y variable. We are going to prove that there exists a limit

$$\gamma(y') = \lim_{t\to\infty} u(y' + t\nu)$$

for each $y' \in \partial H(\nu, y_0)$ and such limits are independent on the choice of y', i.e., $\gamma(y') = \gamma(y_0)$ for all $y' \in \partial H(\nu, y_0)$. Moreover, the above limit behavior is also valid when the equation is given as follows,

$$\begin{cases} F(D^2u, y) = 0 & \text{in } H(\nu, y_0), \\ u(y) = g(y) & \text{on } \partial H(\nu, y_0), \end{cases} \tag{9.2}$$

as long as $F(M, y)$ is uniformly elliptic and periodic in the y variable. Finally, we are going to investigate the application of the above limit behaviors.

10. Variational methods and semilinear elliptic problem

Jinmyoung Seok (KIAS)

[2013. 03. 28. 10:00-11:00 Room 1424]

Abstract. In this talk, I will introduce a basic theory of calculus of variations and its applications to several kinds of semilinear elliptic PDE. Especially, I will focus on nonlinear Schródinger type equations, which arise from diverse areas of mathematical physics and differential geometry and will discuss about the existence of their solutions.

11. Introduction to contact manifolds and their invariants

Otto van Koert (Seoul Nat'l Univ.)

[2013. 01. 15. 17:00-18:00 Room 1423]

Abstract. In this talk we give an introduction to contact geometry and topology. We explain how contact manifolds appear naturally in relation with complex and symplectic manifolds. We also indicate that contact manifolds are flexible in the sense that their automorphism group is large, yet that they also exhibit some rigidity: their global geometry cannot be completely determined by topological means. We give example of this phenomenon by means of contact homology, an invariant of contact manifolds.

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12. Minimal arc presentations of some nonalternating knots or links

Hwa Jeong Lee (KAIST)

[2012. 12. 18. 16:00-17:00 Room 1424]

Abstract. A knot or link L can be embedded in a book with finitely many half planes in $\mathbb{R}^3$ so that each half plane intersects $\mathcal{L}$ in a single arc. We called such an embedding 'arc presentation' of $\mathcal{L}$. The minimal number of half planes among all arc presentations of a link $\mathcal{L}$ is called the arc index of $\mathcal{L}$. It is known that the arc index of alternating knots is the minimal crossing number plus two and that the arc index of prime nonalternating knots is less than or equal to the minimal crossing number. In this talk, we construct minimal arc presentations of some nonalternating knots or links such as Pretzel knots and Montesinos links. Also we compute arc index of them.

13. Conservation laws for CMC surfaces

Sung Ho Wang (KIAS)

[2012. 12. 07. 14:00-15:00 Room 1424]

Abstract. Three different notions of symmetry are introduced for CMC surfaces in a three dimensional space forms. We determine the space of conservation laws, and propose some applications.

14. Eigenvalue Estimates for the generalized Dirac operators with Torsion

Hwajeong Kim (Hannam Univ.)

[2012. 11. 23. 14:00-15:00 Room 1424]

Abstract. In this talk, we study eigenvalue estimates for the Dirac operator on compact Riemannian spin manifolds equipped with a metric connection, the torson of which is non zero. An optimal lower bound for the first eigenvalue of the Dirac operator is found, which generalizes the classical Riemannian estimates. We also determine Killing equations with torsion and discuss the minimum case in the bound.

15. Normal form reduction for unconditional well-posedness of canonical dispersive equations

Soonsik Kwon (KAIST)

[2012. 11. 20. 14:00-15:00 Room 1424]

Abstract. In the well-posedness theory for canonical dispersive equations, harmonic analytic methods, such as Strichartz space or Bourgain's $X^{s,b}$ space, have been extensively used and were successful. However, as one used auxiliary function spaces in the fixed point argument, we know the uniqueness only in the intersection of the auxiliary space and C^s_{tH}. We reprove the local well-posedness with improvement in the aspect of uniqueness via a rather elementary method. We adopt a classical ODE technique, Poincare-Dulac normal form reduction, to infinite dimensional setting and construct the unique solution in C^s_{tH}. I will discuss recent results on mKdV, NLS.

16. c-Isoperimetric Mass

Leobardo Rosales (KIAS)

[2012. 10. 19. 16:00-17:00 Room 1424]

Abstract. We introduce a new mass for currents, through which we ask questions analogous to Plateau's problem of finding the surface with least area given a boundary curve. Connections are drawn to work done on the thread problem by Ecker.

17. Heat Content Asymptotics

Peter Gilkey (Univ. of Oregon)

[2012. 09. 26. 16:00-17:00 Room 1423]

Abstract. Let (M, g) be a compact Riemannian boundary with smooth boundary. Let ϕ denote the initial temperature of the manifold; impose suitable boundary conditions (Dirichlet, Neumann, Robin etc.) to determine the subsequent evolution and define the subsequent temperature $u(x;t)$ on the manifold. Let ρ be the specific heat of the manifold and let $E(t) := \int_M u(x,t)\rho(x)|dvol(x)|$ be the total heat energy content of the manifold. If ϕ and ρ are smooth, there is a complete asymptotic series as $t \downarrow 0$ of the form $E(t) \sim \sum_{n=0}^{\infty} \beta_n t^{n/2}$ where the β_n are certain local invariants of $\{\phi, \rho\}$ and the geometry of the manifold and the boundary condition imposed; if $\{\phi, \rho\}$ are singular near the boundary, there are similar series where the power of t is shifted appropriately. We survey some recent work concerning these invariants. We present combinatorial formulas in several contexts, examine growth estimates, and present results with singular initial data.