Frequency-dependent noise: Charge fluctuations

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(Example of a zero-frequency measurement in the presence of ac-excitation)

Frequency-dependent noise spectra

$$S_{II}(\omega) = ?$$

$$(1/2)\langle I(\omega)I(\omega')+I(\omega')I(\omega)\rangle = 2\pi S_{II}(\omega)\delta(\omega+\omega')$$

Statistical effects

$$f(E)(1 - f(E \pm \hbar\omega))$$

visible when $\hbar\omega >> kT$

S. R. Yang, Solid State Comm. 81, 375 (1992)

M. Buttiker, PRB 45, 3807 (1992)

Intrinsic time-scales

Capacitances, (kinetic)-inductances, RC-times

Energy dependence of scattering matrix

Fundamental:

Experimentally difficult to access: essentially only zero-frequency measurements

Example: charge fluctuations $\Longleftrightarrow$ dephasing $\Longleftrightarrow$ conductance

Example: excite system at Ω measure noise at $\omega \simeq 0$

Dynamic potentials

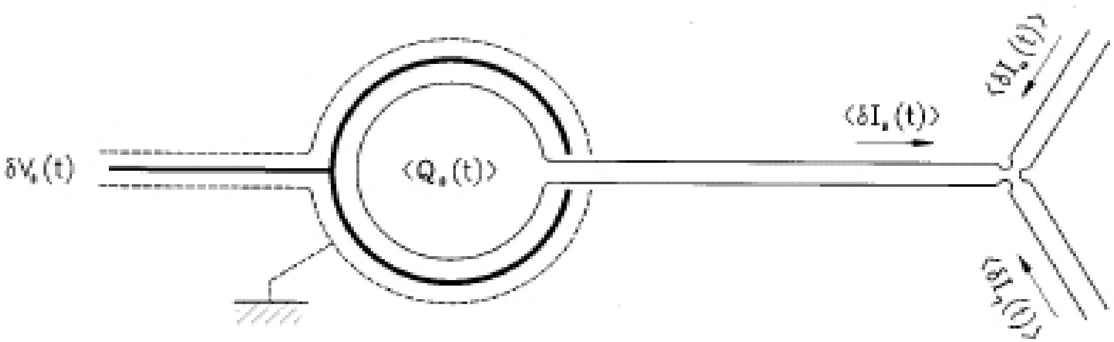
Buttiker, Pretre, Thomas, Phys. Rev. Lett. 70, 4114 (1993)

Linear response to oscillating voltages

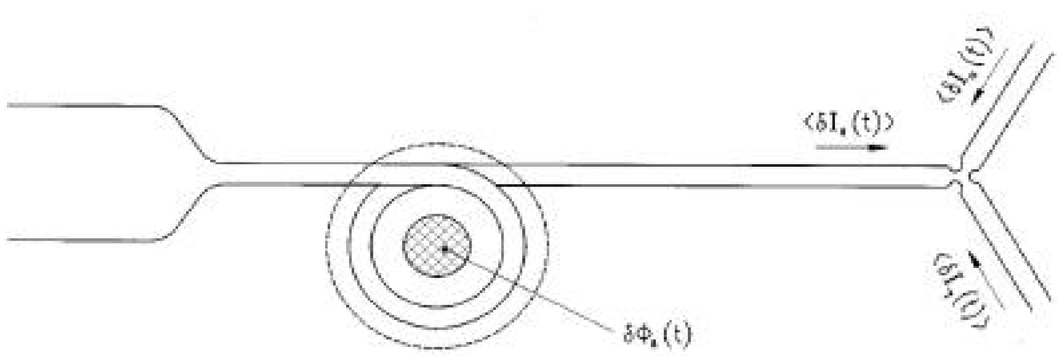
Distinguish:

potentials applied to terminals $dV_\alpha(t) = dV_\alpha(\omega) e^{-i\omega t}$

self-consistent electrostatic potential $dU(\omega, \mathbf{r}) e^{-i\omega t}$



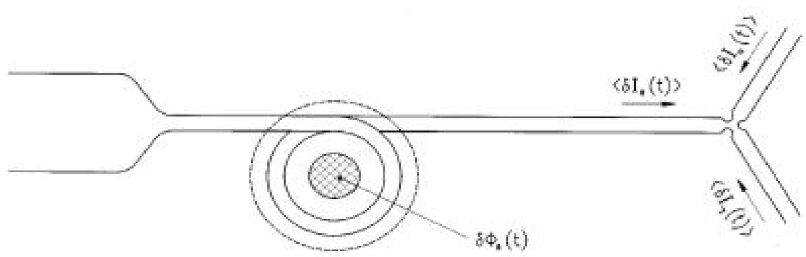
$$H_I = \sum_\alpha Q_\alpha dV_\alpha$$



$$H_I = \sum_\alpha I_\alpha d\Phi_\alpha$$

Response to external potentials

Buttiker, Pretre, Thomas, Phys. Rev. Lett. 70, 4144 (1993)



$$\phi_\alpha(E,t) = \phi_\alpha^+(E)e^{-iEt/\hbar} + c_\alpha \phi_\alpha^+ e^{-iE_+t/\hbar} - c_\alpha \phi_\alpha^+ e^{-iE_-t/\hbar}$$

$$E_\pm = E \pm \hbar\omega \ , \quad c_\alpha = eV_\alpha/\hbar\omega$$

$\widehat{a}_\alpha(E)$; incident state $\qquad \widehat{a}'_\alpha(E)$; reservoir

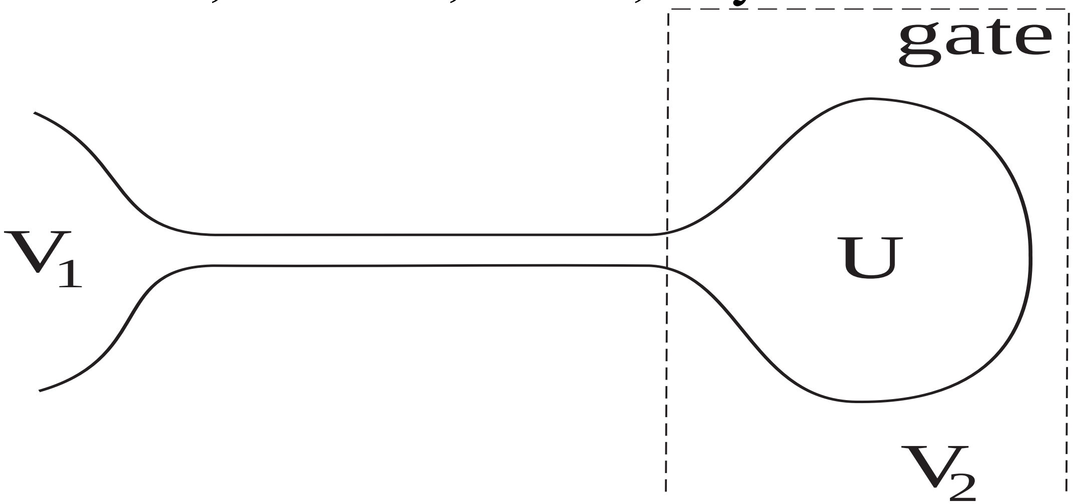
$$\widehat{a}_\alpha(E) = \widehat{a}'_\alpha(E) - c_\alpha \widehat{a}'_\alpha(E_+) + c_\alpha \widehat{a}'_\alpha(E_-)$$

$$\Longrightarrow$$

$$G^{ext}_{\alpha\beta}(\omega) = \frac{e^2}{h} \int dE Tr[A_{\beta\beta}(\alpha, E, E+\hbar\omega)] \frac{f_\beta(E) - f_\beta(E+\hbar\omega)}{\hbar\omega}$$

Mesoscopic Capacitor

Buttiker, Thomas, Pretre, Phys. Lett. A 180, 364 (1993)



single potential U

geometrical capacitance C

$$G^{ext}_{\alpha\beta}(\omega) = \frac{e^2}{h}\int dE\, Tr[A_{\beta\beta}(\alpha, E, E+\hbar\omega)]\frac{f_\beta(E) - f_\beta(E+\hbar\omega)}{\hbar\omega}$$

$$A_{\beta\beta}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} - s^\dagger_{\alpha\beta}(E')s_{\alpha\beta}(E)$$

Internal response

$$G^{ext} dV_1 + i\omega\Pi dU = -i\omega C(dU - dV_2)$$

Invariance under arbitrary potential shift $\Longrightarrow$ $i\omega\Pi = G^{ext}$

$$G^{-1}(\omega) = (-i\omega C)^{-1} + (G^{ext}(\omega))^{-1}$$

Mesoscopic Capacitor

Buttiker, Thomas, Pretre, Phys. Lett. A180, 364 (1993)

$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + ..$$

electrochemical capacitance

$$C_\mu^{-1} = C^{-1} + (e^2 Tr[N])^{-1}$$

charge relaxation resistance

$$R_q = \frac{h}{2e^2}\frac{Tr[N^\dagger N]}{(Tr[N])^2}$$

Eigen channels of s; $\exp(i\phi_n)$; $n = 1, 2, ,$ $\Longrightarrow$

$$Tr[N] = \frac{1}{2\pi i} Tr[s^\dagger \frac{ds}{dE}] = \frac{1}{2\pi}\sum_n \frac{d\phi_n}{dE}$$

$$Tr[N^\dagger N] = (\frac{1}{2\pi})^2 Tr[\frac{ds^\dagger}{dE}\frac{ds}{dE}] = (\frac{1}{2\pi})^2 \sum_n (\frac{d\phi_n}{dE})^2$$

$$R_q = \frac{h}{2e^2}\frac{\sum_n (d\phi_n/dE)^2}{(\sum_n d\phi_n/dE)^2}$$

Universal for n =1;

$$R_q = \frac{h}{2e^2}$$

Charge relaxation resistances

$$R_q = \frac{h}{2e^2}\frac{\sum_n (d\phi_n/dE)^2}{(\sum_n d\phi_n/dE)^2}$$ Universal for n =1; $$R_q = \frac{h}{2e^2}$$

For k degenerate channels

$$R_q = \frac{h}{2e^2}\frac{\sum_n (d\phi_n/dE)^2}{(\sum_n d\phi_n/dE)^2} = \frac{h}{2e^2}\frac{k}{k^2} = \frac{h}{2ke^2}$$

Spin less electrons $R_q = h/2e^2$

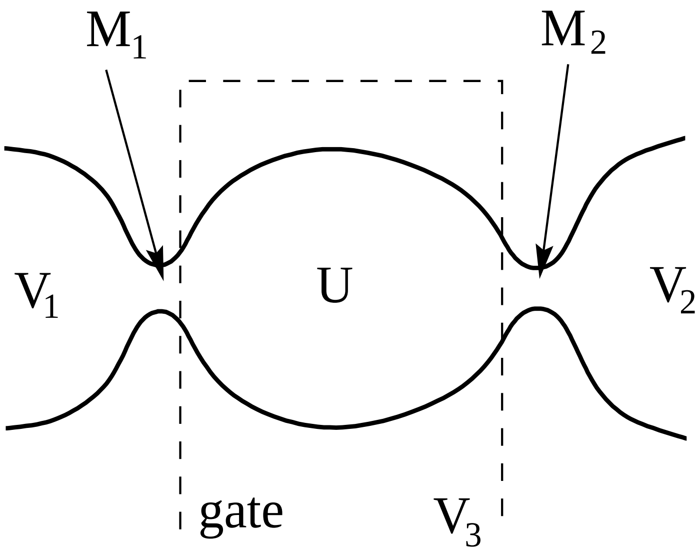
Spin degenerate channel $R_q = h/4e^2$

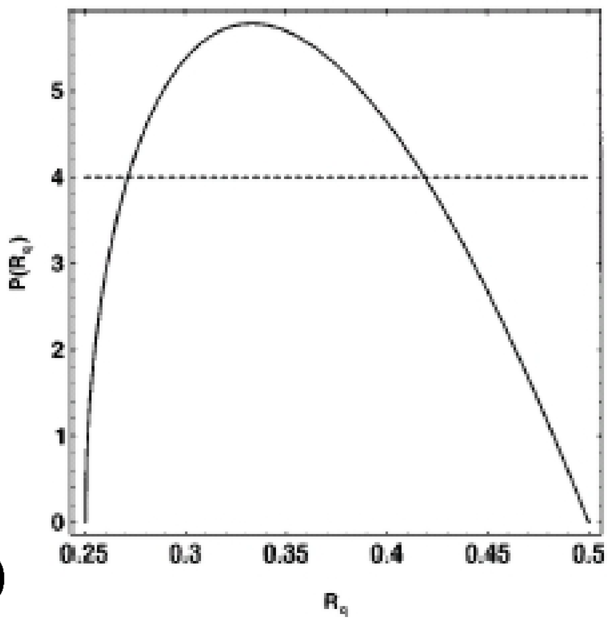
Ideally coupled Carbon Nanotube $R_q = h/16e^2$

Chaotic cavity coupled to two QPC (single channel limit) $P(R_q)$

Charge relaxation resistance distributions

Pedersen, van Langen, Buttiker, PRB57, 1838 (1998)





$$M_1 = M_2 = 1; V_1 = V_2 = V_3 = 0$$

______ orthogonal --------- unitary

Large channel limit:

$$R_q = \frac{h}{2e^2}\frac{1}{M_1 + M_2} \qquad R = \frac{h}{e^2}\left(\frac{1}{M_1} + \frac{1}{M_2}\right)$$

parallel series

Thermal charge fluctuations of a capacitor

$$(1/2)\langle I(\omega)I(\omega')+I(\omega')I(\omega)\rangle = 2\pi S_{II}(\omega)\delta(\omega+\omega')$$

Fluctuation-Dissipation Theorem

$$S_{II}(\omega) = \omega^2 S_{QQ}(\omega) = 2kT Re[G(\omega)]$$

with

$$G(\omega) = -i\omega C_\mu + \omega^2 C_\mu^2 R_q + .. \quad \Longrightarrow$$

Charge fluctuation spectrum

$$S_{QQ}(\omega) = 2kT C_\mu^2 R_q + ..$$

Thermal charge fluctuations of a capacitor

$$\hat{I}_\alpha(t) = \frac{e}{h}\int dE' dE \hat{a}^\dagger_\alpha(E') A_{\alpha\alpha}(\alpha, E', E)\hat{a}_\alpha(E) e^{i(E'-E)t/\hbar}$$

$$\hat{I}_\alpha(\omega) = \frac{e}{h}\int dE \hat{a}^\dagger_\alpha(E) A_{\alpha\alpha}(\alpha, E, E+\hbar\omega)\hat{a}_\alpha(E+\hbar\omega)$$

$$A_{\alpha\alpha}(\alpha, E, E+\hbar\omega) = 1_\alpha - s^\dagger_{\alpha\alpha}(E) s_{\alpha\alpha}(E+\hbar\omega)$$

$$A_{\alpha\alpha}(\alpha, E, E+\hbar\omega) = 2\pi i N \hbar\omega + ...$$

$$N = \frac{1}{2\pi i} s^\dagger \frac{ds}{dE}$$

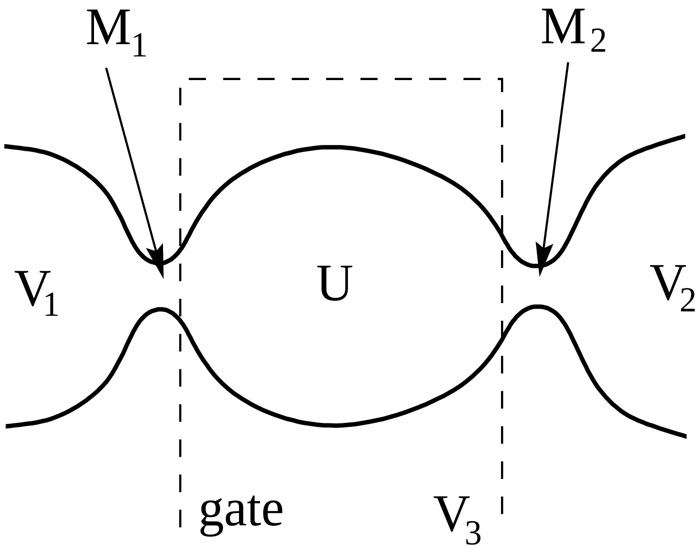
$$\hat{Q}^{ext}(\omega) = e\int dE\, \hat{a}^\dagger(E)\, N\, \hat{a}(E+\hbar\omega)$$

$$d\hat{I} = -i\omega\hat{Q}_{ext}(\omega) + i\omega\Pi d\hat{U} = -i\omega C d\hat{U}$$

$$\hat{Q} = \hat{Q}^{ext} - \Pi d\hat{U} = C\, d\hat{U}$$ screened charge $\Longrightarrow$

$$S_{QQ}(\omega) = 2kTC_\mu^2 R_q + ..$$

Shot noise induced charge fluctuations on a gate



$$V \equiv V_1 - V_2 > 0; kT = 0$$

zero-frequency shot noise

$$S_{II} = 2\frac{e^2}{h}|eV| \sum T_n(1-T_n)$$

What will we see at the gate?

Charge fluctuation sensor

generalized « Wigner-Smith matrix »

$$N_{\gamma\delta} = -\frac{1}{2\pi i} s^{\dagger}_{\alpha\gamma} \frac{ds_{\alpha\delta}}{deU} \qquad R_V = \frac{h}{2e^2} \frac{Tr[N_{12}N_{21}]}{(Tr[N])^2}$$

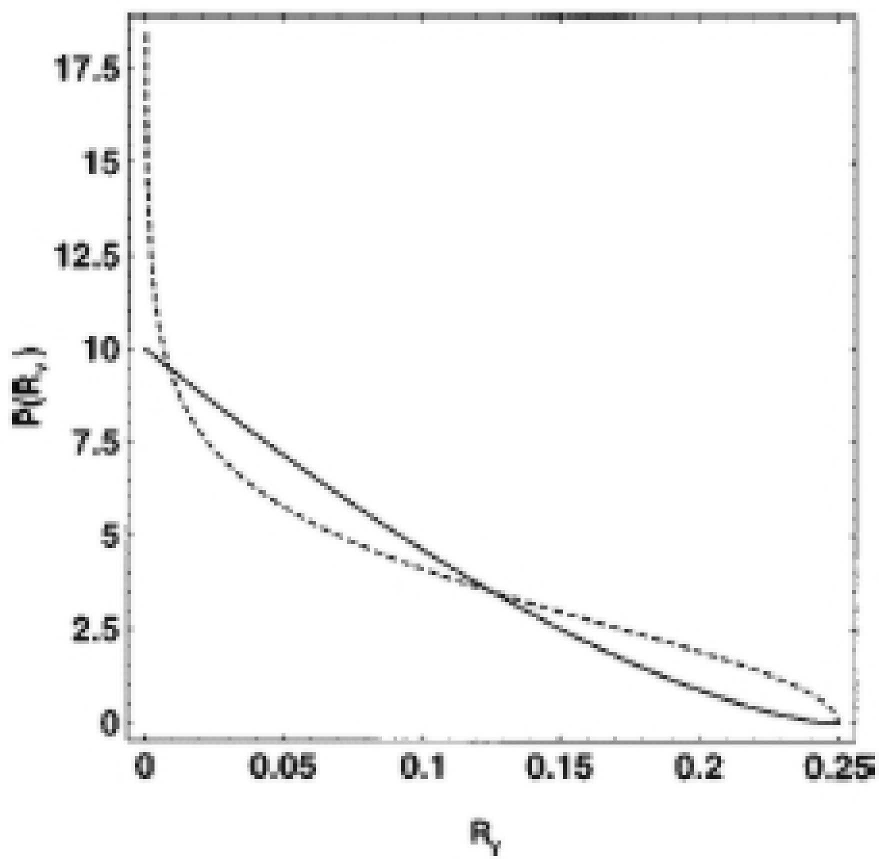
$$S_{I_3 I_3}(\omega) = \omega^2 S_{QQ}(\omega) = 2\omega^2 C_\mu R_V eV$$

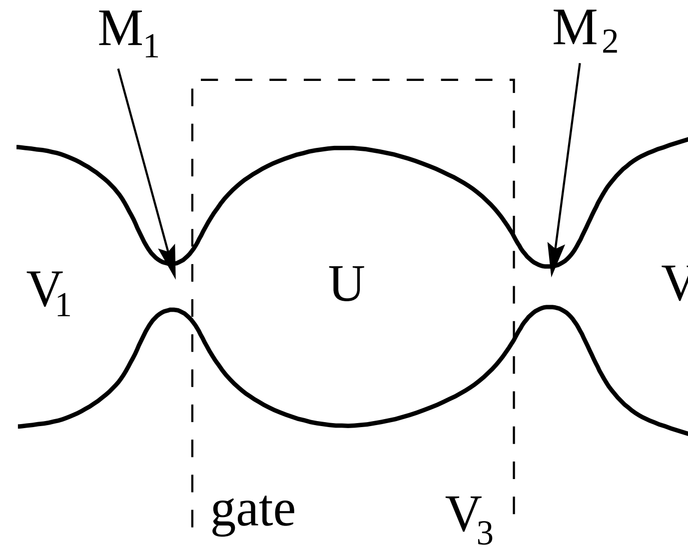
Pedersen, van Langen, Buttiker, PRB 57, 1838 (1998)

Shot-noise induced charge fluctuations

Pedersen, van Langen, Buttiker, PRB 57, 1838 (1998)

$$R_V = \frac{h}{2e^2}\frac{Tr[N_{12}N_{21}]}{(Tr[N])^2}$$





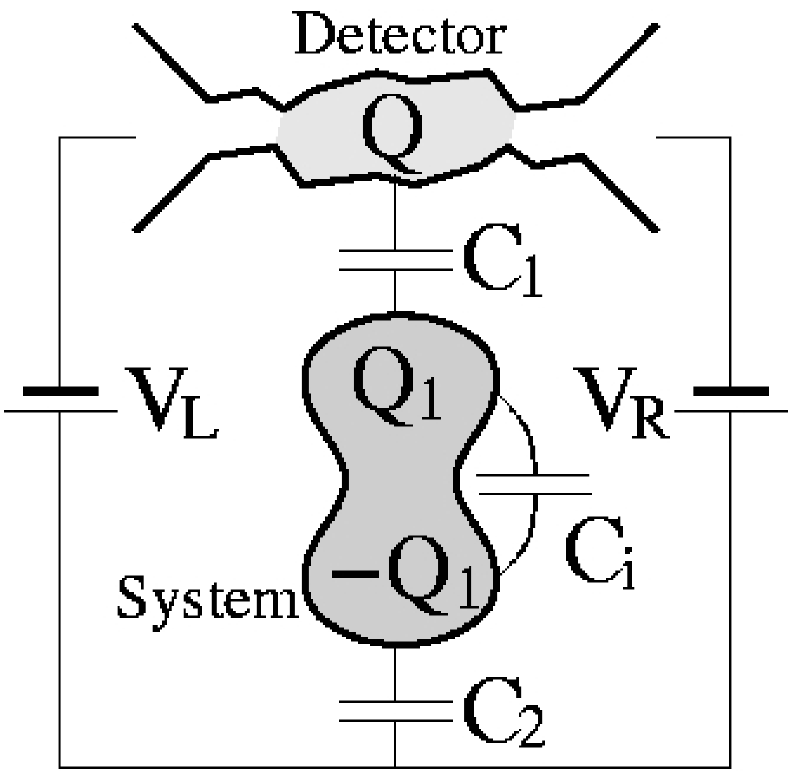
$$M_1 = M_2 = 1$$

--- orthogonal

__ unitary

Mesoscopic detectors

Pilgram and Buttiker, PRL 89, 200401 (2002)



$$D = e^2 Tr[N]$$

$$C_\mu^{-1} = C^{-1} + D^{-1}$$

$$R_q = \frac{1}{2}\frac{Tr[N^2]}{(Tr[N])^2}$$

$$R_v = \frac{1}{2}\frac{Tr[N_{21}N_{21}]}{(Tr[N])^2}$$

$$R_m = \frac{1}{4\pi^2}\frac{(\sum \frac{dT_n}{dU})^2}{(Tr[N])^2(\sum R_n T_n)}$$

$$\Gamma_{rel} = 2\pi\frac{\Delta^2}{\Omega^2}\left(\frac{C_\mu}{C_i}\right)^2 R_q \frac{\Omega}{2} coth\frac{\Omega}{2kT}$$

$$\Gamma_{dec} = 2\pi\frac{e^2}{\Omega^2}\left(\frac{C_\mu}{C_i}\right)^2 (R_q\, kT + R_v e|V|) + \Gamma_{rel}/2\,,$$

$$\Gamma_m = 2\pi\left(\frac{C_\mu}{C_i}\right)^2 R_m e|V|\,,$$

Related work:
Clerk, Girvin, Stone, PRB (2002)

Mesoscopic detectors

Buttiker and Pilgram, Surface Science 532, 617 (2003)

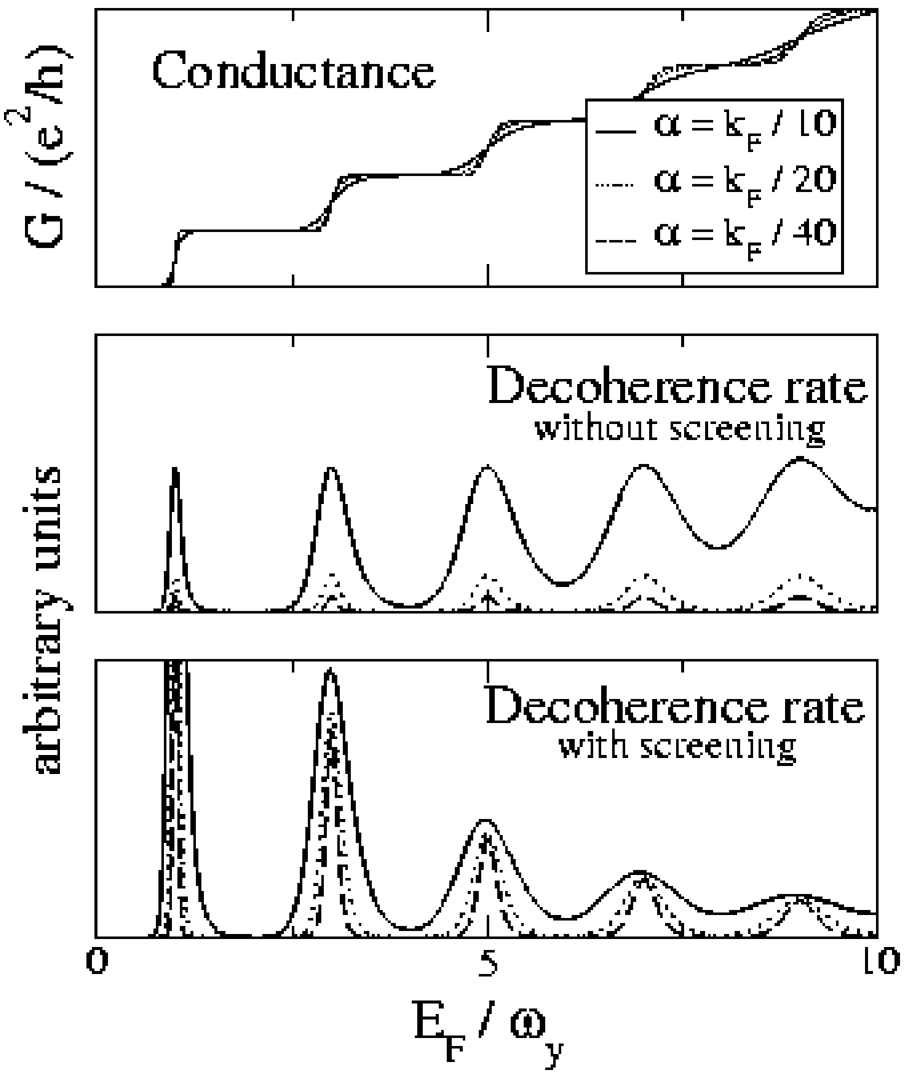
Quantum Point Contact

$$V(x,y) = X(x) + Y(y)$$

$$X(x) = \frac{V_0}{cosh^2(\alpha x)}$$

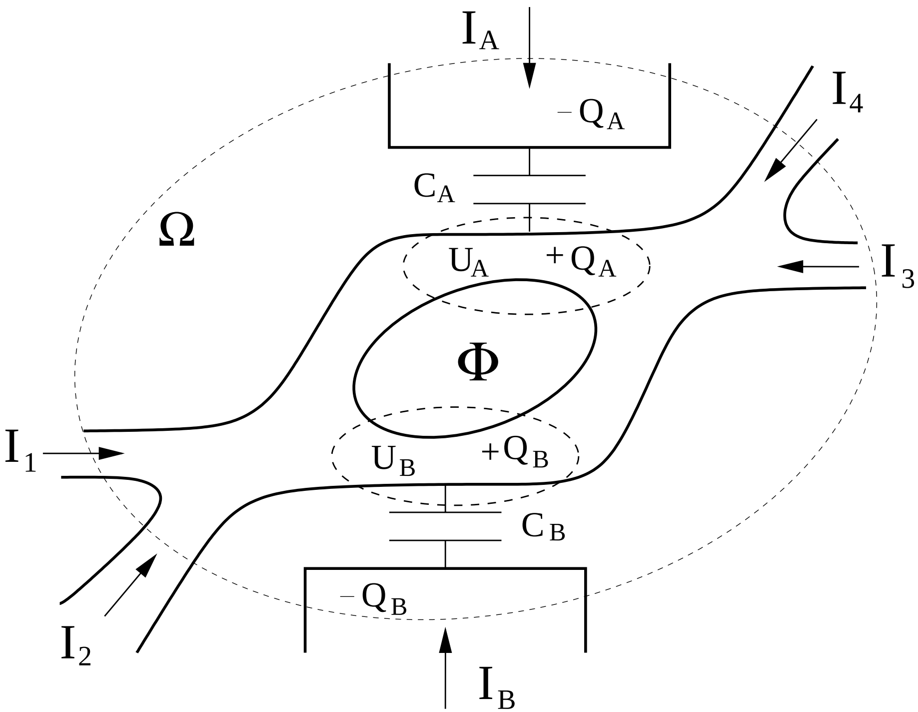
$$Y(y) = \frac{1}{2} m \omega_y y^2$$

$$\Gamma_{dec} \propto R_V \, |eV|$$



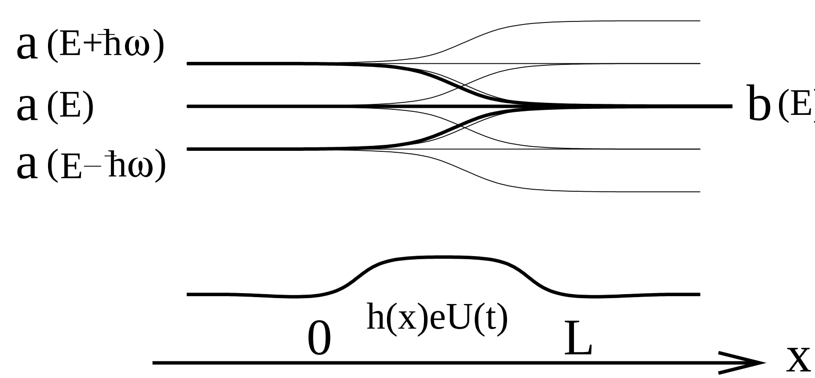
Charge fluctuation induced dephasing in ballistic interferometers

Seelig and Buttiker, PRB 64, 245 313 (2001).



$$g_G = \frac{1}{\left(1 + e^2 D_G / C_G\right)^{1/2}}$$

$$s_{31}(E + \hbar\omega, E)$$



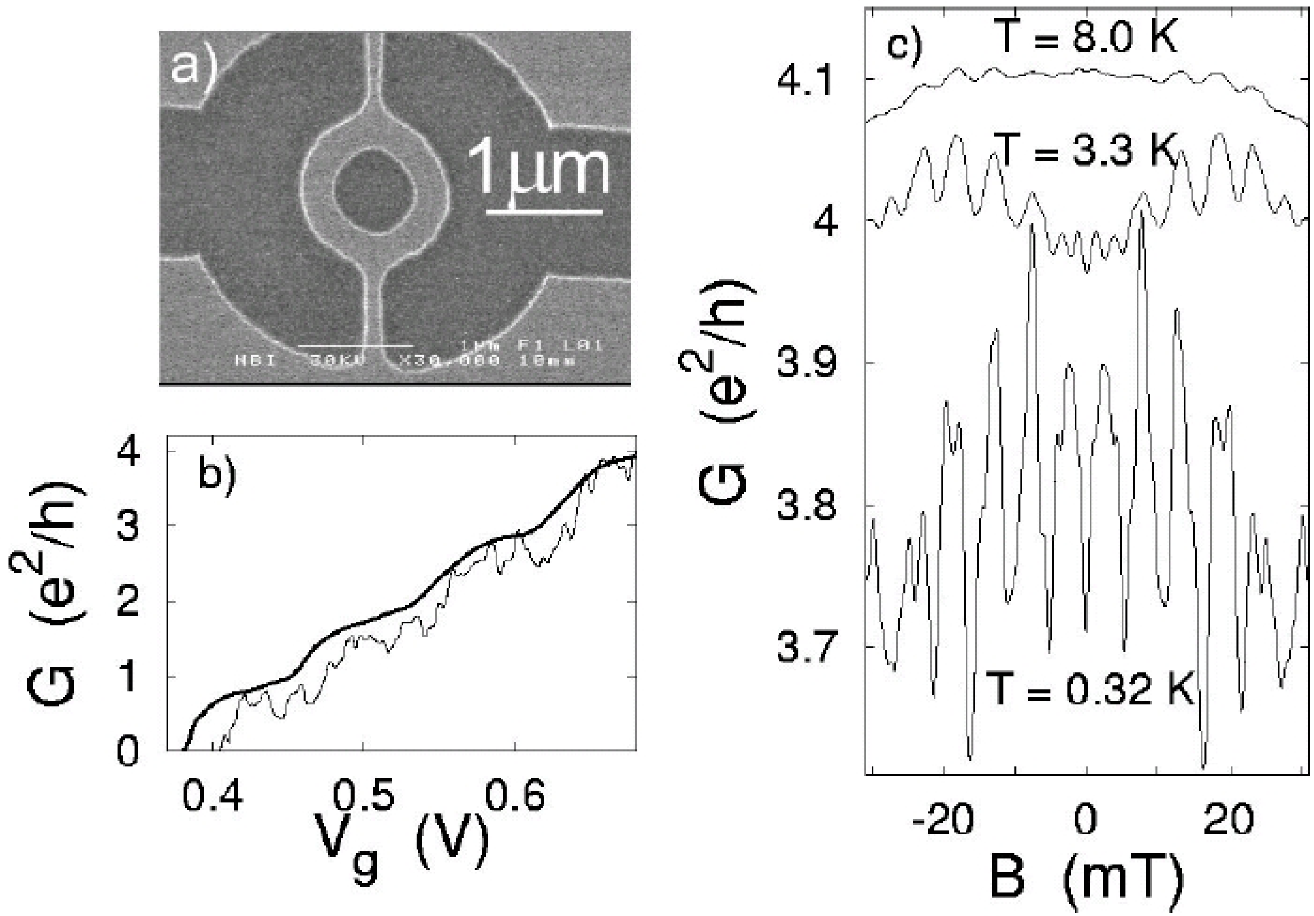
$$T_{31} = \frac{1}{2}[1 + \cos(\Theta(E) - 2\pi\Phi/\Phi_0)]$$

$$T_{31} = \frac{1}{2}[1 + \exp(-\tau/\tau_\phi)\cos(\Theta(E) + \delta\Theta - 2\pi\Phi/\Phi_0)]$$

$$\Gamma_\phi = \left(\frac{2e^2}{3\hbar^2}\right)\left(\frac{C_{\mu,A}^2}{C_A^2}\right) kTR_q = \left(\frac{\pi}{3}\right)\frac{kT}{\hbar}(1 - g^2)^2.$$

Mesoscopic decoherence in Aharonov-Bohm rings

A. E. Hansen et al. PRB 64, 045327 (2001)



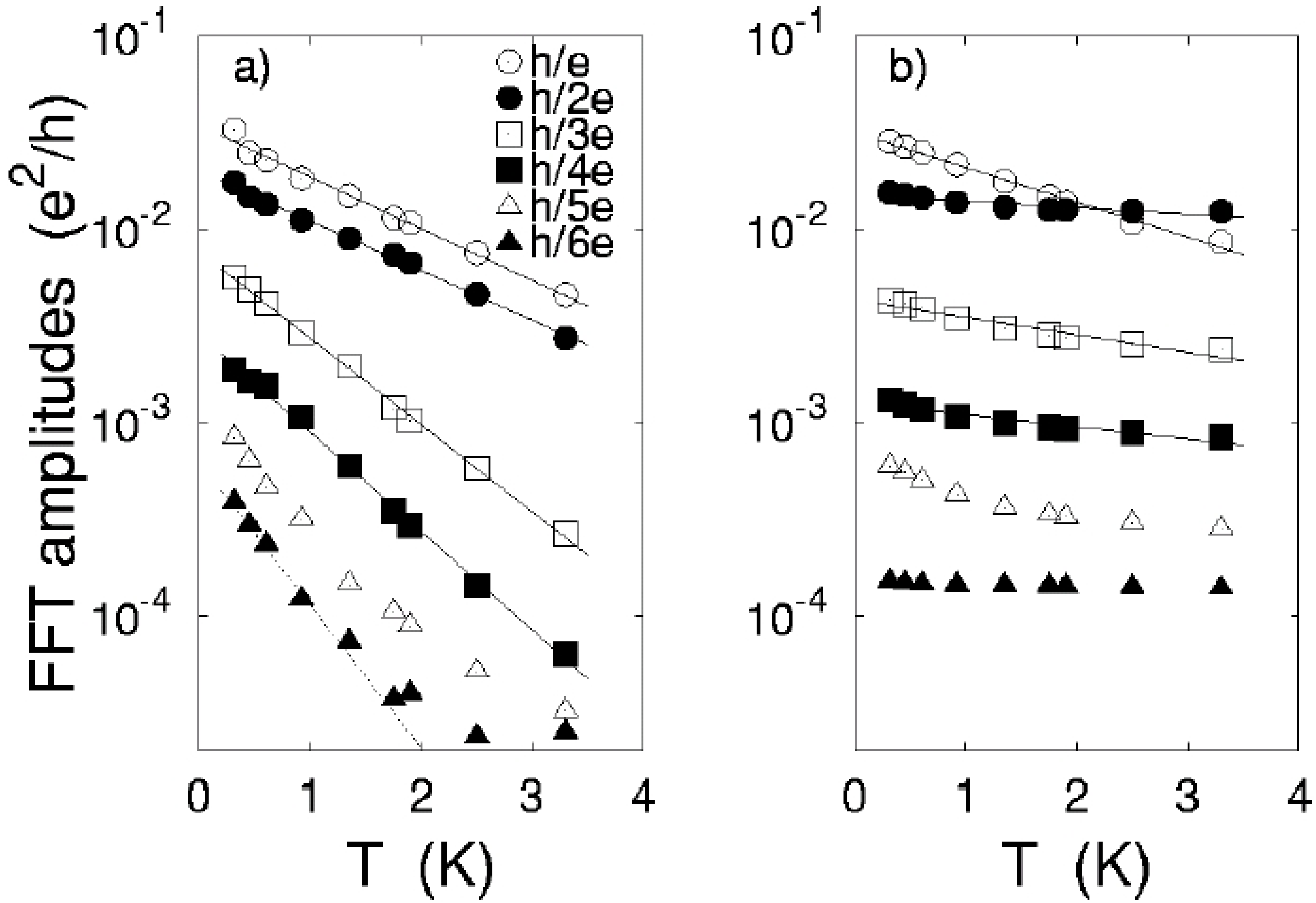
Mesoscopic decoherence in Aharonov-Bohm rings

A. E. Hansen et al. PRB 64, 045327 (2001)

$$\Gamma_\phi = 0.4\frac{kT}{\hbar}$$

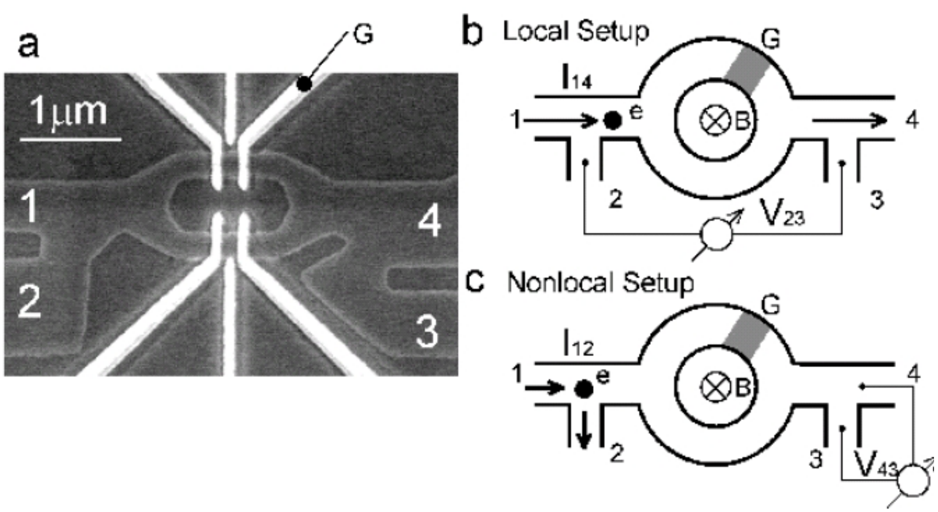
depahsing only

thermal averaging included

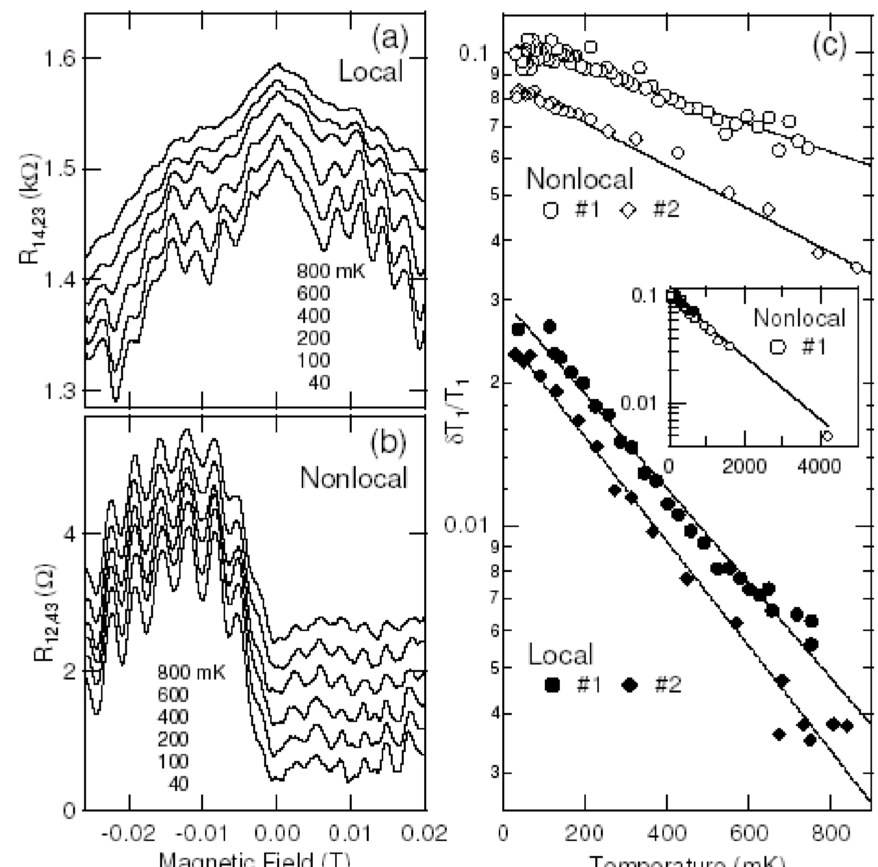


Local and non-local dephasing

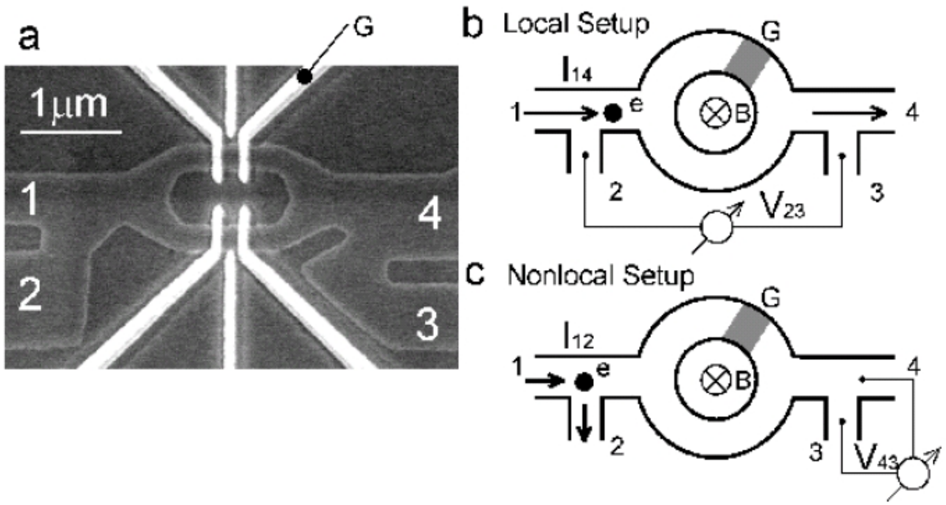
K. Kobayashi et al., J. Phys. Soc. Jpn. 71, 2094 (2002)



$$R_{\alpha\beta,\gamma\delta} = (V_\gamma - V_\delta)/I_{\alpha\beta}$$



Local and non-local dephasing



Current probes:

$$\Delta V_\alpha = 0$$

Voltage probes

$$\Delta I_\alpha = 0$$

$$\Delta Q_i = C\Delta U_i = \Delta Q_i^b - e^2 D U_i + e^2 \sum_{\alpha, I_\alpha = 0} D_\alpha^{(i)} \Delta V_\alpha .$$

$$\Delta I_\alpha = \Delta I_\alpha^b + \sum_\beta G_{\alpha\beta} \Delta V_\beta .$$

Local and non-local dephasing

Seelig, Pilgram, Jordan, Buttiker, PRB 68, 161310 (2003).

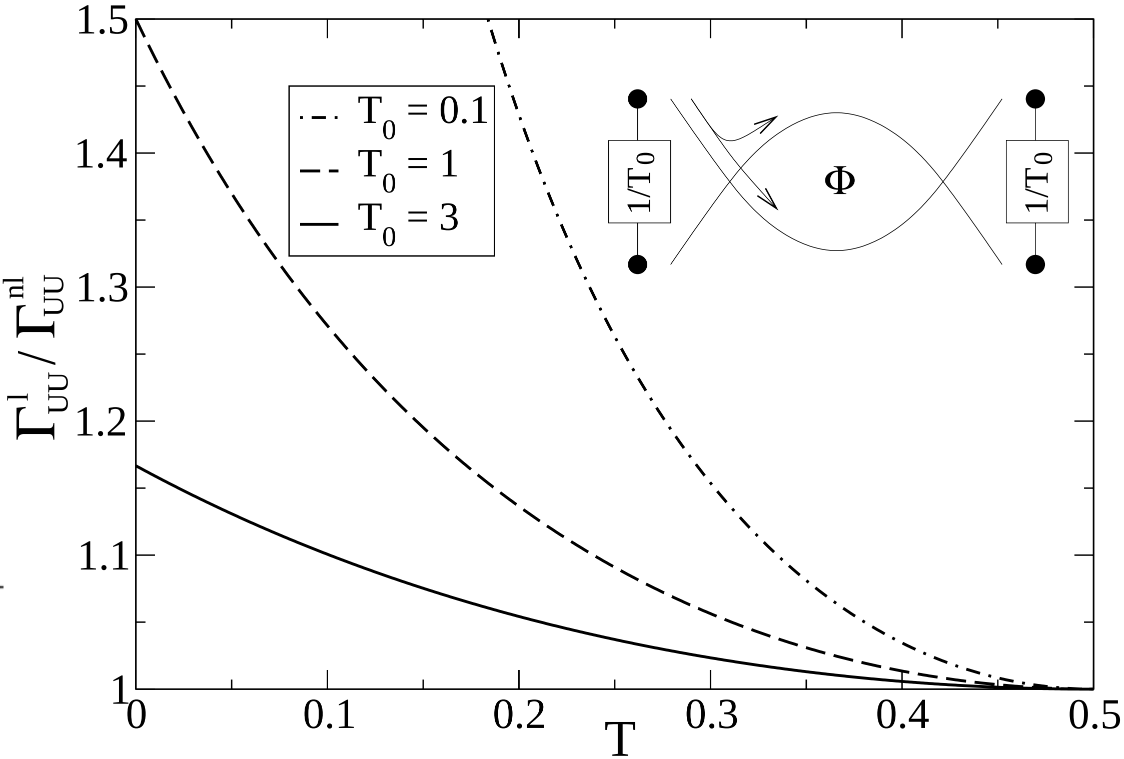
$$\Gamma_\phi^l = \gamma_\phi^0 + \gamma_\phi^l$$

$$\Gamma_\phi^{nl} = \gamma_\phi^0 + \gamma_\phi^{nl}$$

$$\gamma_\phi^0 = \frac{2kTe^2}{\hbar^2}\left(\frac{C_\mu}{C}\right)^2 R_q.$$

$$\gamma_\phi^l = \gamma_\phi^0 \frac{(2\mathcal{T}-1)^2}{2\mathcal{T}(1-\mathcal{T})+T_0}$$

$$\gamma_\phi^{nl} = \gamma_\phi^0 \frac{(2\mathcal{T}-1)^2}{1+2T_0}$$

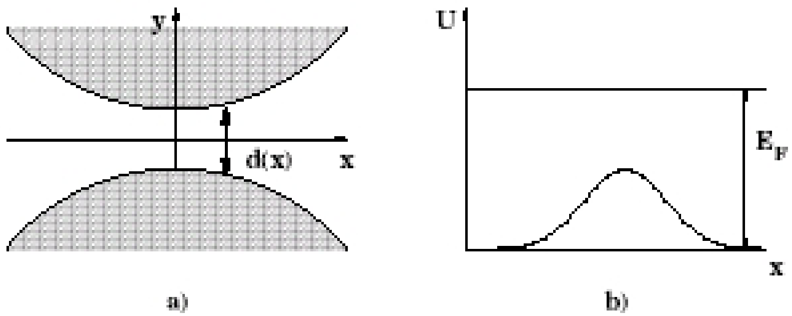


Sample specific dephasing rates

Quantum partition noise of photon-created electron-hole pairs

Reydellet, Roche, Glattli, Etienne, Jin, PRL 90, 176803 (2004)

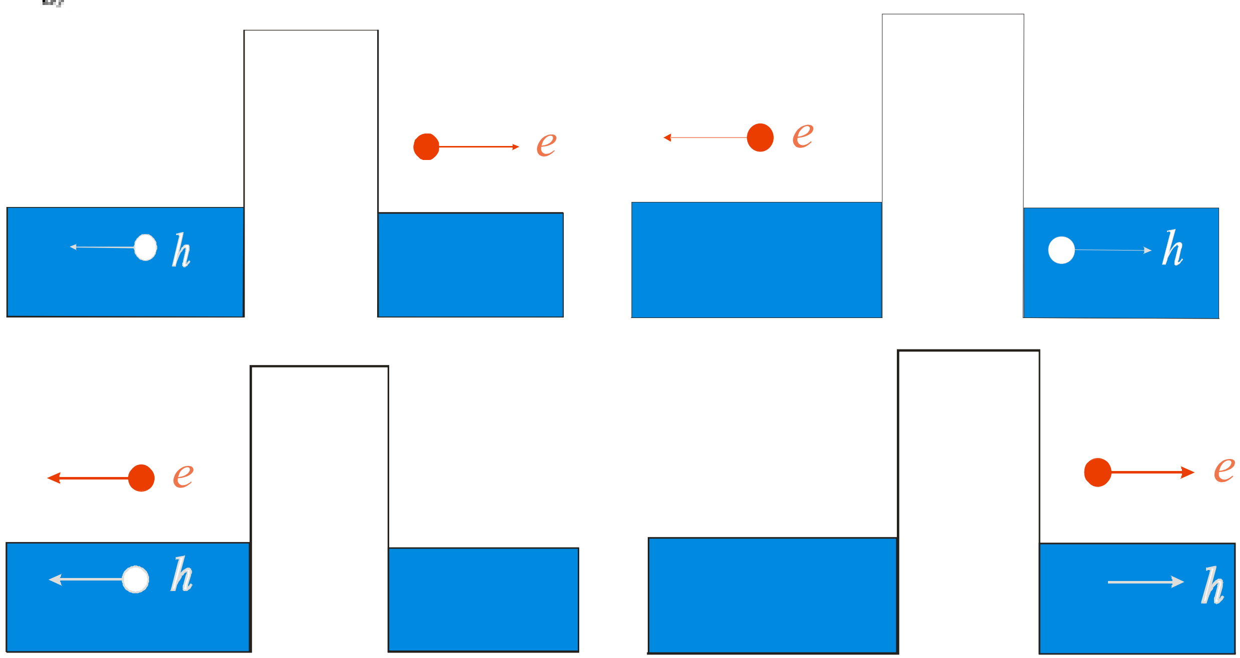
QPC: left contact V_{ac} with frequency ν ; right contact grounded



Ac-voltage excites electron-hole pairs:

electrons with energy $-\epsilon$ below E_F excited to $h\nu - \epsilon > 0$

leaves behind a hole at $-\epsilon$



e-h-interpretation:

Moskalets, Buttiker, PRB 66, 035306 (2002)

Quantum partition noise of photon-created electron-hole pairs

Reydellet, Roche, Glattli, Etienne, Jin, PRL 90, 176803 (2004)

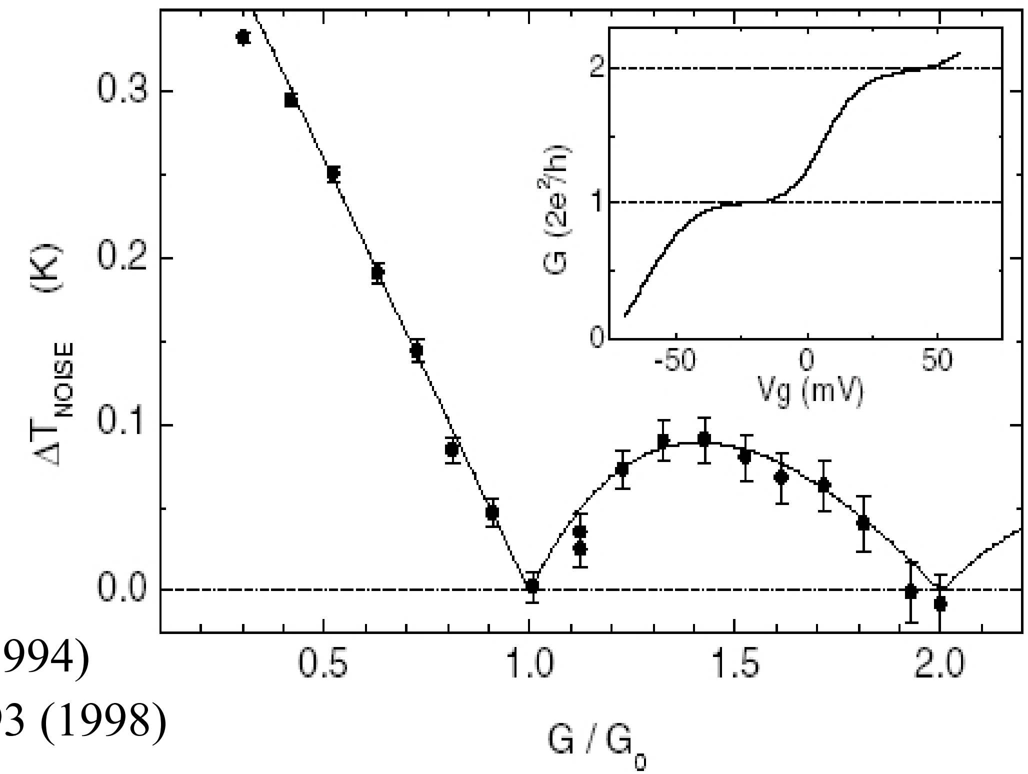
QPC: left contact V_{ac} with frequency ν ; right contact grounded

No dc current

$$k_B T_N = k_B T[J_0^2(\alpha)+\sum_n \frac{T_n^2}{T_n}(1-J_0^2(\alpha))]+\sum_{l=1}^{\infty} lh\nu J_l^2(\alpha)\frac{\sum_{n=1} T_n(1-T_n)}{\sum_{n=1} T_n}$$

$$k_B T_N = S_I/4G$$

$$\alpha = eV_{ac}/h\nu$$



Theory:

Lesovik and Levitov, PRL 72, 538 (1994)

Pedersen and Buttiker, PRB 58, 12993 (1998)

Quantum partition noise of photon-created electron-hole pairs

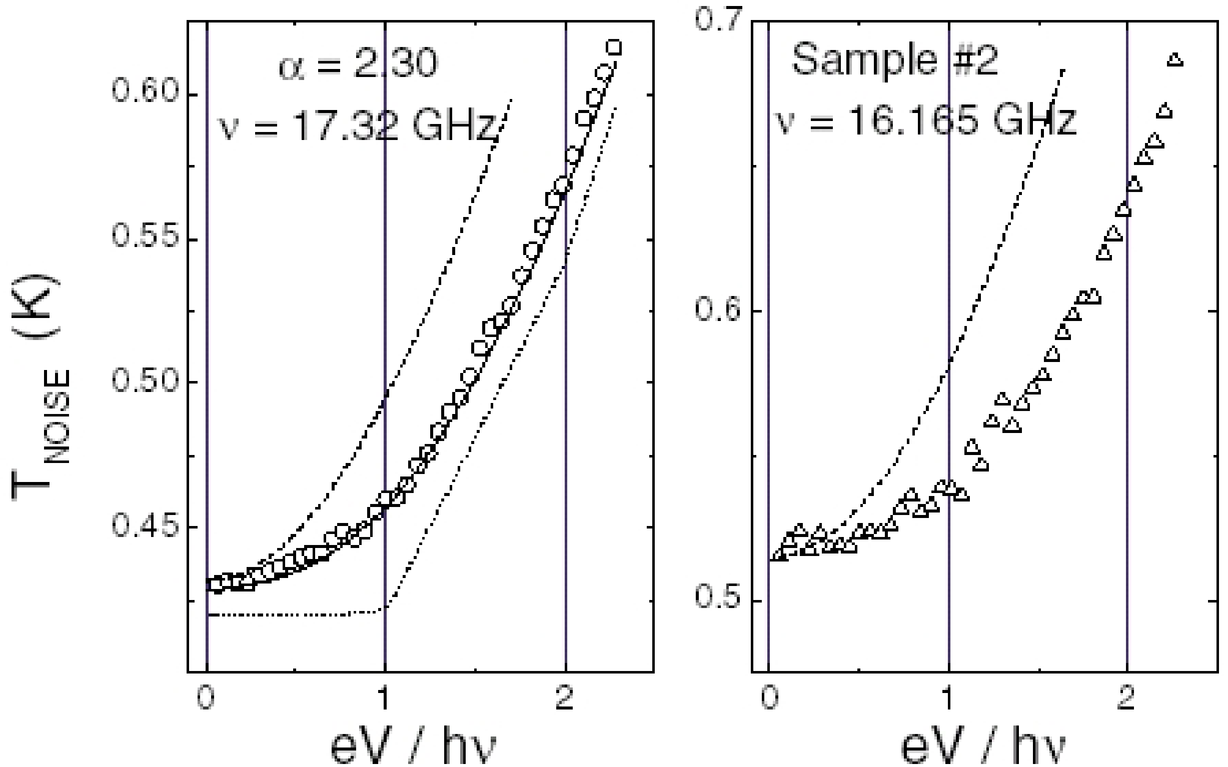
Reydellet, Roche, Glattli, Etienne, Jin, PRL 90, 176803 (2004)

with dc-current (dc-voltage V, ac voltage V_{ac})

$$k_B T_N = k_B T \sum_n \frac{T_n^2}{T_n} + \frac{1}{2} \sum_{\pm} \sum_{l=0}^{\infty} J_l^2(\alpha)\,(eV \pm l h\nu)\coth\left(\frac{eV \pm l h\nu}{2k_B T}\right) \frac{\sum_{n=1} T_n(1-T_n)}{\sum_{n=1} T_n}$$

$$k_B T_N = S_I / 4G$$

$$\alpha = eV_{ac}/h\nu$$



Summary

Spontaneous charge fluctuations

Equilibrium charge fluctuations related to charge relaxation resistance Rq

Determines dephasing rates

Charge fluctuations in the presence of transport related to Rv

Determines back-action dephasing of a mesoscopic detector

Quantum-partition noise of photon created electron-hole pairs

(Example of a zero-frequency measurement in the presence of ac-excitation)

Warning

This is an attempt for a tutorial lecture on shot noise

It is not an overview of the literature on shot noise

It is not even an overview of my own work nor that of my group