

Geneva



Shot Noise in Mesoscopic Conductors

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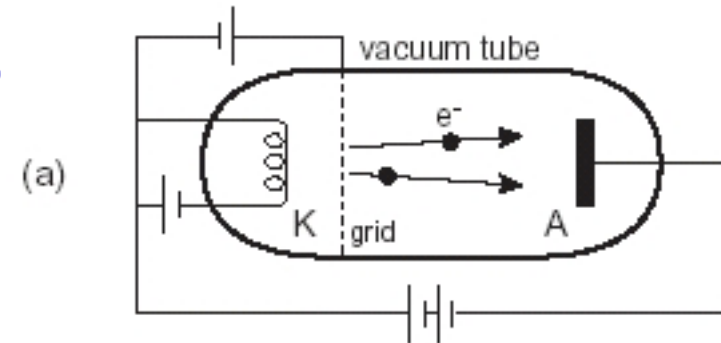


From Schottky to Bell

Classical shot noise:

W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918)

$$\langle (\Delta I)^2 \rangle_\nu = 2e \langle I \rangle$$



Quantum shot noise:

Khlus (1987), Lesovik (1989), Yurke and Kochanski (1989),
Buttiker (1990)

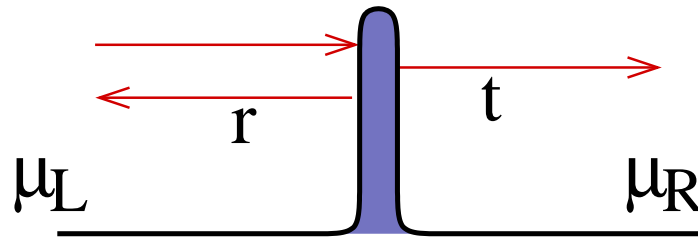
$$|\Psi\rangle_{inc} = e^{ikx}$$

$$|\Psi\rangle_{ref} = r e^{-ikx}$$

$$|\Psi\rangle_{tra} = t e^{ikx}$$

\Rightarrow

$$\langle (\Delta n_T)^2 \rangle = T(1 - T) \quad \Rightarrow \quad \langle (\Delta I)^2 \rangle_\nu = 2e \langle I \rangle (1 - T)$$



Entangled States

J. Bell, Physics 1, 195 (1964)

Lectures

Lecture 1:

Scattering Theory of Thermal and Shot Noise

Lecture 2:

Frequency-dependent noise: charge fluctuations

Single particle interferometers

Lecture 3:

Two-particle interferometers: entanglement

Warning

This is an attempt at a tutorial lecture on shot noise

It is not an overview of the literature on shot noise

It is not even an overview of my own work or that of my group

Scattering Theory of Shot Noise

Fundamental sources of noise

Buttiker, PRB 46, 12485 (1992)

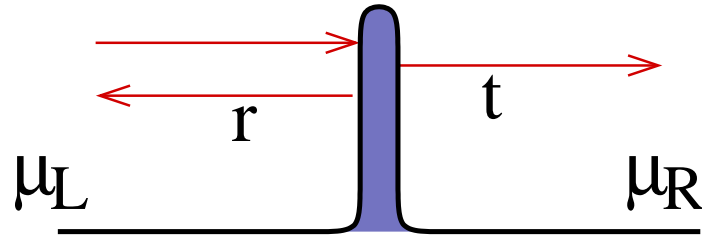
Thermal fluctuations of occupation numbers in the contacts

$$\Delta n(E) = n(E) - \langle n(E) \rangle; \quad f(E) = \langle n(E) \rangle$$

$$\langle (\Delta n)^2 \rangle = \langle n^2 \rangle - \langle n \rangle^2 = f - f^2 = f(1-f) = -kT df/dE$$

\Rightarrow Nyquist-Johnson noise

Quantum partition noise: $kT = 0$



occupation numbers:

n_I : incident beam

n_T : transmitted beam

n_R : reflected beam

averages: $\langle n_I \rangle = 1$; $\langle n_T \rangle = T = |t|^2$; $\langle n_R \rangle = R = |r|^2$;

Each particle can only be either transmitted or reflected:

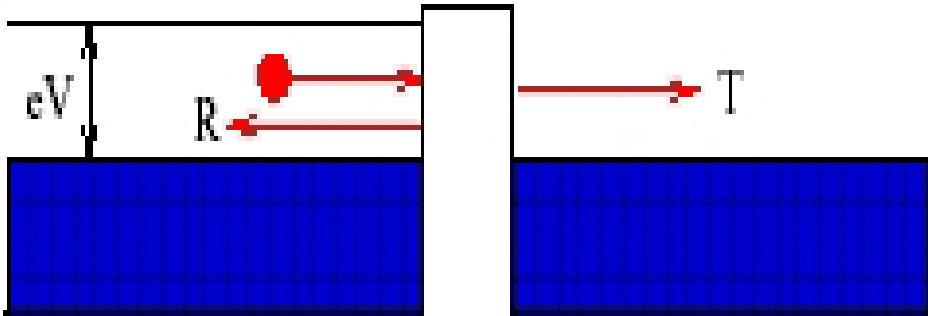
$$\langle n_T n_R \rangle = 0; \Rightarrow$$

$$\langle (\Delta n_T)^2 \rangle = \langle (\Delta n_R)^2 \rangle = -\langle \Delta n_T \Delta n_R \rangle = TR = T(1-T)$$

Conductance from scattering theory

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Heuristic discussion



Fermi energy left contact $\mu + eV$

Fermi energy right contact μ ,

μ applied voltage eV ,

transmission probability T ,

reflection probability R ,

incident current

$$I_{in} = ev_F \Delta \rho$$

density

$$\Delta \rho = (d\rho/dE) eV$$

density of states

$$d\rho/dE = (d\rho/dk) (dk/dE) = (1/2\pi) (1/\hbar v_F)$$

\Rightarrow

$$I_{in} = (e/h) eV \text{ independent of material !!}$$

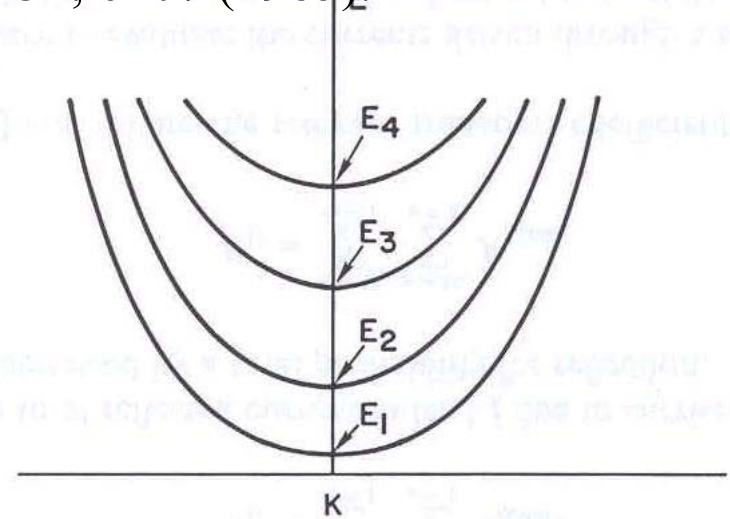
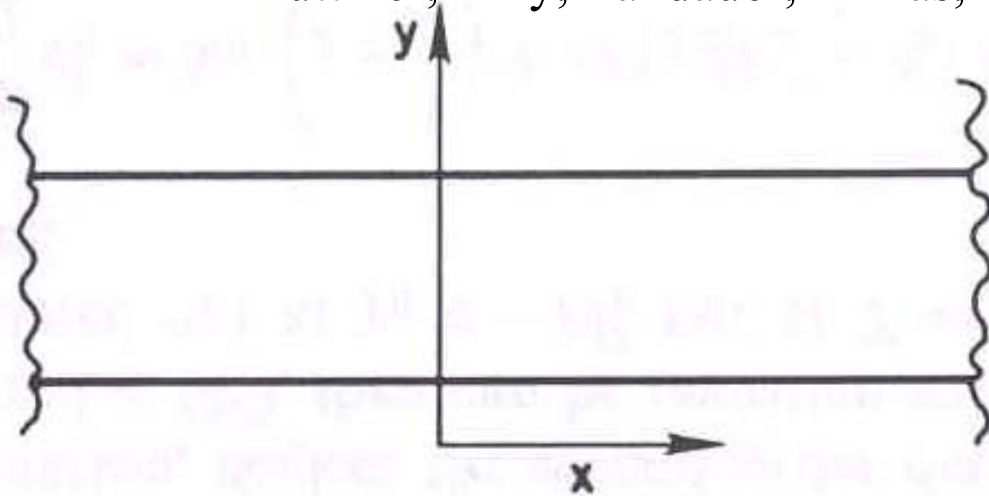
$$I = (e/h) T eV \Rightarrow$$

$$G = dI/dV = \frac{e^2}{h} T$$

Landauer

Multi-channel conductance: leads

Buttiker, Imry, Landauer, Pinhas, PRB 31, 6207 (1985)



asymptotic perfect translation invariant potential

$$V(x, y) = V(y) \implies$$

separable wave function

$$\phi_{\alpha n}^{\pm}(\mathbf{r}, E) = e^{\pm i k_n(E) x} \chi_{\alpha n}(y)$$

energy of transverse motion E_n channel threshold

energy for transverse and longitudinal motion

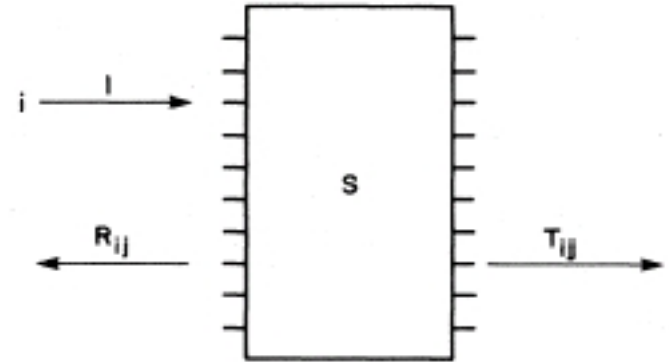
$$E = E_n + \hbar^2 k^2 / 2m \iff \text{scattering channel}$$

Multi-channel conductance: scattering state

channel dispersion

$$E_n(k) = E_n + \hbar^2 k^2 / 2m$$

$$v_n(k) = \hbar^{-1} dE_n(k) / dk$$



scattering state:

incident + reflected wave in channel n in contact α

$$\phi_{\alpha n}(\mathbf{r}, E) = \frac{1}{\sqrt{v_{\alpha n}}} [e^{ik_n(E)x} + s_{\alpha\alpha,nn} e^{-ik_n x}] \chi_{\alpha n}(y)$$

reflected waves in channel m in contact α

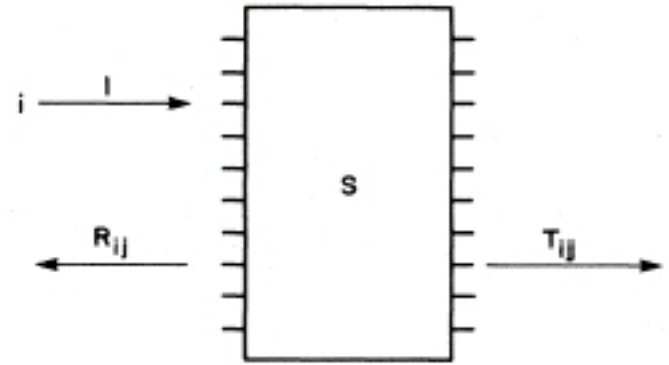
$$\phi_{\alpha n}(\mathbf{r}, E) = \frac{1}{\sqrt{v_{\alpha m}}} s_{\alpha\alpha,mn} e^{-ik_m x} \chi_{\alpha m}(y)$$

transmitted wave in channel m in contact

$$\phi_{\alpha n}(\mathbf{r}, E) = \frac{1}{\sqrt{v_{\beta m}}} s_{\beta\alpha,mn} e^{-ik_m x} \chi_{\beta m}(y)$$

Multi-channel conductor: scattering matrix

$$\begin{aligned}
 s_{\alpha\alpha,nn} , \quad s_{\alpha\alpha,mn} &\implies s_{\alpha\alpha} , \\
 s_{\beta\alpha,mn} &\implies s_{\beta\alpha}
 \end{aligned}$$



$$s = \begin{pmatrix} r & t \\ t & r' \end{pmatrix} \quad s = \begin{pmatrix} r_{11} & t_{12} \\ t_{21} & r_{22} \end{pmatrix} \quad \text{unitary}$$

reflection probabilities

transmission probabilities

$$R_{\alpha\alpha,nm} = |s_{\alpha\alpha,nm}|^2 , \quad T_{\beta\alpha,mn} = |s_{\beta\alpha,mn}|^2 ,$$

Multi-channel conductance, $kT = 0$, two terminal

$$I = (e/h)eV \sum_{mn} T_{RL,mn} = (e^2/h)VT \quad \implies \quad G = \frac{e^2}{h}T$$

Eigen channels

$$T = \sum_{mn} T_{\beta\alpha, mn} = \sum_{mn} |s_{\beta\alpha, mn}|^2 = \text{Tr}[s_{\alpha\beta}^\dagger s_{\alpha\beta}] = \text{Tr}[t^\dagger t]$$

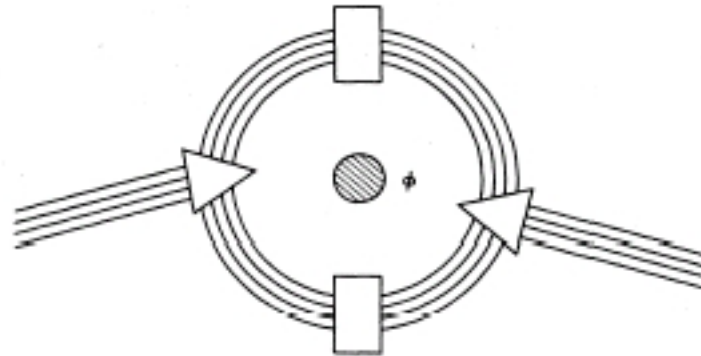
$t^\dagger t$ hermitian matrix; real eigenvalues T_n

$r^\dagger r$ hermitian matrix; real eigenvalues R_n

$$T = \text{Tr}[t^\dagger t] = \sum_n T_n$$

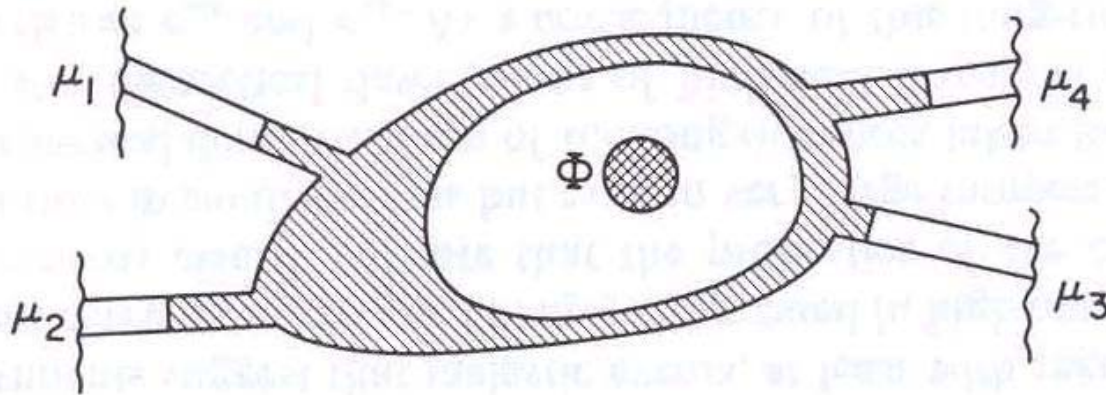
$$G = \frac{e^2}{h} \sum_n T_n$$

T_n are the genetic code of mesoscopic conductors !!



Multiprobe Conductors

Buttiker, PRL 57, 1761 (1986); IBM J. Res. Developm. 32, 317 (1988)



$$\mu_\alpha = \mu_0 + eV_\alpha$$

$$I_\alpha = \frac{e}{h} [(N_\alpha - R_{\alpha\alpha}) \mu_\alpha - \sum_{\beta \neq \alpha} T_{\alpha\beta} \mu_\beta] \quad \Rightarrow$$

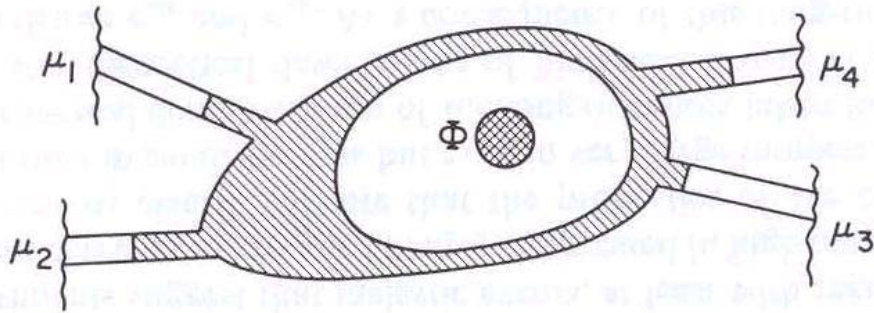
$$G_{\alpha\alpha} = dI_\alpha/dV_\alpha = \frac{e^2}{h} (N_\alpha - R_{\alpha\alpha}) = \frac{e^2}{h} \sum_{\beta \neq \alpha} T_{\alpha\beta}$$

$$G_{\alpha\beta} = dI_\alpha/dV_\beta = -\frac{e^2}{h} T_{\alpha\beta}$$

$$I_\alpha = \sum_{\beta} G_{\alpha\beta} V_\beta ; \quad \sum_{\alpha} G_{\alpha\beta} = 0 ; \quad \sum_{\beta} G_{\alpha\beta} = 0$$

Multi-probe conductors: scattering matrix

Buttiker, PRL 57, 1761 (1986); IBM J. Res. Developm. 32, 317 (1988)



$$s = \begin{pmatrix} s_{11} & s_{12} & s_{13} & s_{14} \\ s_{21} & s_{22} & s_{23} & s_{24} \\ s_{31} & s_{32} & s_{33} & s_{34} \\ s_{41} & s_{42} & s_{43} & s_{44} \end{pmatrix}$$

$$T_{\beta\alpha} = \sum_{mn} T_{\beta\alpha,mn} = \sum_{mn} |s_{\beta\alpha,mn}|^2 = \text{Tr}[s_{\beta\alpha}^\dagger s_{\beta\alpha}]$$

$$R_{\alpha\alpha} = \sum_{mn} R_{\alpha\alpha,mn} = \sum_{mn} |s_{\alpha\alpha,mn}|^2 = \text{Tr}[s_{\alpha\alpha}^\dagger s_{\alpha\alpha}]$$

magnetic field symmetry $s_{\beta\alpha,mn}(B) = s_{\alpha\beta,nm}(-B)$

$$T_{\alpha\beta}(B) = T_{\beta\alpha}(-B); \quad R_{\alpha\alpha}(B) = R_{\alpha\alpha}(-B)$$

$$G_{\alpha\beta}(B) = G_{\beta\alpha}(-B); \quad G_{\alpha\alpha}(B) = G_{\alpha\alpha}(-B)$$

Occupation number and current amplitudes

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Buttiker, PRB 46, 12485 (1992)

Incident current at $kT = 0$

$$I_{in} = (e/h)eV$$

Incident current at $kT > 0$

$$dI_{in} = (e/h) f(E) dE$$

Occupation number

$$f(E) = \langle n(E) \rangle$$

$\langle \rangle =$ statistical average

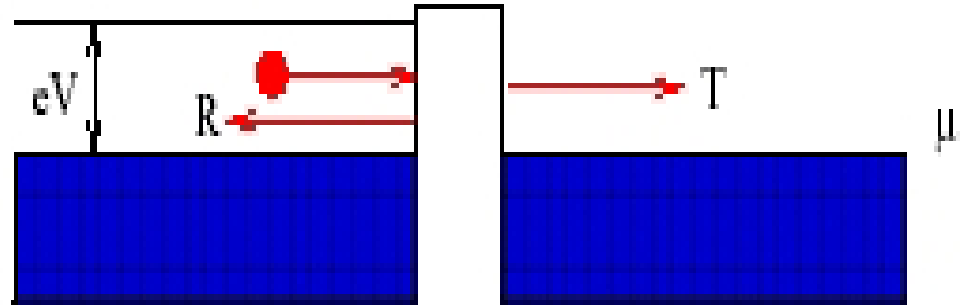
Creation and annihilation operators

$$\langle \hat{a}^\dagger(E) \hat{a}(E') \rangle = f(E) \delta(E - E')$$

« Incident current » « Current amplitude » $\hat{a}(E)$

$$\hat{I}_{in}(t) = (e/h) \int dE \int dE' \hat{a}^\dagger(E) \hat{a}(E') e^{i(E-E')t/\hbar}$$

$$\hat{I}_{in}(t) = (e/h) \int dE \hat{n}(E, t)$$



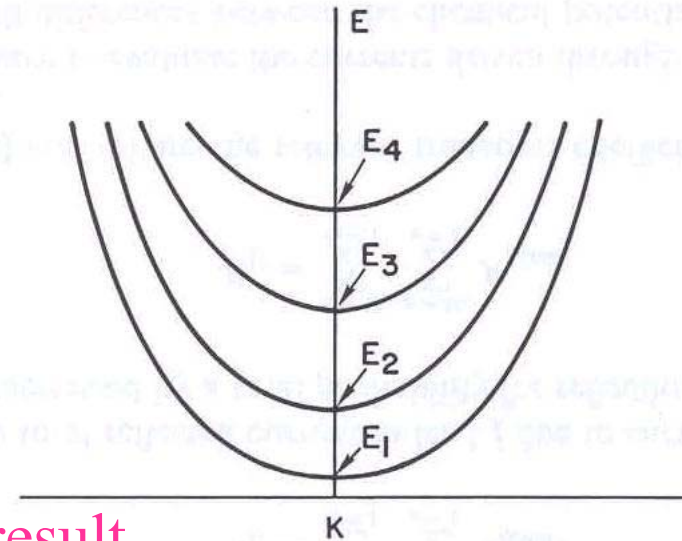
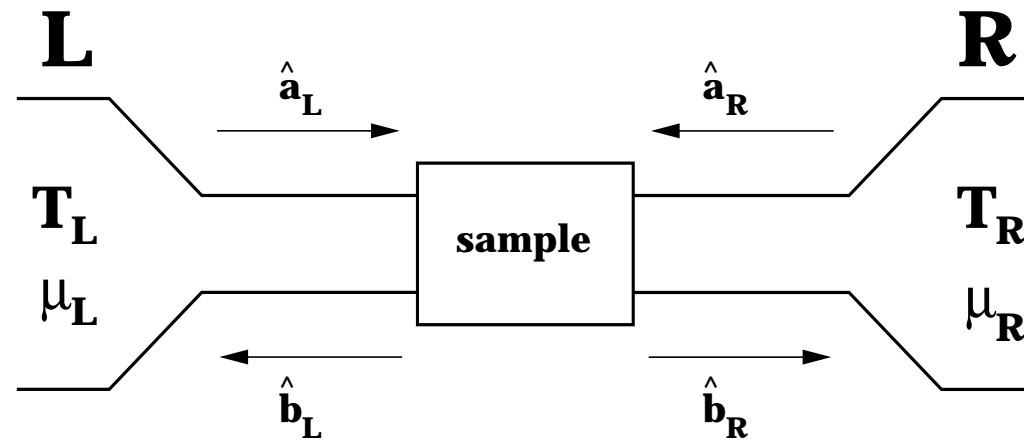
Buttiker, PRL 65, 2901 (1990)

Current in contact \propto single channel result

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE [\hat{n}_{\alpha,in}(E, t) - \hat{n}_{\alpha,out}(E, t)]$$

current amplitude: $\hat{a}_\alpha(E)$ (incoming) $\hat{b}_\alpha(E)$ (outgoing)

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE [\hat{a}_\alpha^\dagger(E') \hat{a}_\alpha(E) - \hat{b}_\alpha^\dagger(E') \hat{b}_\alpha(E)] e^{i(E'-E)t/\hbar}$$



Current in contact \propto multi-channel channel result

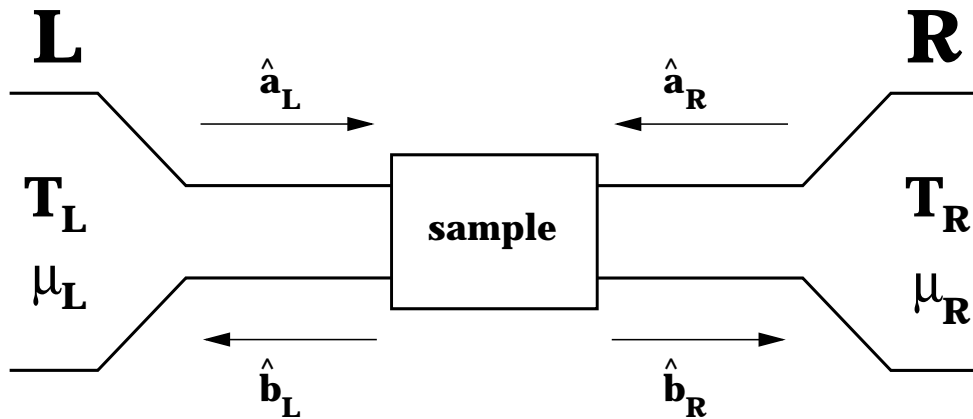
$$\hat{I}_\alpha(t) = \frac{e}{h} \sum_n \int dE' dE [\hat{a}_{\alpha n}^\dagger(E') \hat{a}_{\alpha n}(E) - \hat{b}_{\alpha n}^\dagger(E') \hat{b}_{\alpha n}(E)] e^{i(E'-E)t/\hbar}$$

Current operator

Buttiker, PRL 65, 2901 (1990)

$\hat{a}_\alpha(E), \hat{b}_\alpha(E)$: N_α component vectors

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE [\hat{a}_{\alpha n}^\dagger(E') \hat{a}_\alpha(E) - \hat{b}_\alpha^\dagger(E') \hat{b}_\alpha(E)] e^{i(E' - E)t/\hbar}$$



$$\mathbf{b}_\alpha = \sum_{\beta} \mathbf{s}_{\alpha\beta} \mathbf{a}_\beta$$

$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE \sum_{\beta, \gamma} \hat{a}_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) \hat{a}_\gamma(E) e^{i(E' - E)t/\hbar}$$

$$A_{\beta\gamma}(\alpha, E', E) = 1_\alpha \delta_{\alpha\beta} \delta_{\alpha\gamma} - s_{\alpha\beta}^\dagger(E') s_{\alpha\gamma}(E)$$

quantum statistical average

$$\langle \hat{a}_\beta^\dagger(E) \hat{a}_\gamma(E') \rangle = \delta_{\beta\gamma} \delta(E - E') f_\beta(E) \quad \Rightarrow \quad \text{average current, conductance}$$

Noise spectral density

Spectral density S (noise power)

$$(1/2)\langle I_\alpha(\omega)I_\beta(\omega') + I_\beta(\omega')I_\alpha(\omega) \rangle = 2\pi S_{\alpha\beta}(\omega)\delta(\omega + \omega')$$

Use

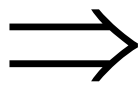
$$\hat{I}_\alpha(t) = \frac{e}{h} \int dE' dE \sum_{\beta, \gamma} \hat{a}_\beta^\dagger(E') A_{\beta\gamma}(\alpha, E', E) \hat{a}_\gamma(E) e^{i(E' - E)t/\hbar}$$

quantum statistical average of four creation and annihilation op. \Rightarrow

zero-frequency spectrum (white noise limit)

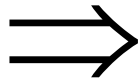
$$S_{\alpha\beta} = 2 \frac{e^2}{h} \sum_{\gamma\delta} \int dE T r [A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_\gamma(E) (1 - f_\delta(E))$$

equilibrium



fluctuation-dissipation theorem

non-equilibrium



shot-noise

Buttiker, PRL 65, 2901 (1990); PRB 46, 12485 (1992)

Equilibrium current fluctuations

Use

$$S_{\alpha\beta} = 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE T r[A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_{\gamma}(E)(1-f_{\delta}(E))$$

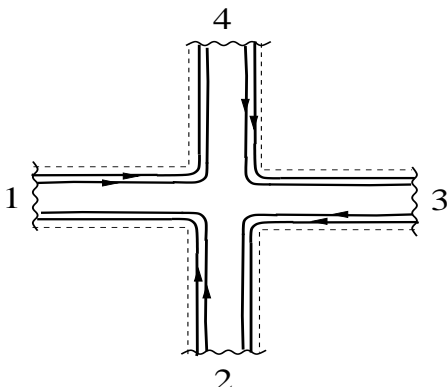
with $f_{\alpha}(E) = f(E)$ for all $\alpha = 1, 2, 3, \dots \implies$

auto-correlation $\langle I_{\alpha}^2 \rangle_{\nu}$

$$S_{\alpha\alpha} = 2kTG_{\alpha\alpha} = 2kT\frac{e^2}{h} \int dE (-df/dE)(N_{\alpha} - R_{\alpha\alpha});$$

cross-correlation $\langle I_{\alpha} I_{\beta} \rangle_{\nu}$

$$S_{\alpha\beta} = kT[G_{\alpha\beta} + G_{\beta\alpha}] = -kT\frac{e^2}{h} \int dE (-df/dE)[T_{\alpha\beta} + T_{\beta\alpha}]$$



QHE-plateau N : $S_{\alpha\alpha} = 2kT\frac{e^2}{h} N;$

$$S_{\alpha+3,\alpha} = -kT\frac{e^2}{h} N;$$

$$S_{\alpha+2,\alpha} = S_{\alpha+1,\alpha} = 0$$

Shot-noise: two-terminal

$$S_{\alpha\beta} = 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE \text{Tr}[A_{\gamma\delta}(\alpha)A_{\delta\gamma}(\beta)] f_{\gamma}(E)(1-f_{\delta}(E))$$

Consider $kT = 0$, $V > 0$, and a two-terminal conductor:

$$S = S_{11} = -S_{12} = -S_{21} = S_{22};$$

Quantum partition noise

$$S = 2\frac{e^2}{h} |eV| \text{Tr}[tt^{\dagger}rr^{\dagger}] = 2\frac{e^2}{h} |eV| \sum_n T_n(1 - T_n)$$

If all $T_n \ll 1 \implies$

$$S = 2e\left(\frac{e^2}{h} \sum_n T_n\right) |V| = 2e |I|$$

Shottky (Poisson)

Fano factor

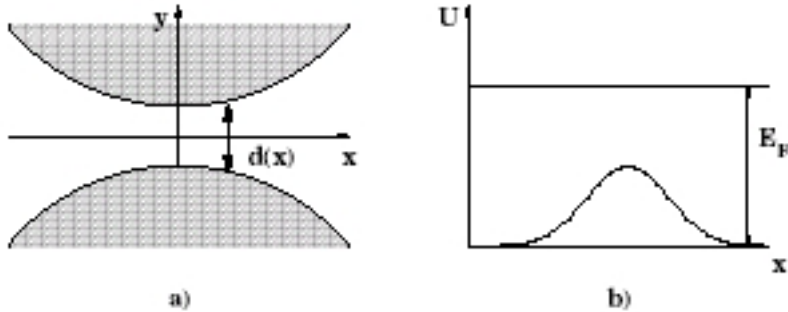
$$F = \frac{S}{S_P} = \frac{\sum_n T_n(1 - T_n)}{\sum_n T_n}$$

Khlus (1987)

Lesovik (1989)

Buttiker (1990)

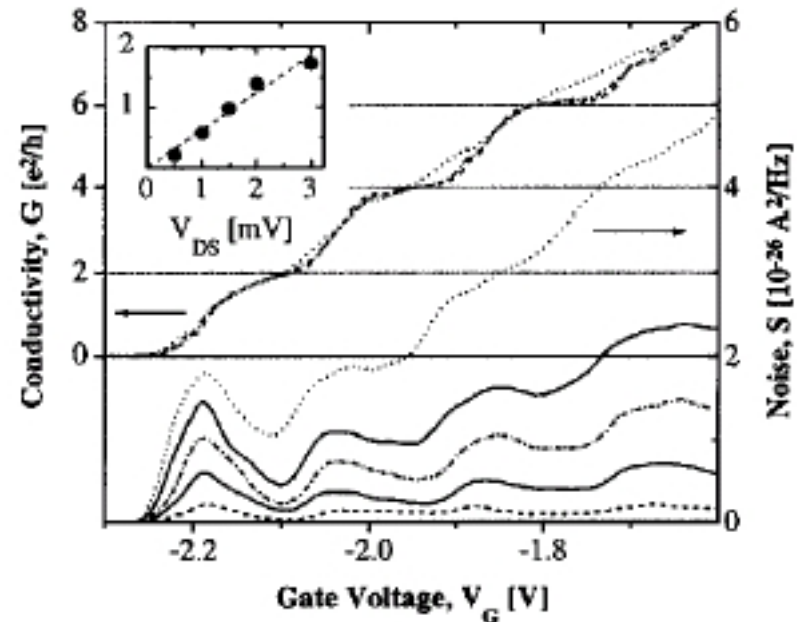
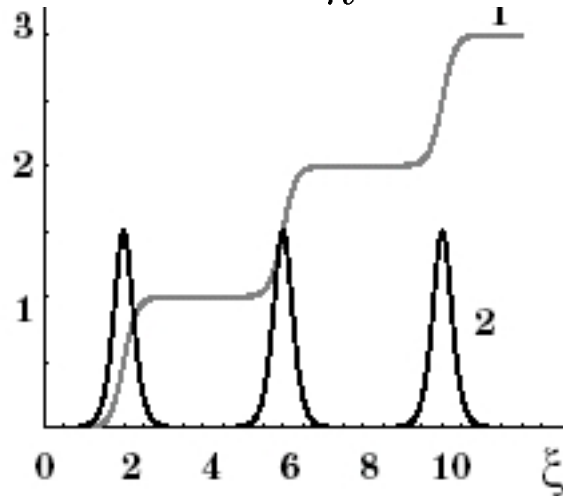
Shot-noise: Quantum point contact



A. Kumar, L. Saminadayar, D. C. Glattli,
Y. Jin, B. Etienne, PRL 76, 2778 (1996)

M. I. Reznikov, M. Heiblum, H. Shtrikman,
D. Mahalu, PRL 75, 3340 (1996)

$$S = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$



Ideally only one channel contributes

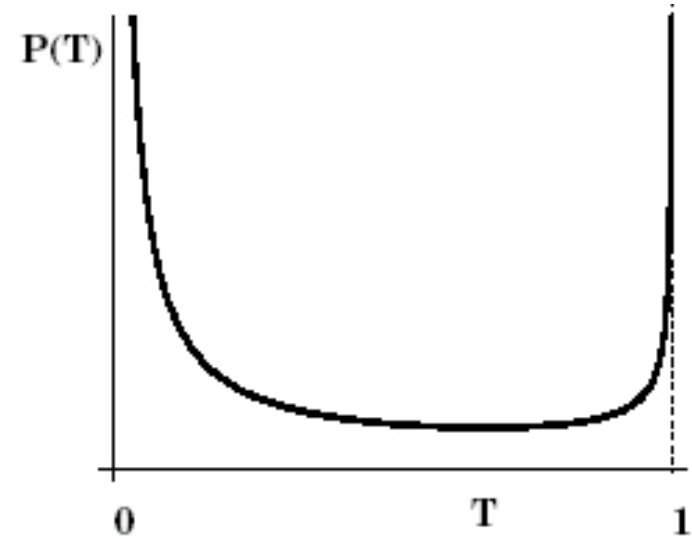
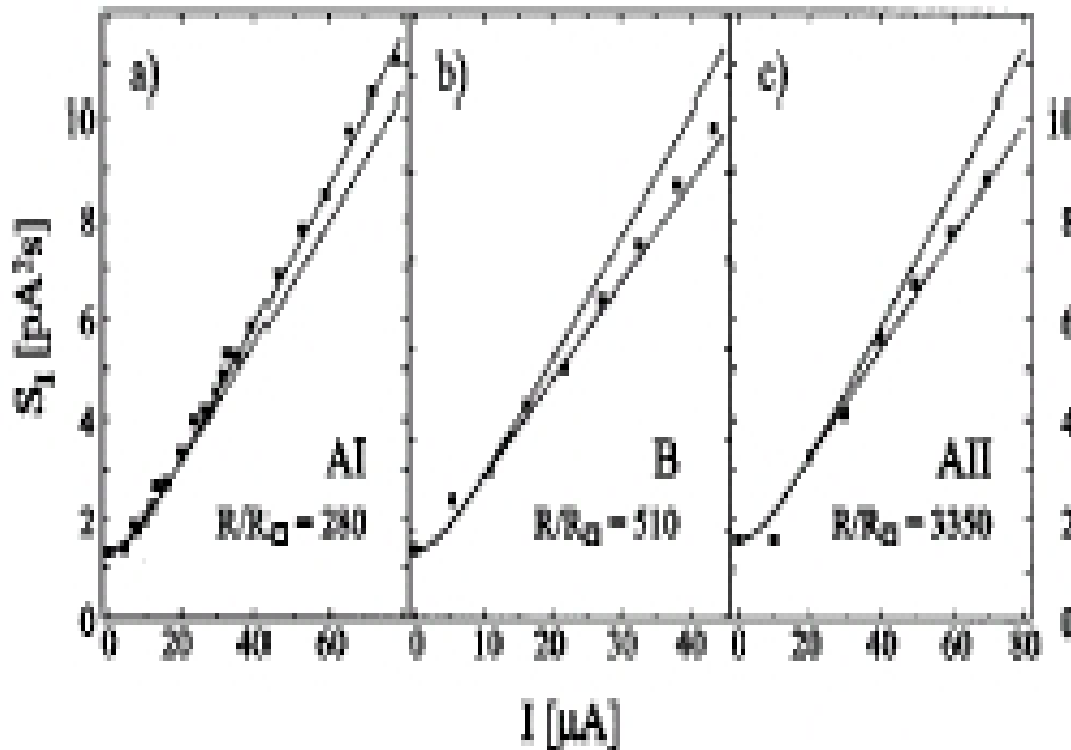
Shot-noise: Metallic diffusive wire

Beenakker and Buttiker, PRB 46, 1889 (1992)

$$G = \frac{e^2}{h} \sum_n T_n$$

$$S = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$

Henny et al. PRB 59, 2871 (1999)



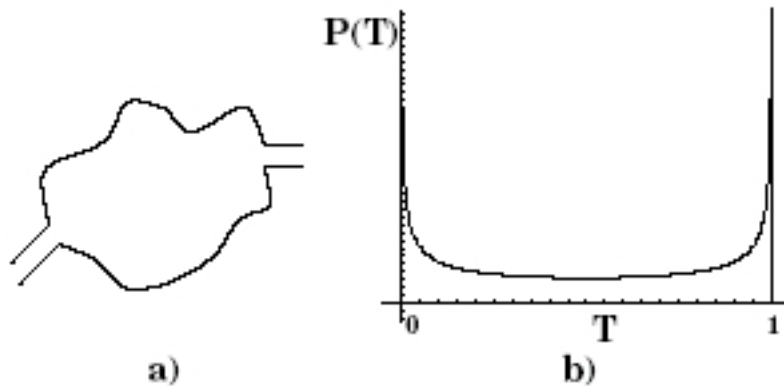
$$p(T) = \frac{l_e}{2L} \frac{1}{T \sqrt{(1-T)}}$$

$$\langle G \rangle = \frac{e^2}{h} N \frac{l_e}{L}$$

$$\langle S \rangle = \frac{1}{3} 2e|I|$$

Shot-noise: Chaotic cavity

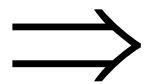
Jalabert, Pichard and Beenakker, Europhys. Lett. 27, 255 (1994)



$$G = \frac{e^2}{h} \sum_n T_n$$

$$S = 2 \frac{e^2}{h} |eV| \sum_n T_n (1 - T_n)$$

$$p(T) = \frac{1}{\pi} \frac{1}{\sqrt{T(1-T)}}$$

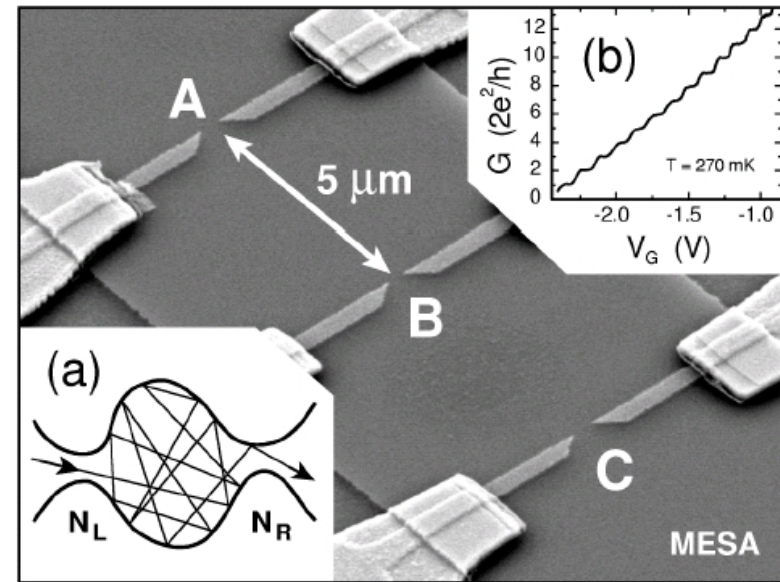


$$\langle G \rangle = \frac{e^2 N}{h 4}$$

$$\langle S \rangle = \frac{1}{4} 2e |I|$$

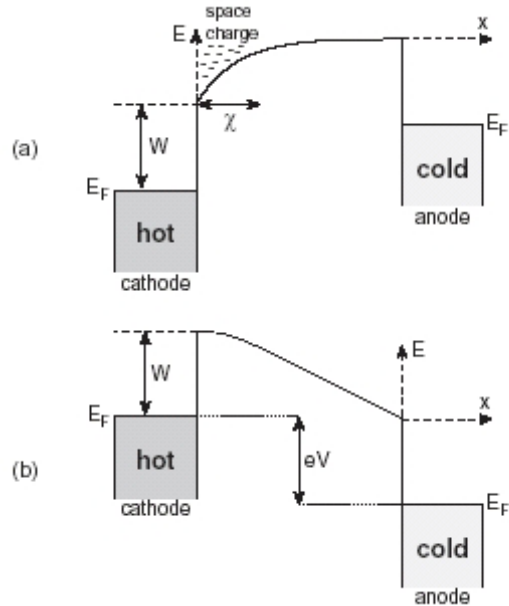
for symmetric cavity with $N_1 = N_2 \gg 1$

Oberholzer et al., PRL 86, 2114 (2001)

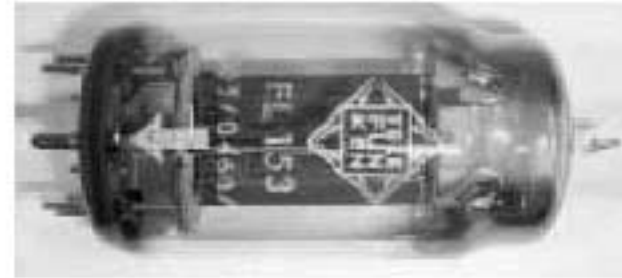


Connection with Schottky

Schoenenberger et al. cond-mat



(b)



$$I = \frac{e}{h} \sum_n \int dE T_n (f_1 - f_2)$$

$$S = 2 \frac{e^2}{h} \sum_n \int dE [T_n f_1 (1 - f_1) + T_n f_2 (1 - f_2) + T_n R_n (f_1 - f_2)^2]$$

$$f_1(E) \propto \exp[-(E - E_F)/kT] ; f_1^2 \simeq 0 ; f_2 = 0 \quad \Rightarrow$$

$$S = 2eI$$

Schottky noise = thermionic emission noise

Is shot noise quantum or classical?

metallic diffusive wire

$$\langle S \rangle = \frac{1}{3} 2e|I| ; \quad \langle I \rangle = \langle G \rangle V ; \quad \langle G \rangle = \frac{e^2}{h} N \frac{l_e}{L}$$

Scattering approach: Beenakker and Buttiker, PRB 46, 1889 (1992)

Langevin approach: Nagaev, Phys. Lett. A 169, 103 (1992)

Drude conductance $\propto N$

Quantum corrections to Drude conductance
(weak localization, UCF) $\propto 1$

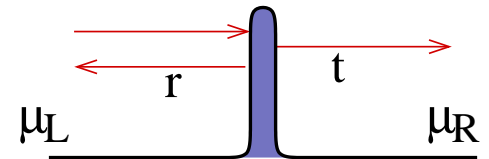
Shot noise spectrum $\propto N$

Quantum correction to shot noise $\propto 1$

Fano factor for metallic diffusive wire or for chaotic (many) channel cavity give no information on long range coherence **but** short range coherence, **quantum diffraction** is necessary

Diffraction can be switched off in chaotic cavities

Ehrenfest time $\langle S \rangle \Rightarrow 0$



$$S_{\alpha\beta} = 2\frac{e^2}{h} \sum_{\gamma\delta} \int dE T r [A_{\gamma\delta}(\alpha) A_{\delta\gamma}(\beta)] f_{\gamma}(E) (1 - f_{\delta}(E))$$

Consider multi-terminal conductor at $kT = 0$,

M source contacts with distribution f at voltage eV

All other contacts grounded with distribution f_0

Correlation measured between two grounded contacts:

$$S_{\alpha\beta} = -2\frac{e^2}{h} \sum_{\gamma\delta} \int dE T r [B_{\alpha\beta}^{\dagger} B_{\alpha\beta}] ; B_{\alpha\beta} = \sum_{\gamma=1}^M s_{\alpha\gamma}^{\dagger} s_{\beta\gamma} (f - f_0)$$

M=1, partition noise

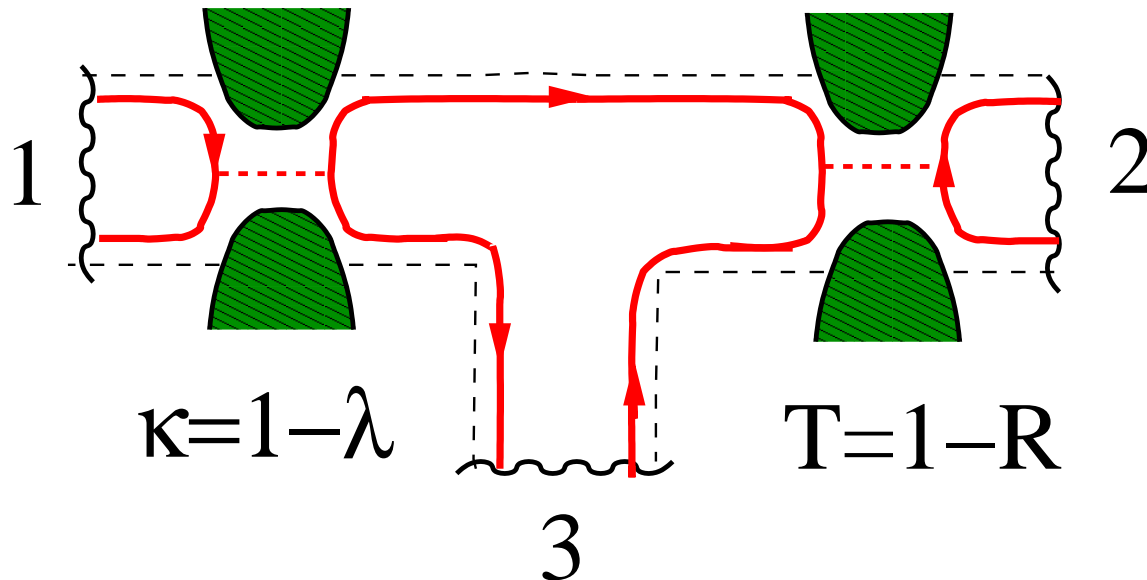
M=2, exchange effects,

two particle Aharonov-Bohm effect,

orbital entanglement, violation of Bell inequality

Current correlations, Oberholzer experiment

Oberholzer et al. Physica E6, 314 (2000)



Bias configuration: $\mu_1 = \mu_0 + eV$, $\mu_2 = \mu_3 = \mu_0$

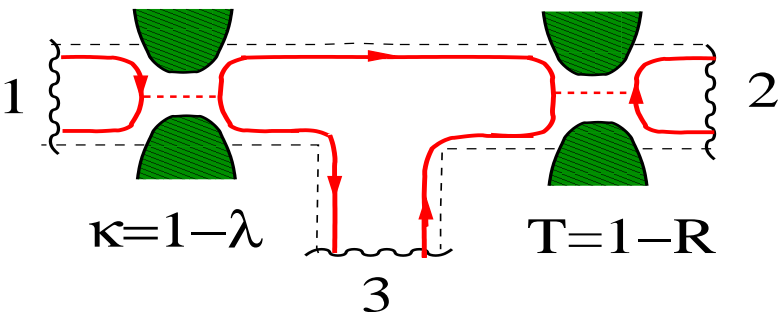
$$S_{23} = -2 \frac{e^2}{h} |eV| \kappa^2 T R$$

$$S_{22} = 2 \frac{e^2}{h} |eV| \kappa T (1 - \kappa T)$$

$$S_{33} = 2 \frac{e^2}{h} |eV| \kappa R (1 - \kappa R)$$

Oberholzer et al. Experiment

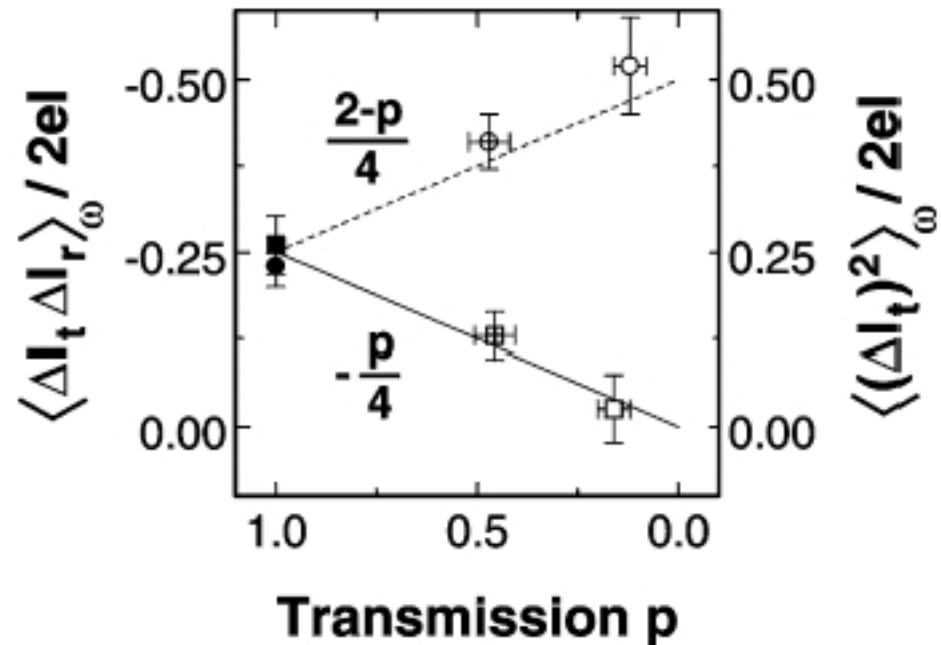
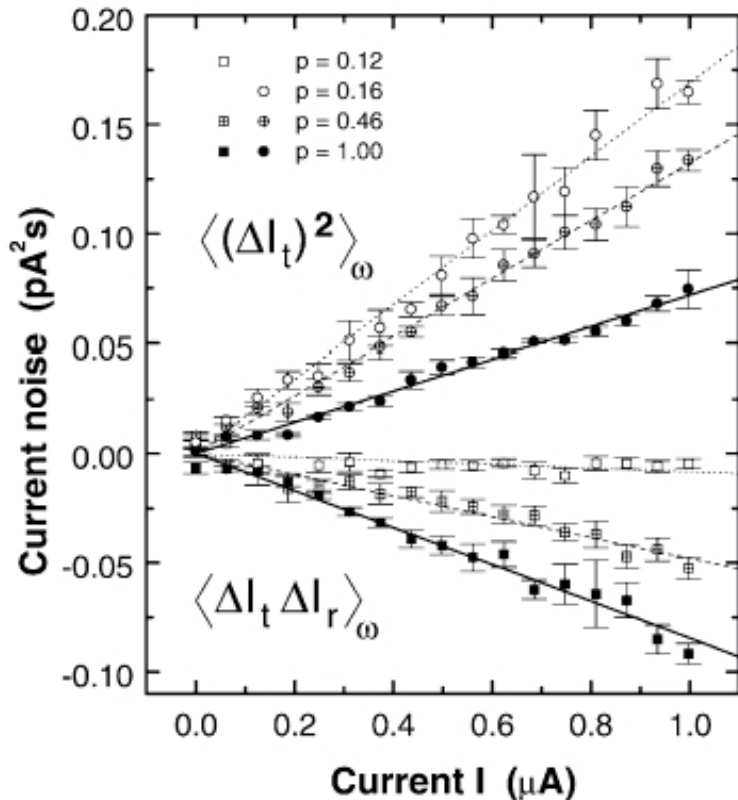
Oberholzer et al, Physica E6, 314 (2000)



$$S_{22} = 2 \frac{e^2}{h} |eV| \kappa T (1 - \kappa T)$$

$$S_{33} = 2 \frac{e^2}{h} |eV| \kappa R (1 - \kappa R)$$

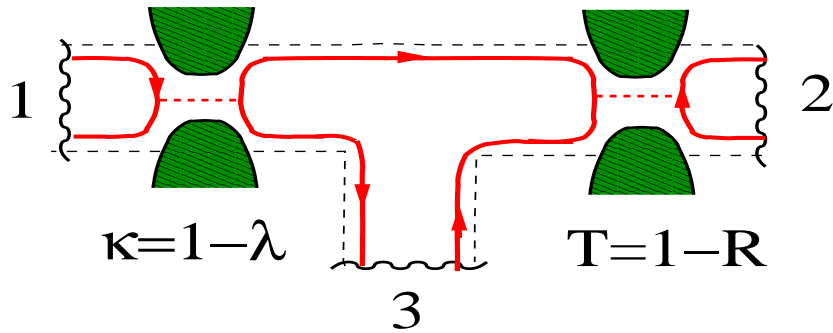
$$S_{23} = -2 \frac{e^2}{h} |eV| \kappa^2 T R$$



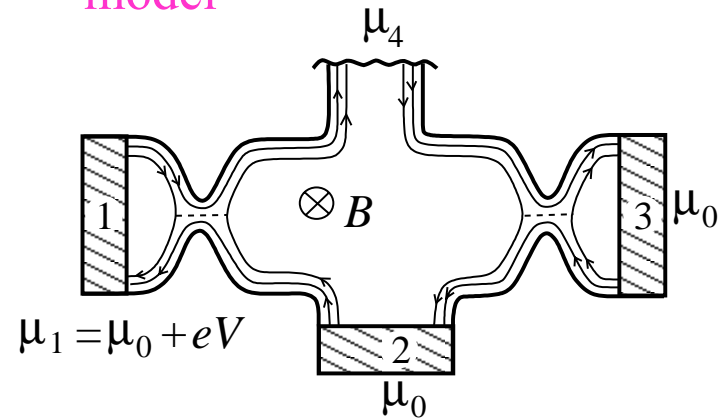
Inelastic and quasi-elastic scattering

Texier and Buttiker, PRB 62, 7454 (2000)

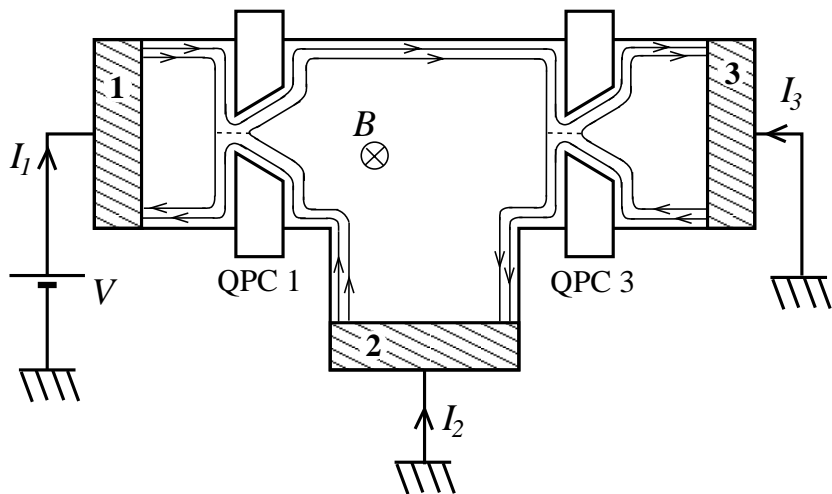
Single channel, no effect



model

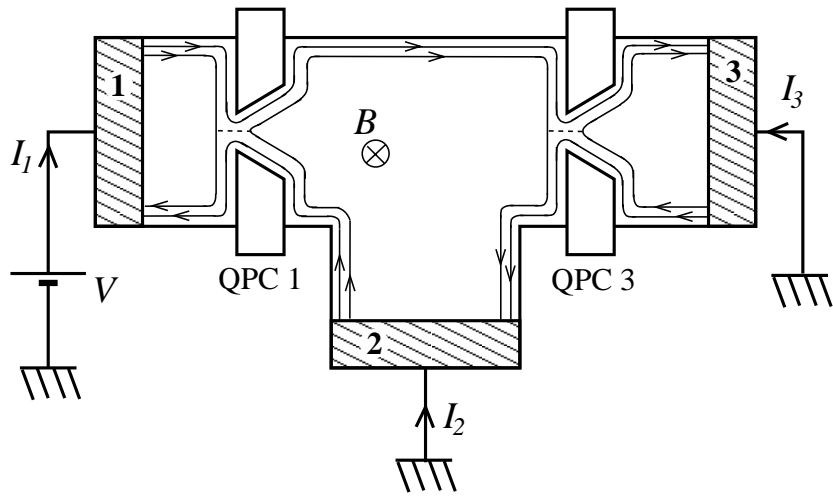


Two-channel



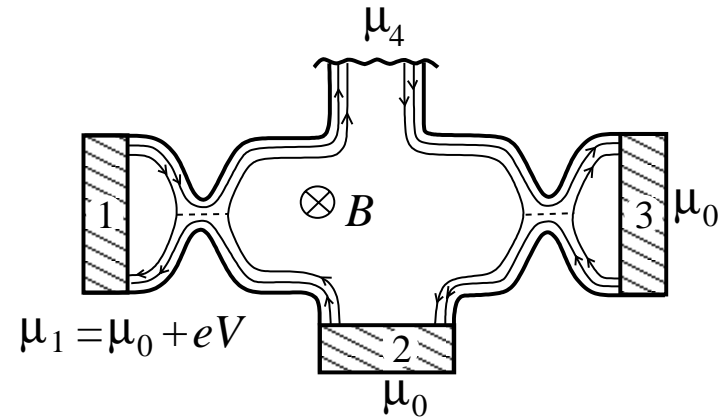
Quasi-elastic and inelastic inter-edge scattering: effect on conductance and noise

Inelastic scattering



Consider $T_3 = 0$;

$$G = \frac{e^2}{h} \begin{pmatrix} (1 + T_1) & -(1 + T_1) & 0 \\ -T_1 & (1 + T_1) & -1 \\ -1 & 0 & 1 \end{pmatrix}$$



Voltage probe 4:

Maintains zero-net current

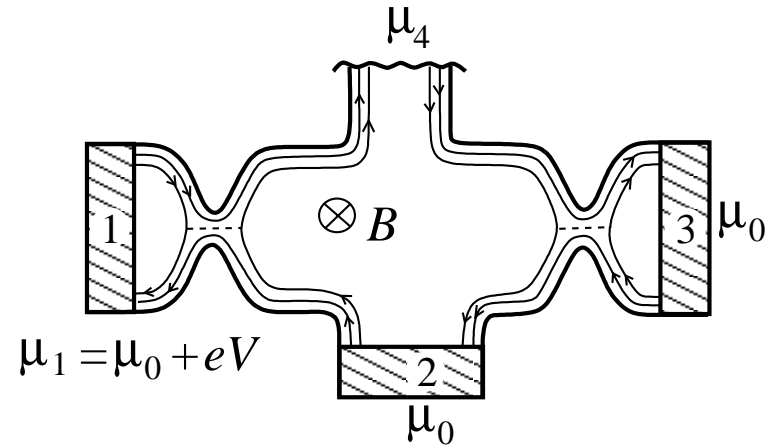
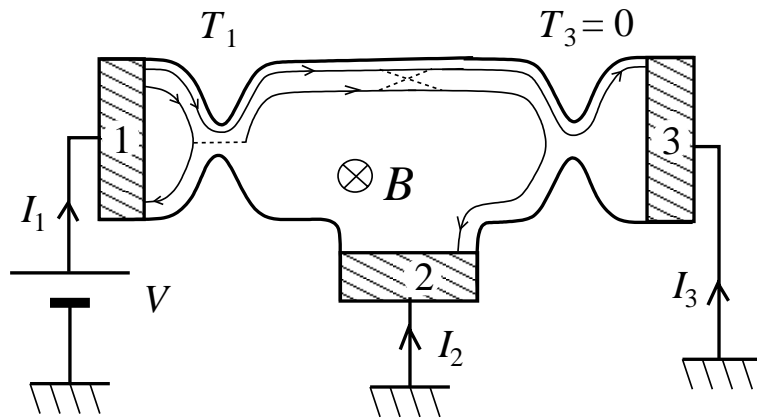
$$I_\alpha = \sum_{\beta} G_{\alpha\beta} V_\beta ;$$

$$I_4 = 0 ; \Rightarrow$$

$$\mu_4 = (1/2)[(1 + T_1)\mu_1 + R_1\mu_2] ;$$

$$G = \frac{e^2}{h} \begin{pmatrix} (1 + T_1) & -(1 + T_1) & 0 \\ -(1/2)(1 + T_1) & (2 - (1/2)R_1) & -1 \\ -(1/2)(1 + T_1) & -(1/2)(1 - T_1) & 1 \end{pmatrix}$$

Shot noise correlation: elastic versus inelastic scattering



$$I_4(t) = 0; \Rightarrow$$

$$\mu_4(t) = \langle \mu_4 \rangle + \delta \mu_4(t)$$

$$\Delta I_\alpha = G_{\alpha 4} \delta V_4 + \delta I_\alpha;$$

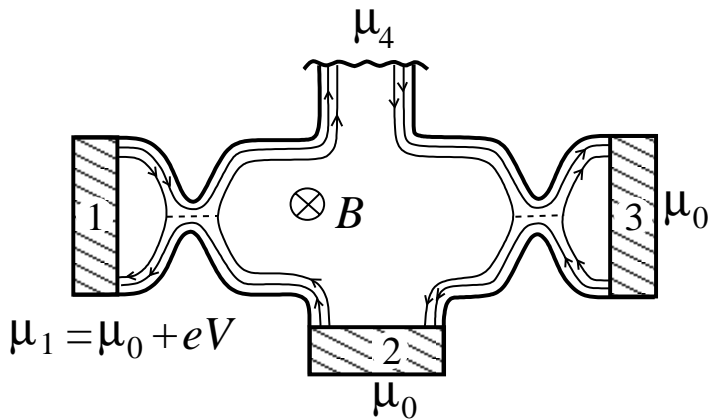
$$S_{23} = -2 \frac{e^2}{h} |eV| \epsilon (1 - \epsilon) R_1^2$$

$$S_{23} = + \frac{e^2}{h} |eV| T_1 R_1 / 2$$

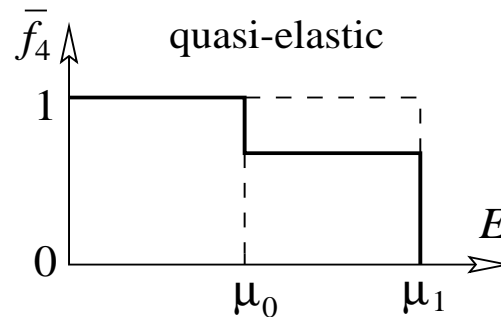
Positive !!

Shot noise correlation: quasielastic versus inelastic scattering

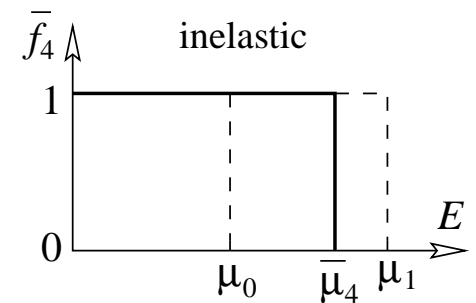
Texier and Buttiker, PRB 62, 7454 (2000)



de Jong and Beenakker
quasi-elastic scatt



inelastic scattering



$$I_4 = \int dE j_4(E); \quad j_4(E) = 0;$$

$$f_4 = (1/2)[(1 + T_1)f_1 + (1 - T_1)f_2];$$

inelastic

fluctuations of potential

$$S_{23} = +\frac{e^2}{h}|eV|T_1R_1/2$$

quasi-elastic

fluctuations of distribution

$$S_{23} = -\frac{e^2}{h}|eV|R_1^2/4$$

Review on Shot Noise

« Shot Noise in Mesoscopic Conductors »

Ya. M. Blanter and M. Buttiker,

Phys. Rep. 336, 1 (2000)

Conference Proceedings

« Quantum Noise in Mesoscopic Physics »

Y.V. Nazarov, (Proceedings of NATO ARW,

Delft, The Netherlands, June 2-4, 2002), Springer

ISBN 1-4020-1