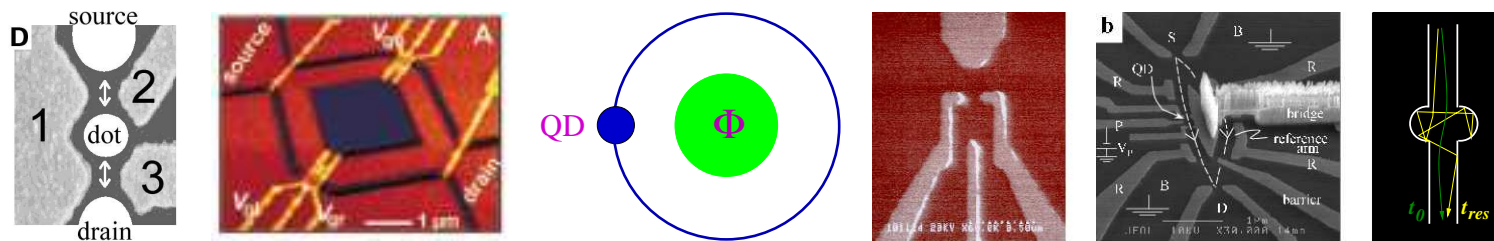


Kondo Effect and Phase-Coherent Transport in Quantum Dots



Kicheon Kang

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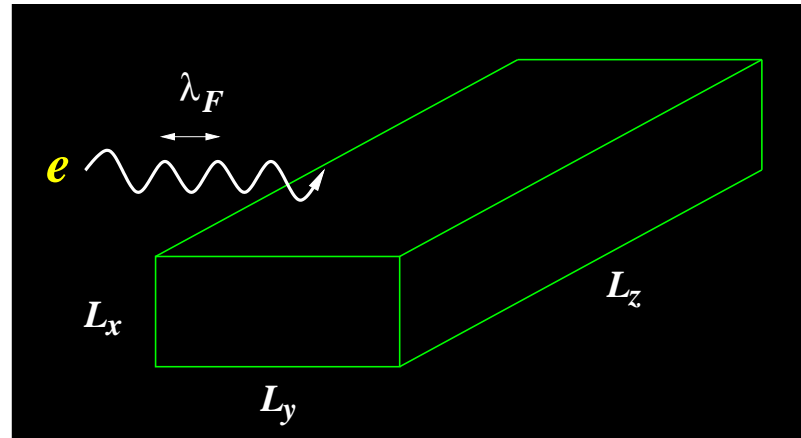
Outline

- **Coherent transport in quantum dots**
- **Kondo effect in quantum dots**
- **Phase coherence of the Kondo effect**
- **“Mesoscopic” Kondo effect and spin-charge separation**

Outline

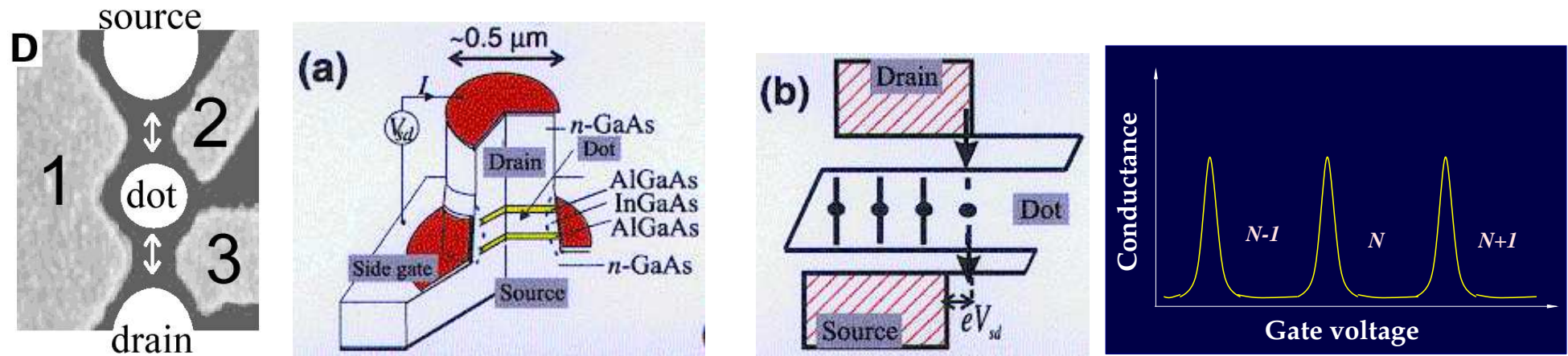
- **Coherent transport in quantum dots**
- Kondo effect in quantum dots
- Phase coherence of the Kondo effect
- “Mesoscopic” Kondo effect and spin-charge separation

Fermi Wavelength (λ_F) and Dimensionality



- $\lambda_F \ll L_x, L_y, L_z$: 3-dimension
- $L_x < \lambda_F \ll L_y, L_z$: 2-dimension (2-D electron gas)
- $L_x, L_y < \lambda_F \ll L_z$: 1-dimension (quantum wire)
- $L_x, L_y, L_z < \lambda_F$: 0-dimension (quantum dot)

Quantum Dot (Artificial Atom)



Charge and energy quantization

- E_c : single electron charging energy, Δ : energy level discreteness
- $E_c \equiv e^2/2C \gg k_B T$: Coulomb blockade, single electron tunneling (SET)
- $\Delta \gg k_B T$: Quantum confinement, resonant tunneling \rightarrow phase-coherent process

“Cotunneling” in the Coulomb Blockade Region

Averin & Nazarov (1990) - Theory; Eiles et al., PRL (1992) - Experiment

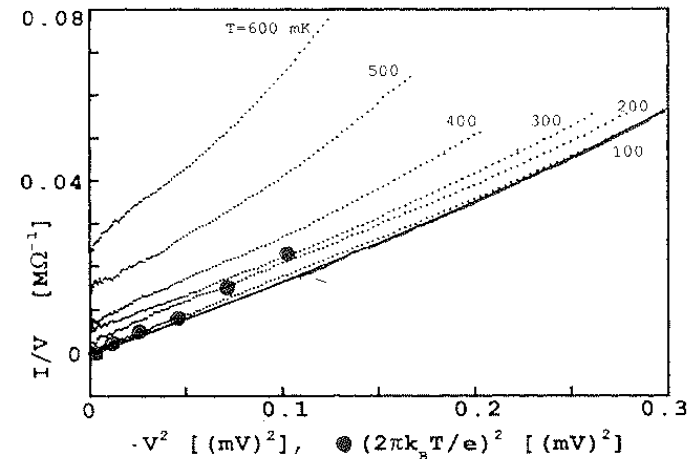
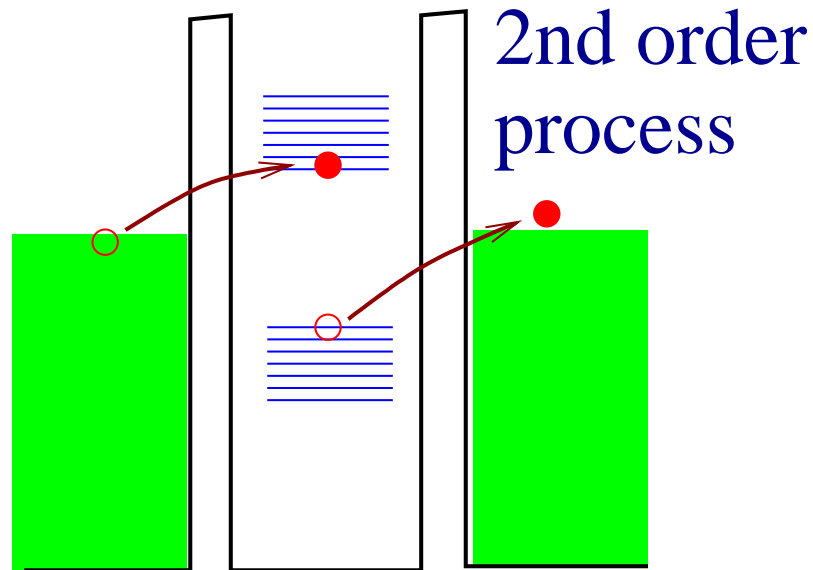


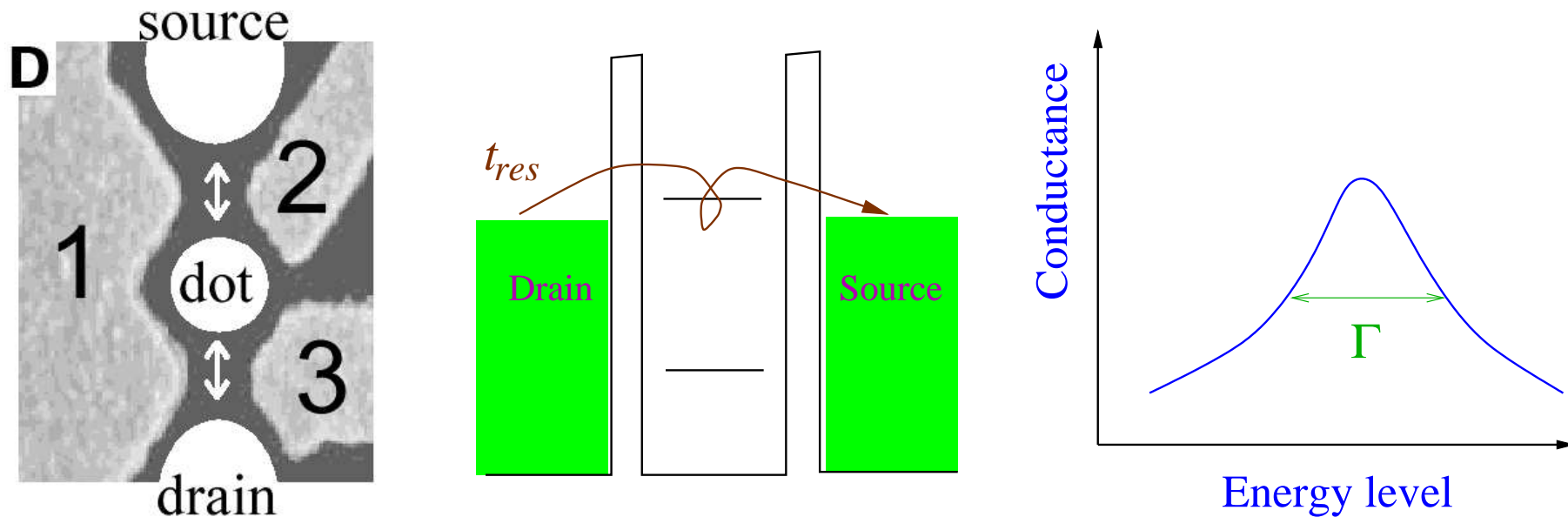
FIG. 3. Plot of I/V vs V^2 for $q=0.55$. Experimental data (solid points) are plotted for six values of T . Open circles are I/V as $V \rightarrow 0$ obtained from the six temperatures plotted vs $(2\pi k_B T/e)^2$. The solid line is the cotunneling prediction of Eq.

- **Cotunneling**: 2nd or higher order (virtual) process
 - Macroscopic quantum tunneling of charge

$$G \propto T^2, V^2 \quad \text{for } \Delta \rightarrow 0 \quad (\text{Inelastic})$$

- For $\Delta \gg k_B T, eV$ inelastic cotunneling current is strongly suppressed
(See e.g., Kang & Min, PRB (1997))

Resonant Tunneling through a Quantum Dot



- “Coherent” resonant tunneling through a single impurity level (ε_0) for $\Gamma \gg k_B T$

$$G = \frac{2e^2}{h} |t_{res}(E_F)|^2 = G_{max} \frac{1}{e_0^2 + 1} \quad \left(e_0 \equiv \frac{2}{\Gamma} (\varepsilon_0 - E_F) \right)$$

- Phase coherence of the transmission amplitude $t_{res} = |t_{res}| e^{i\gamma}$ cannot be directly addressed (Conductance measures $|t_{res}|$ only)

Detecting the Phase Coherence I

- 2-Terminal Aharonov-Bohm (AB) Interferometer (*Yacoby et al., PRL (1995)*)

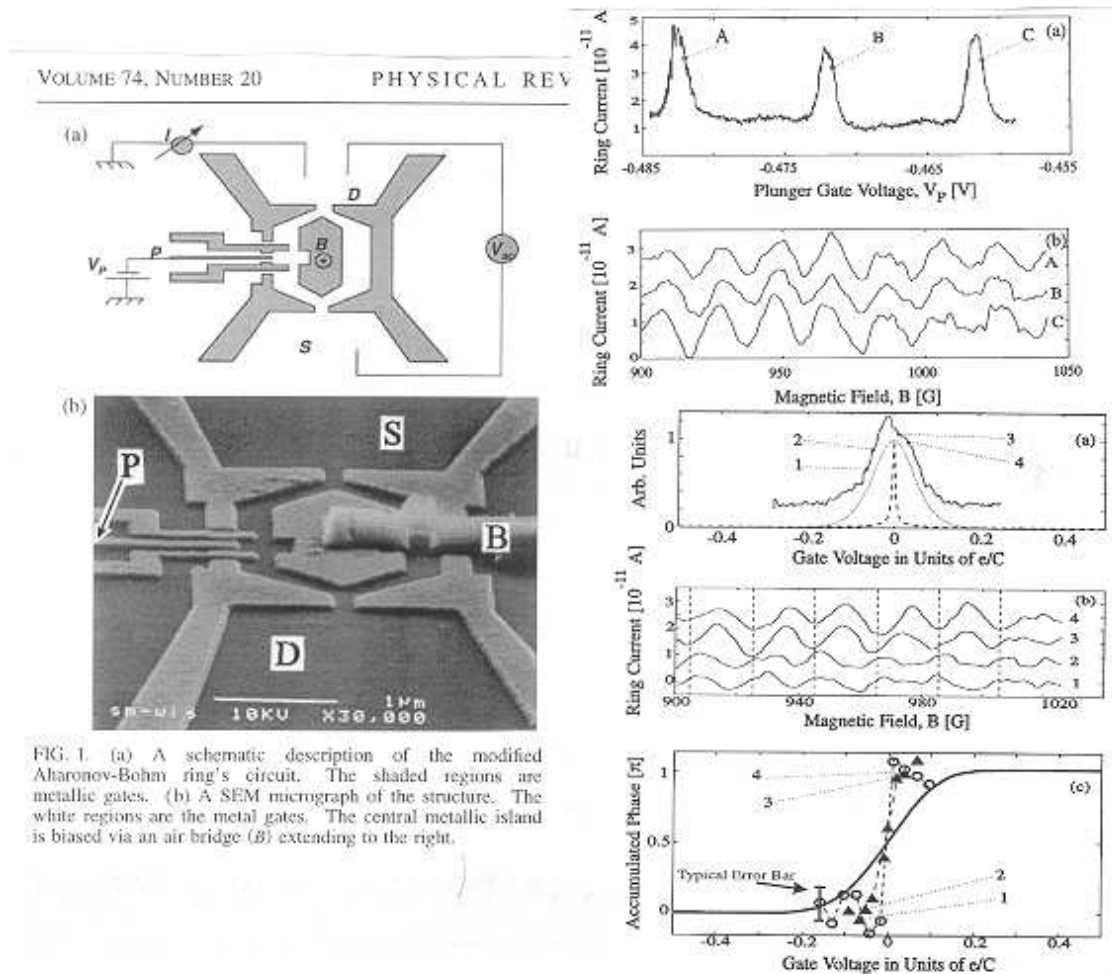
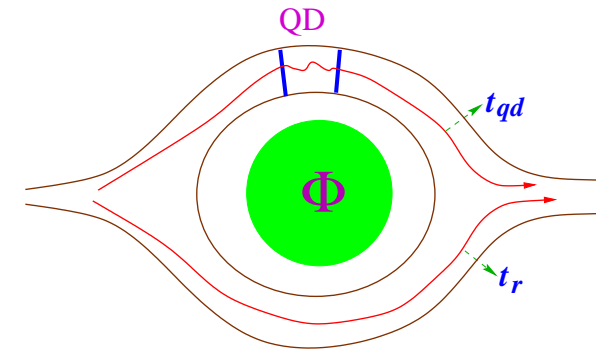


FIG. 1. (a) A schematic description of the modified Aharonov-Bohm ring's circuit. The shaded regions are metallic gates. (b) A SEM micrograph of the structure. The white regions are the metal gates. The central metallic island is biased via an air bridge (B) extending to the right.



$$G \sim |t_{qd} + t_r|^2$$

$$= T_0 + 2|t_{qd}| |t_r| \cos(\varphi_{qd} - \varphi_{AB})$$

Transmission Amplitude:

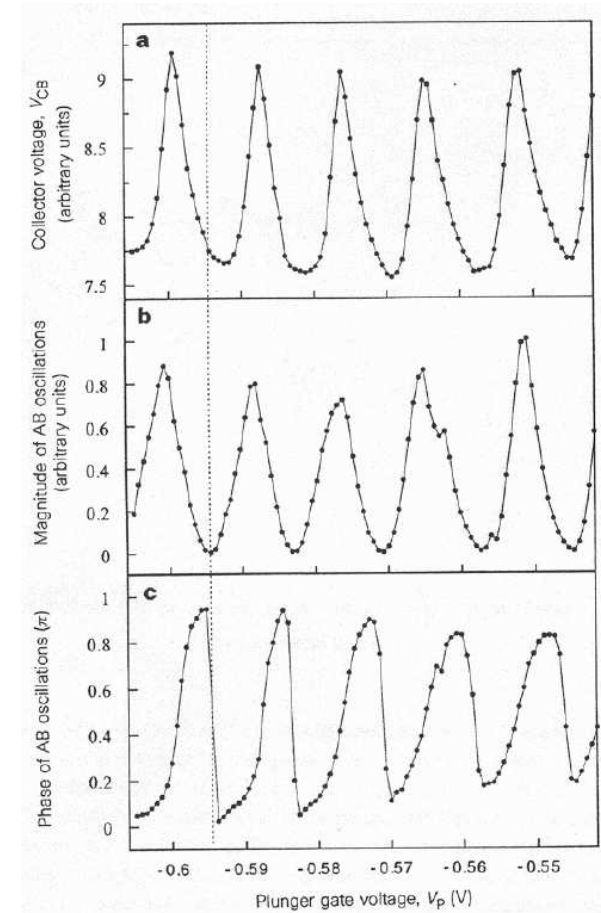
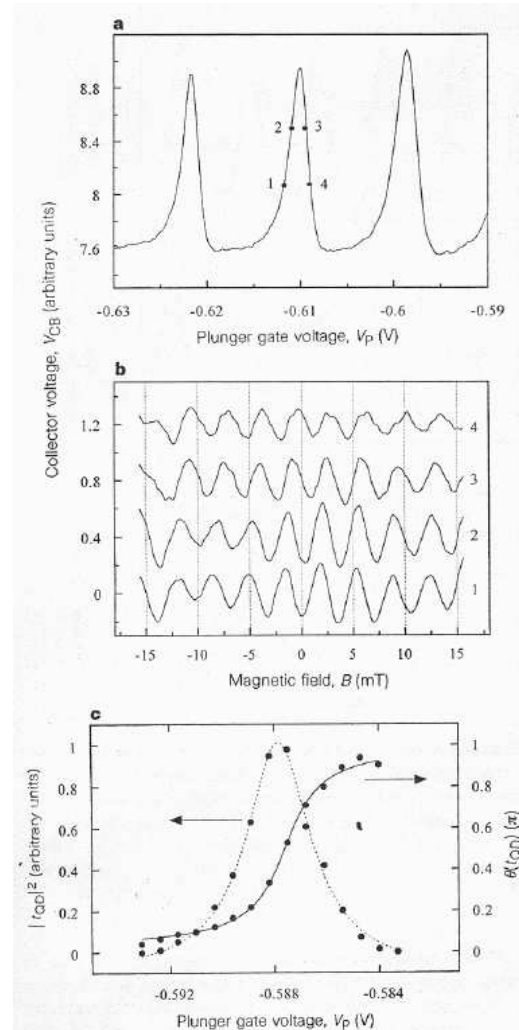
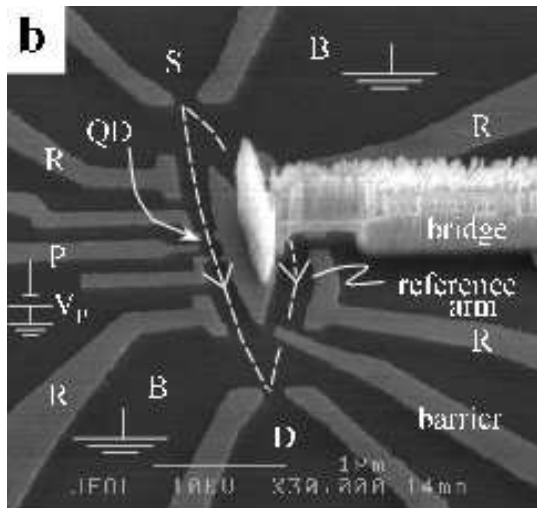
$$t_{qd} = |t_{qd}| \exp(i\varphi_{qd})$$

(Conventional SET experiment measures only $|t_{qd}|^2$)

- AB oscillation of the conductance \rightarrow **Phase coherence** of transmission through a QD
- Onsager's relation $G(-B) = G(B) \rightarrow$ **Phase rigidity** ($\varphi_{qd} = 0$ or π)

Phase Measurement of a Quantum Dot

- AB Interferometer with Open Geometry (Schuster *et al.*, Nature (1997))

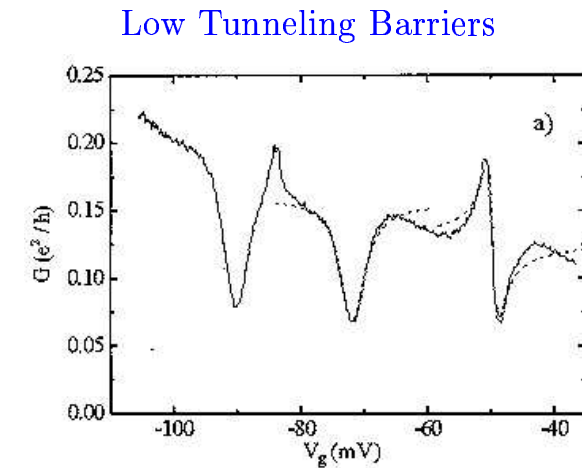
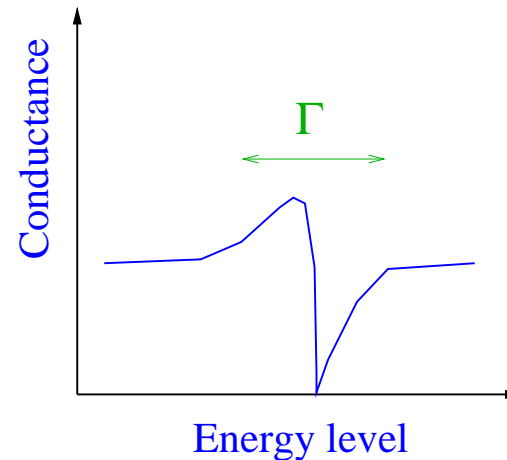
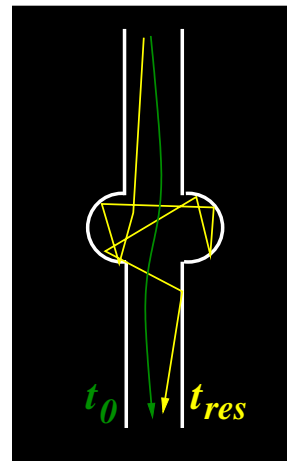
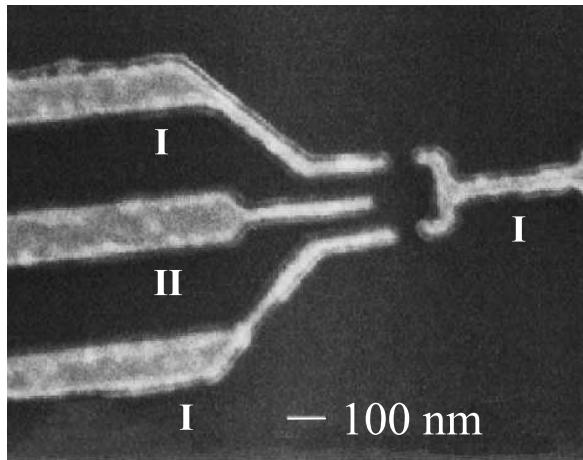


- Open geometry allows two-path interference and the continuous phase evolution of the QD
- Abrupt phase drop (π) accompanied by transmission zero / In-phase resonances

Detecting the Phase Coherence II

Coherence of the resonant tunneling

- Fano Resonances (Göres *et al.*, PRB (2000))



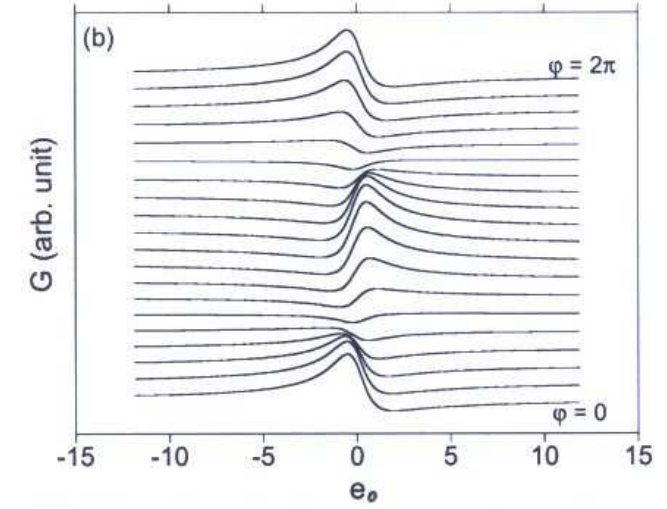
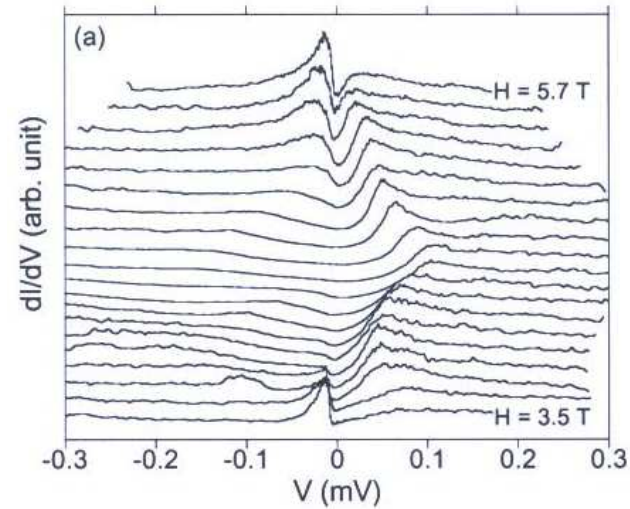
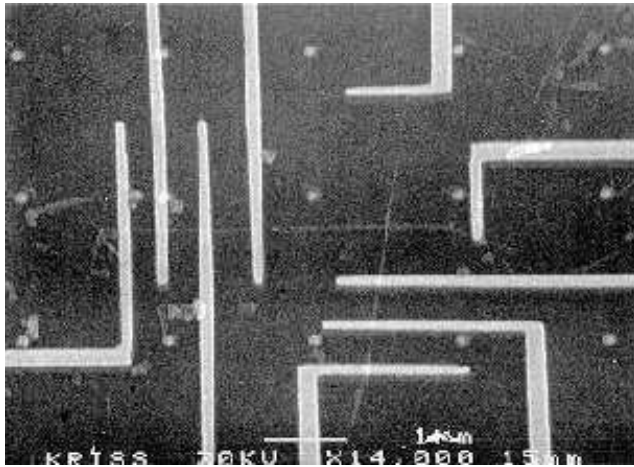
- Fano-resonance as a result of the interference between t_{res} and t_0

$$G \simeq \frac{2e^2}{h} |t_0 + t_{res}|^2 = G_0 \frac{|e_0 + Q|^2}{e_0^2 + 1}$$

- Asymmetric line shape : Evidence of the phase coherence

Fano Resonance in Carbon Nanotubes

(J. Kim, Kicheon Kang, and coworkers, PRL (2003))



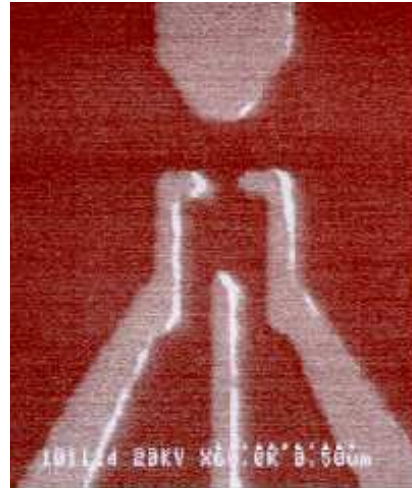
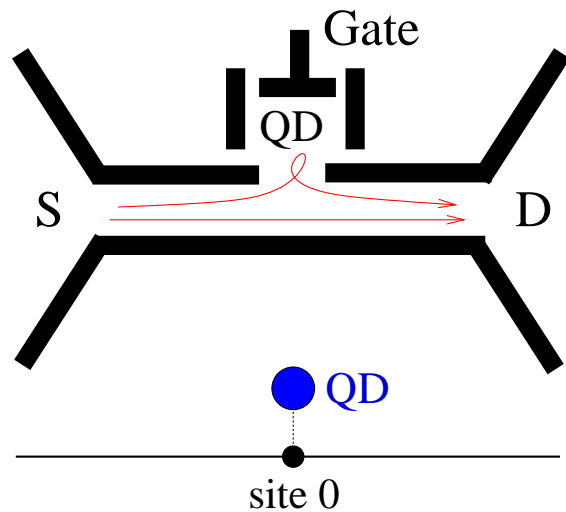
- Resonant “cavity” at the contact region
- “Aharonov-Bohm oscillation” of the Fano resonances
- Agrees well with the theoretical prediction (K. Kang, unpublished)

$$G \simeq G_0 \frac{|e_0 + Q|^2}{e_0^2 + 1}, \quad Q = Q_R \cos \varphi + iQ_I \sin \varphi$$

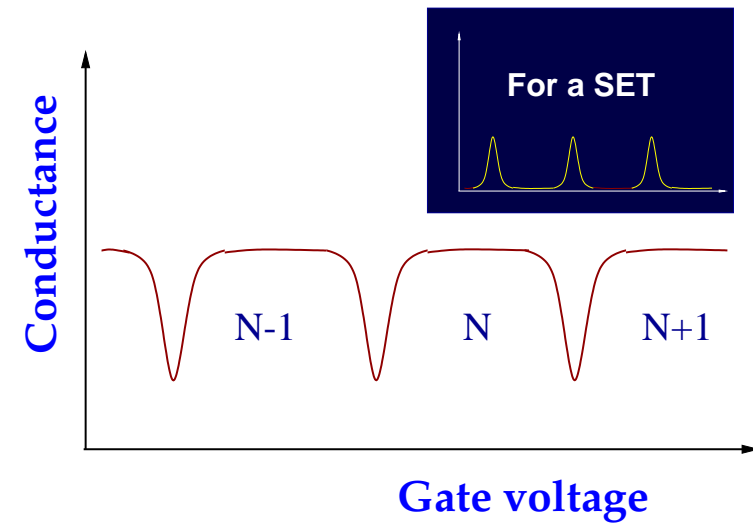
(See also Hofstetter *et al.* (2001), Kim *et al.* (2002))

“Anti-Coulomb-Blockade” ?

Kicheon Kang *et al.*, PRB(2001)



ETRI (2002)



- Theoretical prediction for 1D wire + dot:

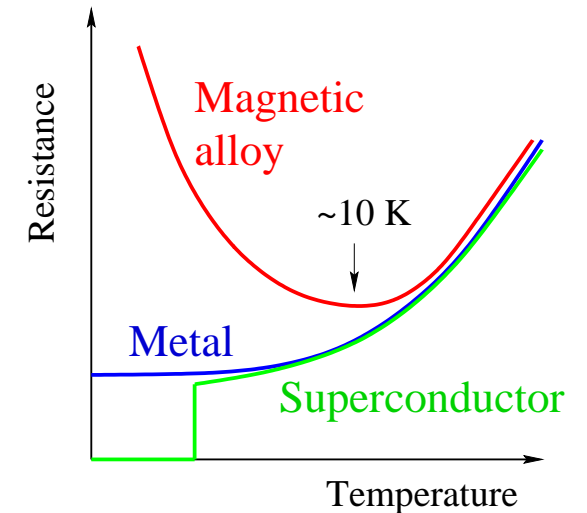
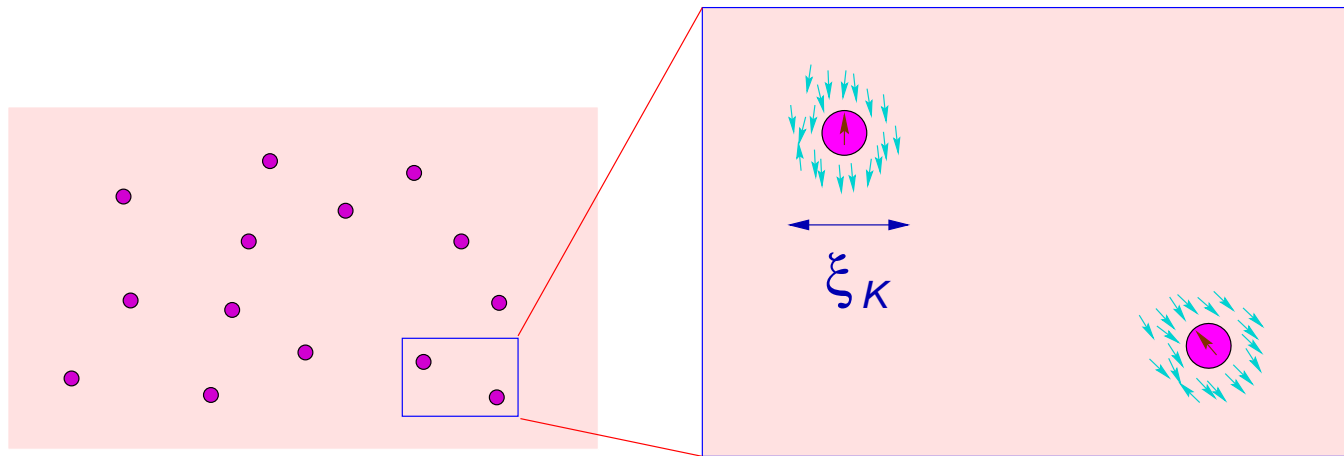
$$G = (e^2/h) \cos^2 \pi n_d$$

- $G = e^2/h$ for $n_d = N$ (integer)
- Anti-resonances around $n_d = N + 1/2$ ← “destructive interference”
- Quasi-periodic “Coulomb-blockade anti-resonance” oscillation is expected
- Experimental challenge to verify the prediction (ETRI)

Outline

- Coherent transport in quantum dots
- **Kondo effect in quantum dots**
- Phase coherence of the Kondo effect
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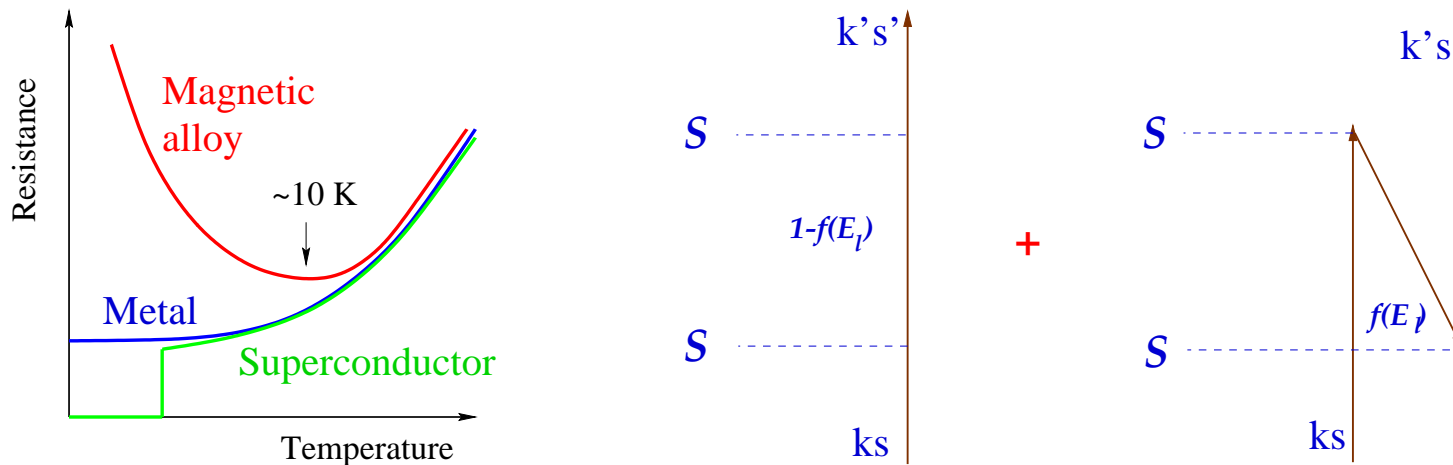
Kondo Effect in Dilute Magnetic Alloys



- Antiferromagnetic exchange interactions between the impurity spin and the conduction electron spin
- **(Entangled) spin singlet ground state** is formed between the impurity and the conduction electrons
- Screening cloud around the impurity (with its size ξ_K and the binding energy T_K) that screens the local magnetic moment
- Enhanced resistivity at low temperatures due to the large spin-flip scattering cross section

Origin of the Resistance Anomaly

J. Kondo (1964) (Cf. D. J. Kim, *Many-body theory of metallic magnetism*)



- Spin-exchange Hamiltonian

$$H = -J \sigma \cdot S$$

- 2nd-order spin-exchange scattering amplitude:

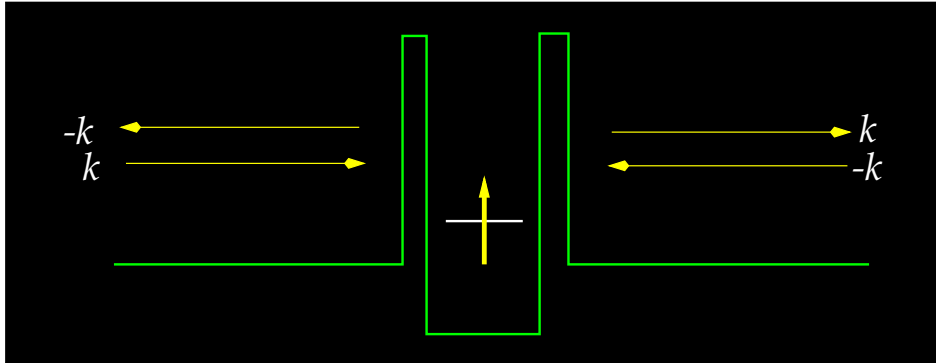
$$\sim J^2 \sum_l \frac{f(E_l)}{E_F - E_l} \sim -J^2 N(E_F) \log(k_B T/W)$$

- Logarithmic singularity ← **time-reversal asymmetry** of the two processes!

Kondo-Resonant Transmission

Coherence of the Kondo resonance

Langreth (1966), Glazman & Reich (1988), Ng & Lee (1988)



$$|\text{out}\rangle = S|\text{in}\rangle$$
$$S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

- Generalized Friedel sum rule: No inelastic scattering takes place at $T = 0$

$$|t|^2 = \sin^2 \frac{\pi}{2} n$$

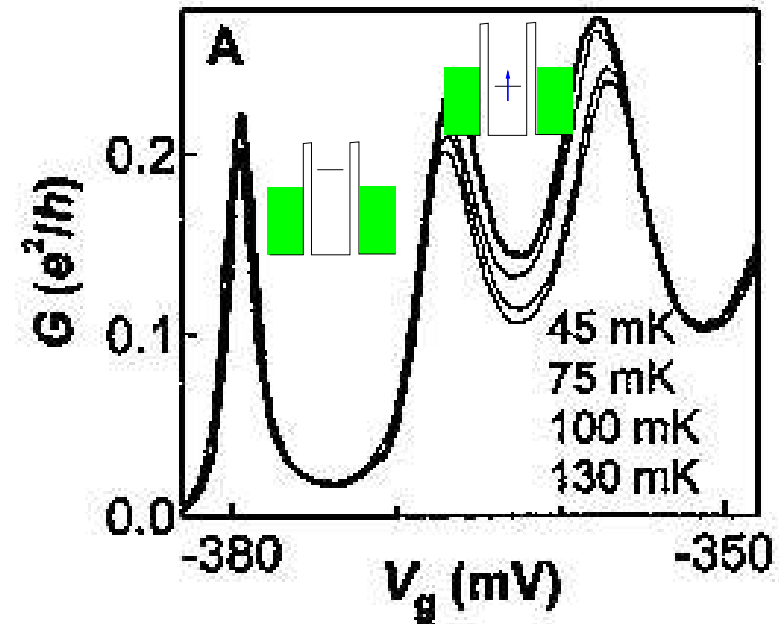
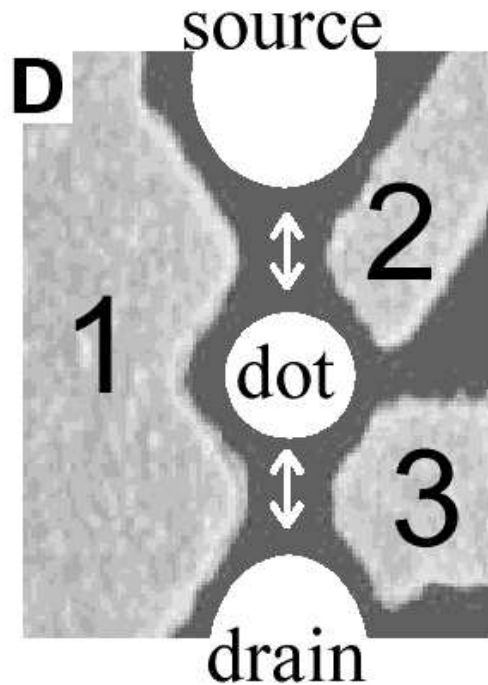
- Conductance formula

$$G = \frac{2e^2}{h} |t|^2 = \frac{4e^2}{h} \tilde{\Gamma} \rho(\mathbf{E}_F) \longrightarrow 2e^2/h$$

in the (unitary) Kondo limit

Kondo Effect in Quantum Dots

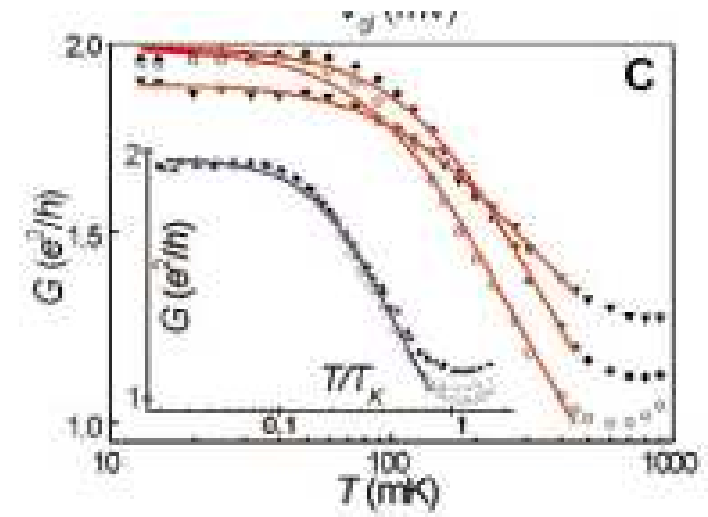
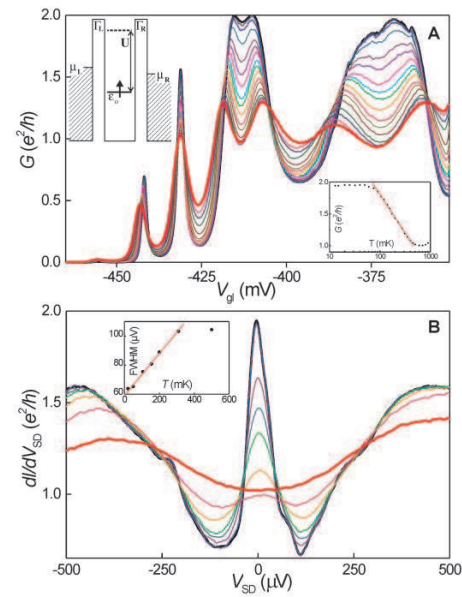
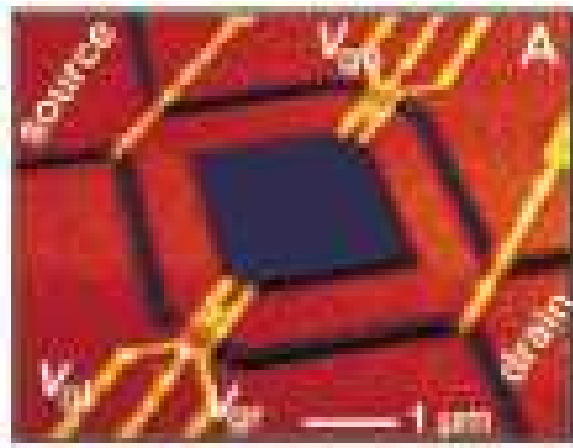
Goldhaber-Gordon *et al.* (Nature 1998), Cronenwett *et al.* (Science 1998), and more...



- Strong even-odd effect
- Conductance enhanced at low temperatures in the “Kondo” valley

Kondo Effect in the Unitary limit

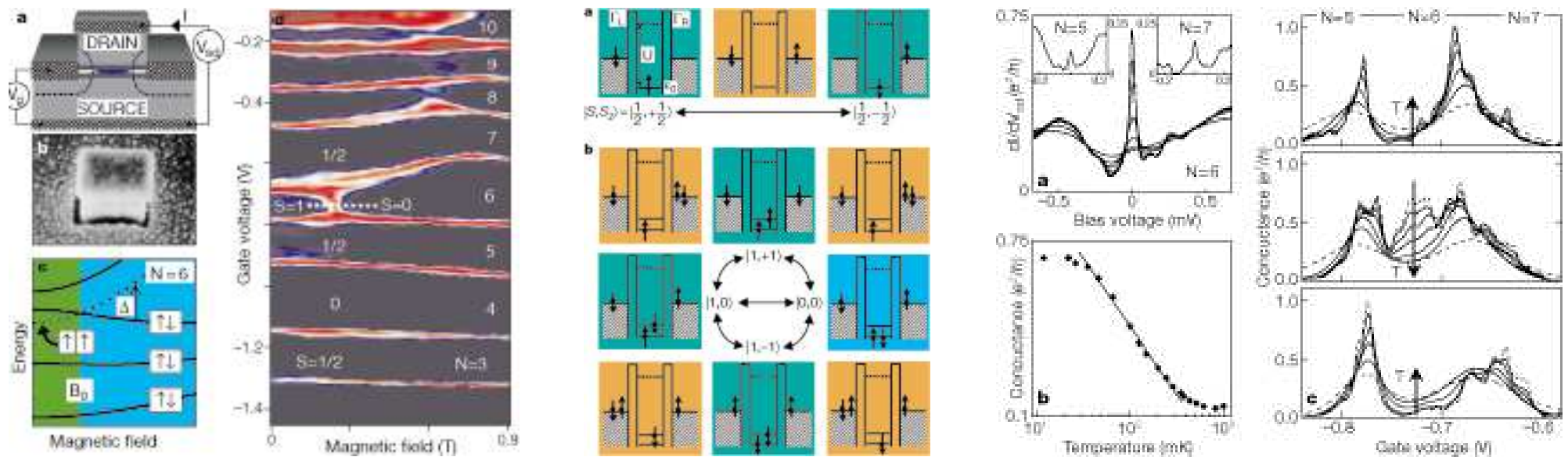
W. van der Wiel *et al.* (Science 2000)



- Unitarity / universal scaling

Kondo Effect in an Integer-spin QD

S. Sasaki *et al.* (Nature 2000)



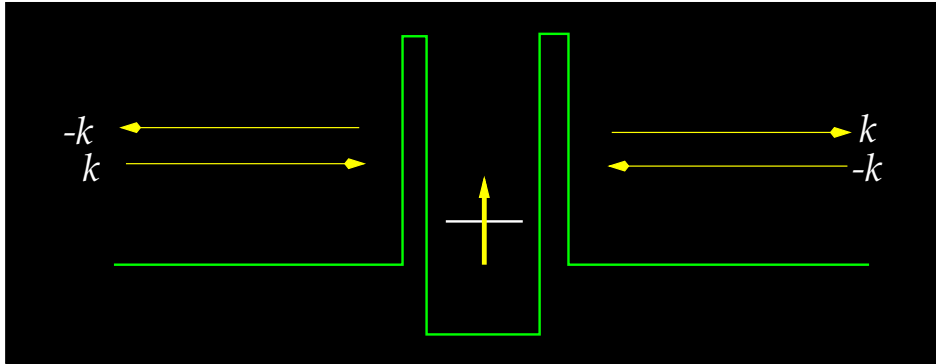
- Enhanced Kondo effect at degeneracy between $S = 0$ and $S = 1$

Outline

- Coherent transport in quantum dots
- Kondo effect in quantum dots
- **Phase coherence of the Kondo effect**
- “Mesoscopic” Kondo effect and spin-charge separation

Kondo Resonance as a Scattering Problem

Langreth (1966), Glazman & Reich (1988), Ng & Lee (1988)



$$|\text{out}\rangle = S|\text{in}\rangle$$
$$S = \begin{pmatrix} r & t \\ t' & r' \end{pmatrix}$$

- Generalized Friedel sum rule: No inelastic scattering takes place at $T = 0$

$$t = -i \sin \gamma e^{i\gamma}, \quad \gamma = \frac{\pi}{2}n$$

- $G = (2e^2/h) \sin^2 \gamma$: Conductance enhanced in the Kondo limit ($n = 1$)
- Phase shift of $\Delta\gamma = \pi/2$

Phase Evolution in a Kondo System

Y. Ji *et al.*, Science (2000); PRL (2002)

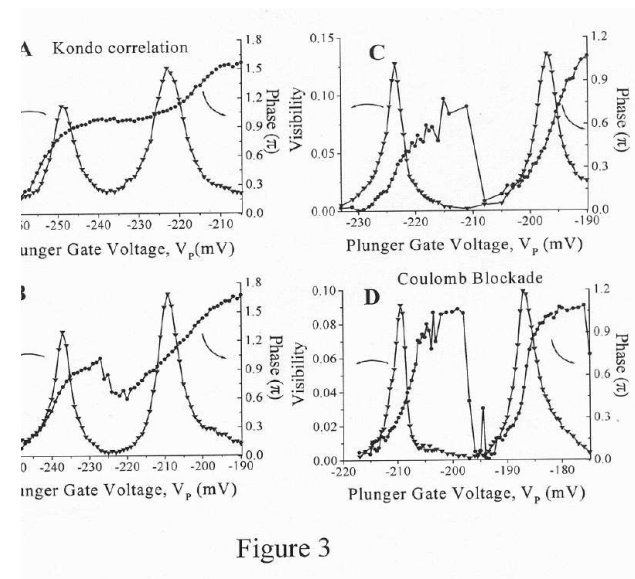
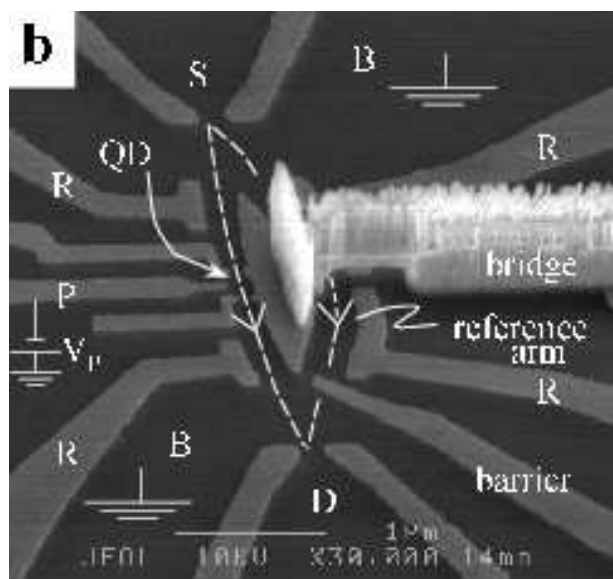
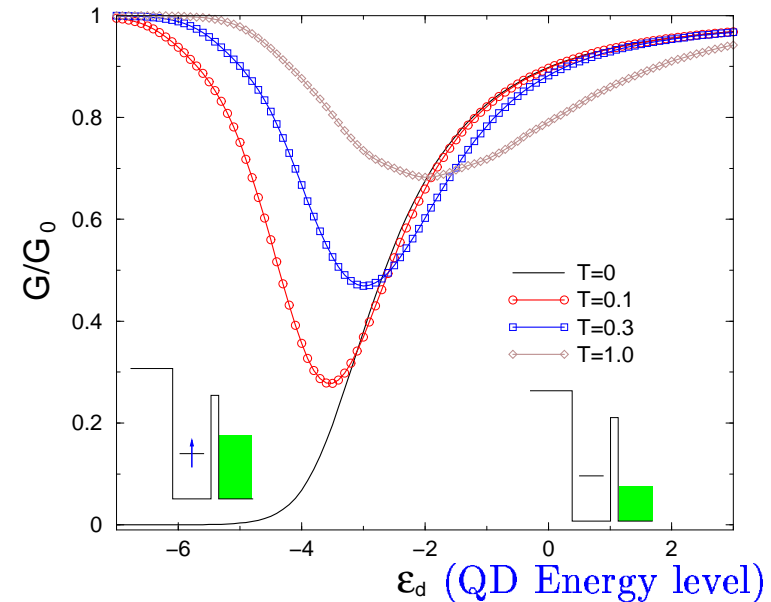
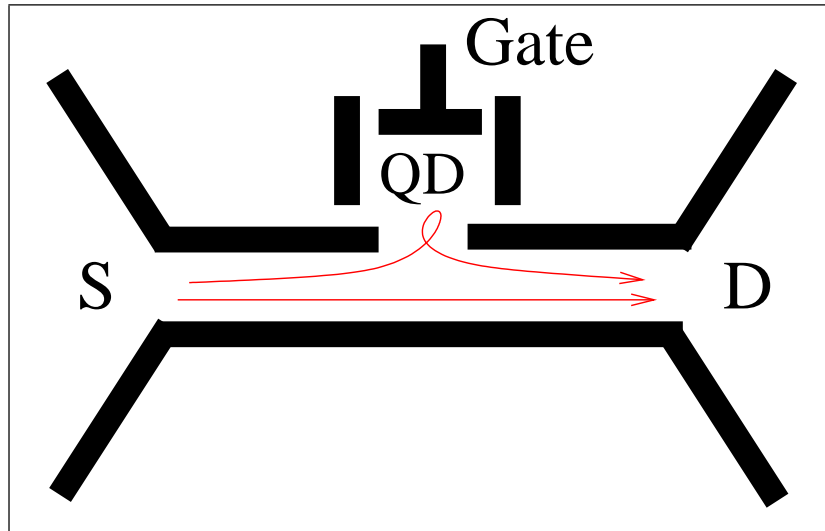


Figure 3

- Phase-coherent transmission: From the Coulomb blockade to the Kondo limit
- Anomalous Kondo plateaus with $\Delta\gamma \sim \pi$ instead of $\pi/2$

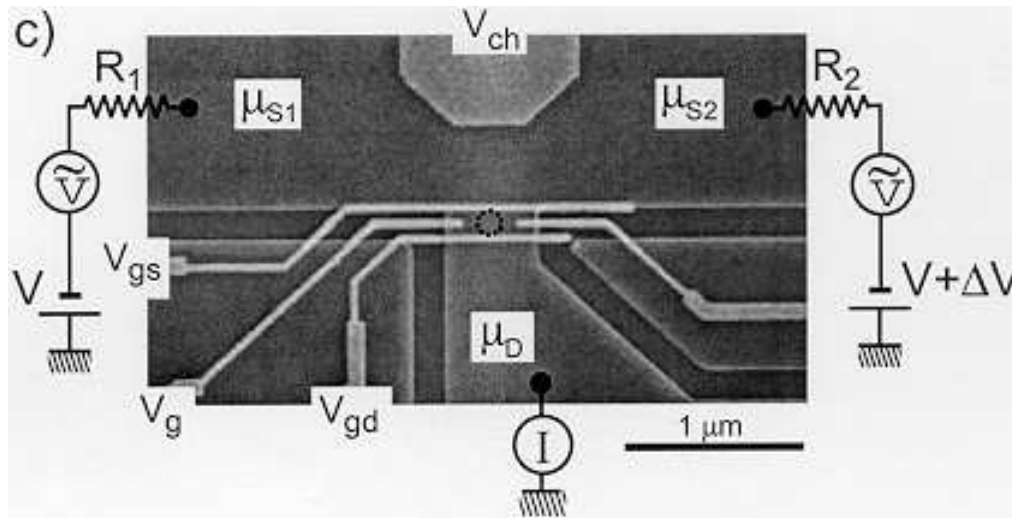
Anti-Kondo Resonance

K. Kang *et al.*, PRB(2001)

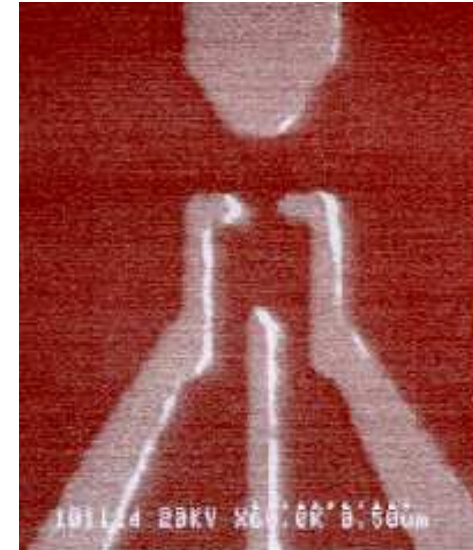


- Theoretical prediction : $G = (2e^2/h) \cos^2 \pi n_d / 2$ (for $T = 0$)
- For an isolated one-dimensional ballistic wire: $G = G_0 \equiv 2e^2/h$
- For $S = 0$, $G \rightarrow G_0$: The wire and the dot is effectively decoupled
- For $S = 1/2$, $G \rightarrow 0$: Destructive interference between the ballistic channel and the **spin** (Kondo) channel
- Anti-resonance behavior at finite temperatures

Anti-Kondo Resonance : Experimental Challenges



Franceschi et al. (2002)



ETRI (2002)

- Delft University (Leo Kouwenhoven, PRL (2002))
- Nanoelectronic Devices Team, ETRI, in progress

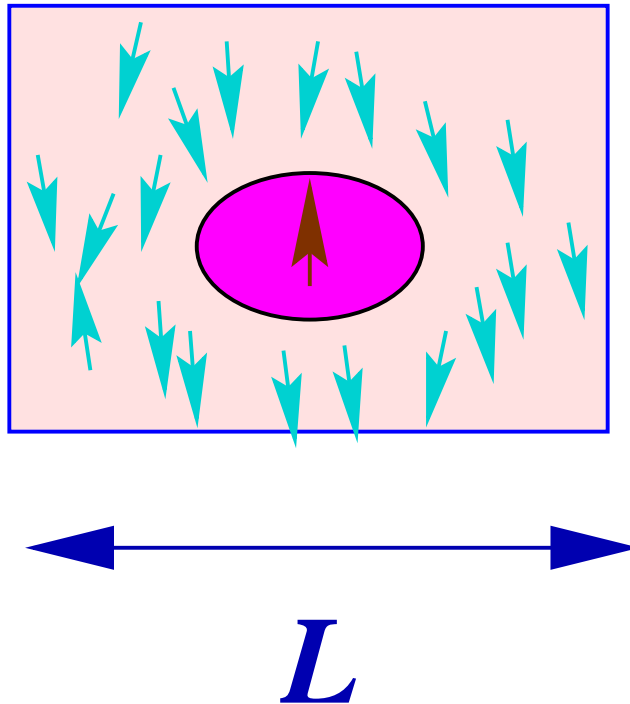
Outline

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Mesoscopic Kondo Effect

What Happens If $\xi_K \sim > L$ (or $T_K \sim < \delta$) ?

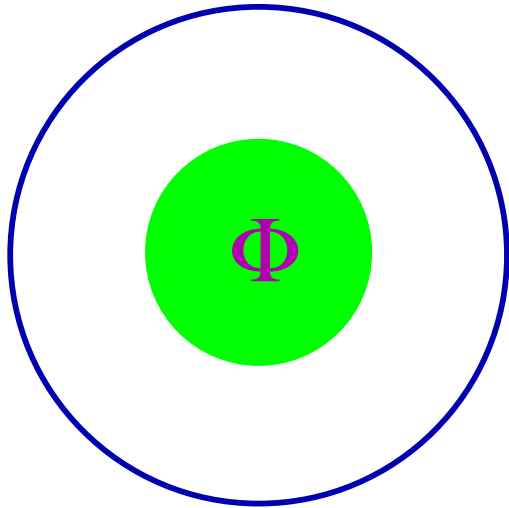
$$(\delta = \hbar v_F / L, T_K = \hbar v_F / \xi_K)$$



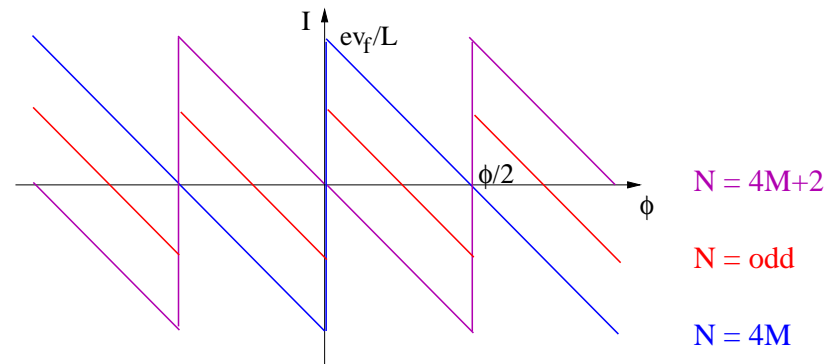
→ *Mesoscopic Kondo Effect*

- Kondo Box: A single Kondo impurity embedded in a mesoscopic box (Thimm *et al.*, PRL (1999))
- Composite QD - AB ring: More systematic study is possible as a function of the AB phase etc.

Persistent Current in a Perfect 1D Ring



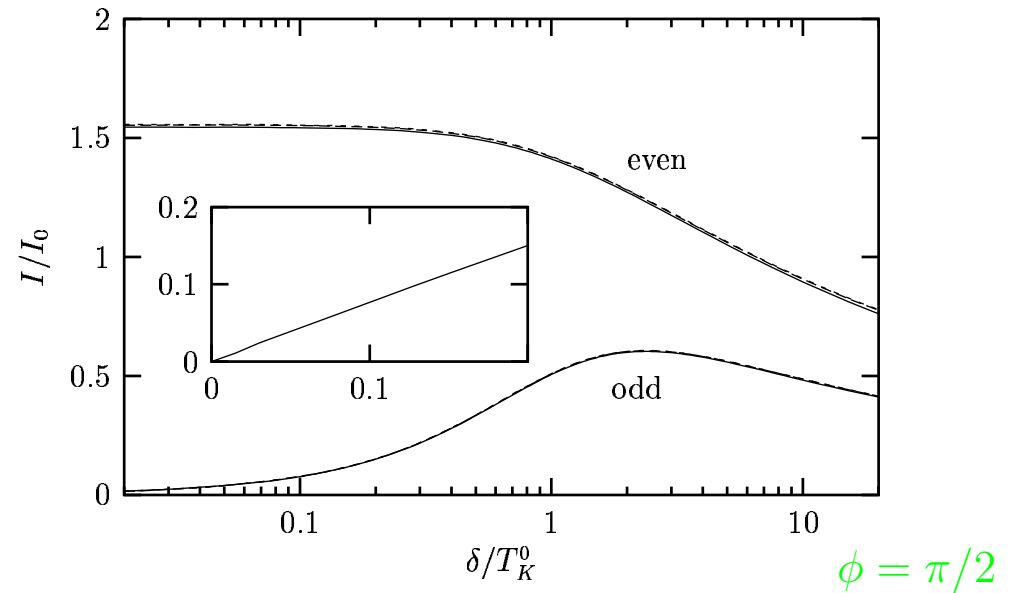
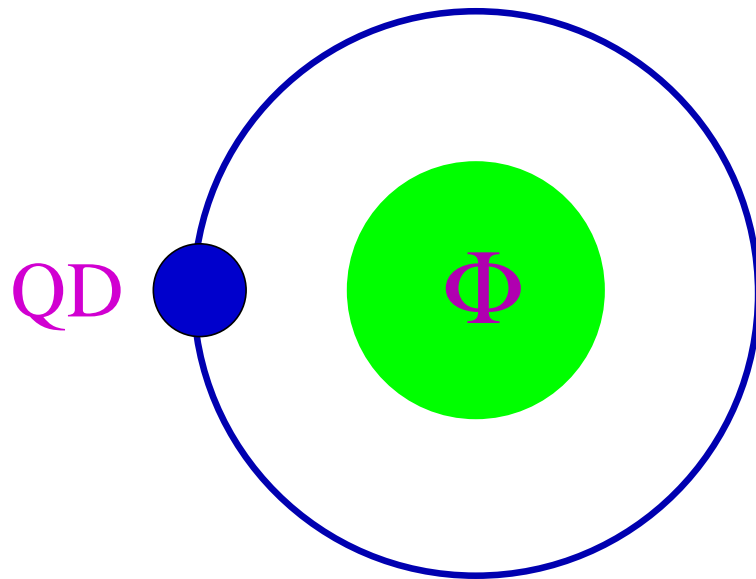
Spin 1/2 (Loss and Goldbart (1991))



- Persistent current (PC): Indicator of **quantum coherence** in mesoscopic scale
- Elastic scattering (e.g. due to geometric imperfection) reduces the current, but does not destroy the phase-coherence ($L_\phi \neq l$)

Kondo Effect and the Persistent Current I

Kang & Shin, PRL (2000)



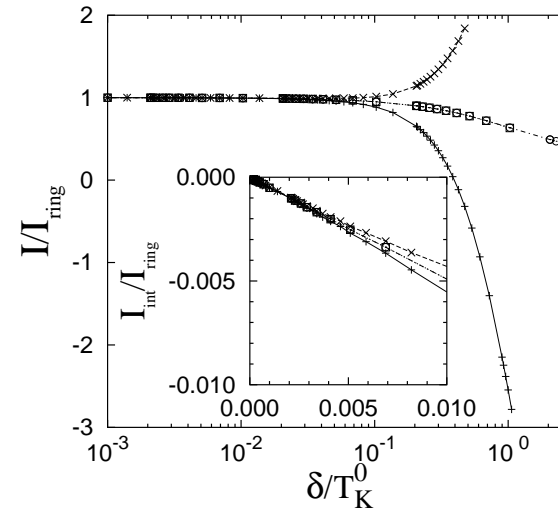
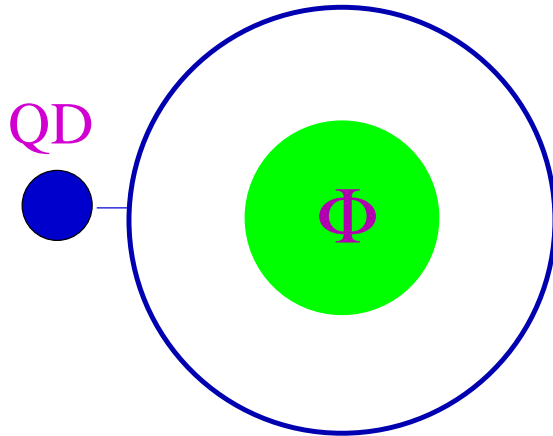
- Leading order $1/N_s$ expansion (N_s : Spin degeneracy)
- I_0 : current amplitude for an ideal ring
- Kondo assisted tunneling through the QD provides persistent current (I)
- Current suppressed for $\delta/T_K \gg 1$: Destruction of the screening cloud \rightarrow *Mesoscopic Kondo effect*
- Universal scaling with strong even-odd dependence
- Suppression of the current for odd parity : because $I_{\downarrow} \sim -I_{\uparrow}$

Leggett's Theorem

- For any 1D mesoscopic ring with N spinless electrons, the persistent current is
 - Diamagnetic (Ground state at $\Phi = n\Phi_0$) for odd N , and
 - Paramagnetic (Ground state at $\Phi = (n + 1/2)\Phi_0$) for even N ,regardless of interactions, disorder, etc.

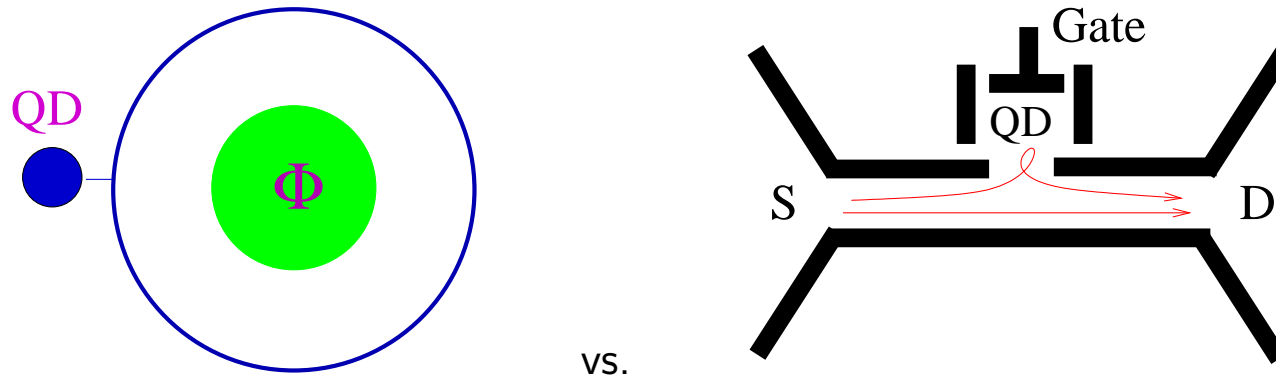
Kondo Effect and the Persistent Current II

Cho, Kang and coworkers, PRB (2001)



- $I(\phi) = I_{\text{ring}}(\phi) + I_{\text{int}}(\phi)$ (I_{ring} : from the ideal ring, I_{int} : from the interaction)
- $I \rightarrow I_{\text{ring}}$ in the continuum limit ($\delta/T_K \rightarrow 0$):
Agrees with the exact Bethe ansatz result (H.P. Eckle *et al.*, PRL (2001))
- Linear reduction of the normalized current for small δ/T_K : $I/I_{\text{ring}} = I_{\text{ring}} - c \cdot \delta/T_K$
- For $\delta \gg T_K$, $I \rightarrow I_{\text{ring}}^{N-1}$ (PC of an ideal ring with an electron missing):
Effective decoupling of the ring and the dot

Remarks on Some Theoretical Debates



Behavior of the Persistent Current (PC) in the Limit of $L \rightarrow \infty$

- For any 1D noninteracting system, a correspondence exists between the PC and transport in an open system ([Gogolin \(1994\)](#)): If $T(E_F) = 1$, then $I/I_{\text{ring}} = 1$
- A Kondo-correlated QD does not seem to meet this condition

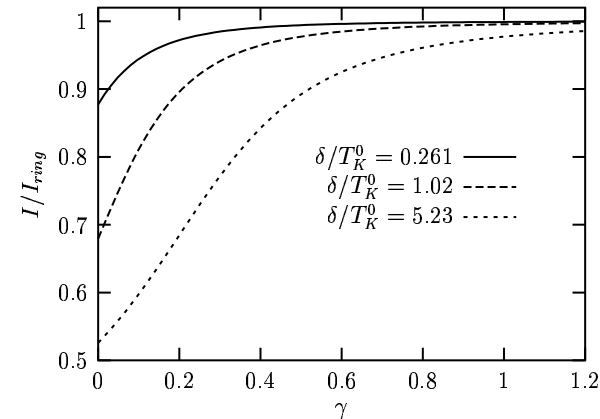
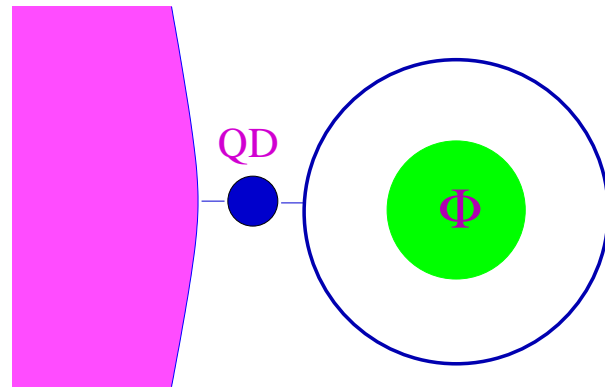
$$T(E_F) = 1, \quad \text{but } I/I_{\text{ring}} = 0$$

- Transport counts “electrons”, but PC does not! [Cho et al., PRB \(2001\)](#), [Eckle et al., PRL \(2001\)](#), etc.
- Opposite result by [Affleck et al., PRL \(2001\)](#) based on RG

$$T(E_F) = 1, \quad I/I_{\text{ring}} = 1$$

Mesoscopic Kondo Effect and Dephasing?

K. Kang *et al.*, PRB (2002)



$$\phi = 0.1\pi$$

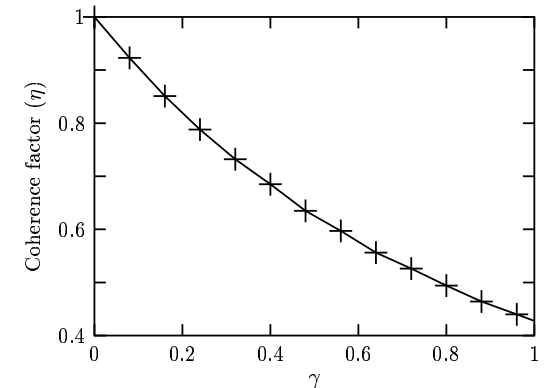
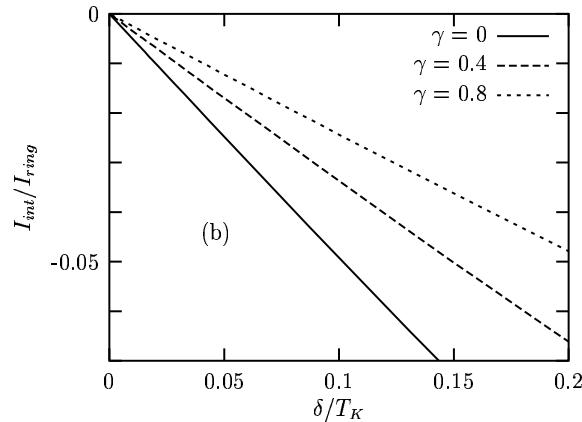
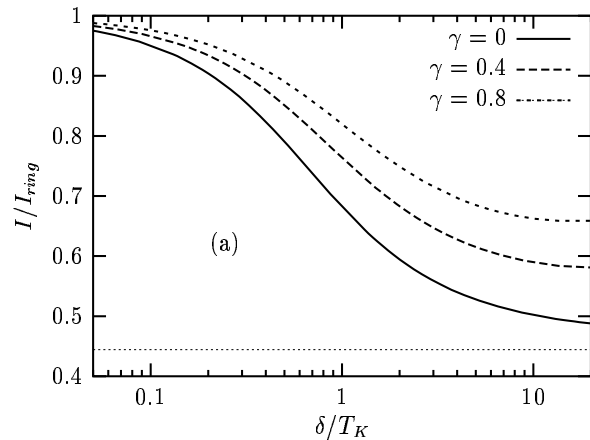
γ : relative coupling strength of dot-reservoir to dot-ring

- $I(\phi) = I_{ring}(\phi) + I_{int}(\phi)$: Universal function of δ/T_K^0 , γ
- The PC increases as γ increases: Counterintuitive!

Effects of the Dot-Reservoir Coupling

- To enhance the Kondo energy scale T_K (and thus reduces δ/T_K) \rightarrow Enhances the PC
- To induce dephasing ?

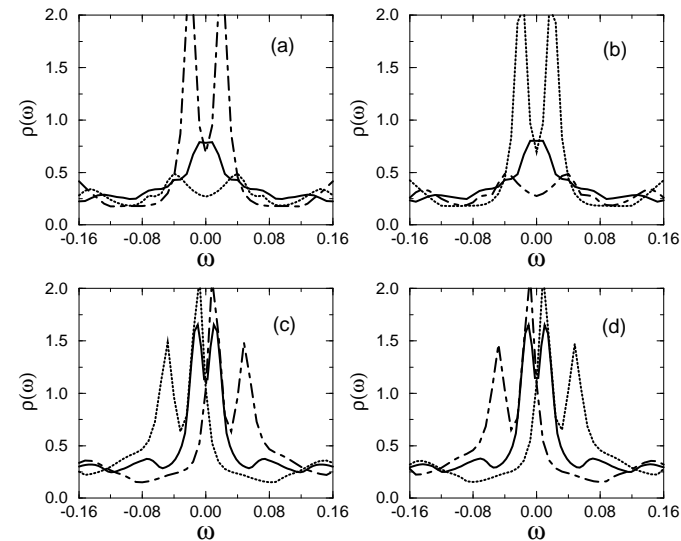
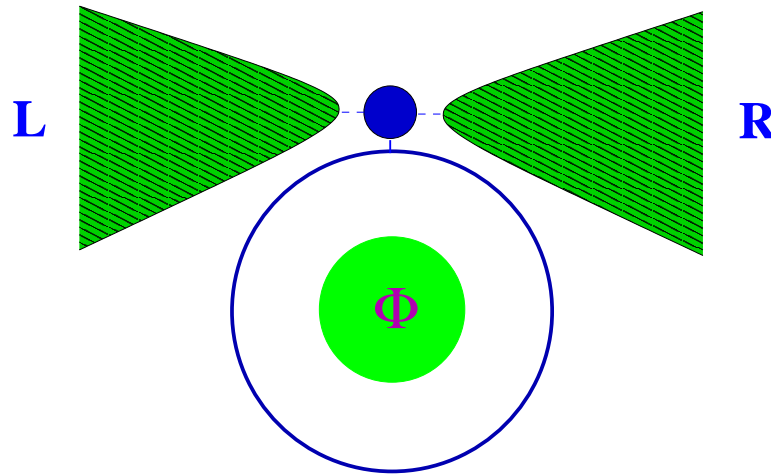
Spin Fluctuation Induced Dephasing



- Current as a function of δ/T_K : The effect of the rescaled Kondo energy extracted
- Even after subtracting the effect of renormalized T_K , the PC increases due to the dot-reservoir coupling
 → Dephasing tends to **enhance** the AB oscillation !!
- Dot-reservoir coupling reduces I_{int} only: $I \rightarrow I_{\text{ring}}$ for $\gamma \gg 1$
- Can be interpreted as **spin-charge separation**: The reservoir degrades the coherence of spin degree of freedom (I_{int}) only, not affecting the charge one (I_{ring})
- **Coherence factor** defined by the relative strength of the slope in I_{int} vs. δ/T_K : $\eta \equiv c(\gamma)/c(0)$

Detecting the Mesoscopic Kondo Effect by Transport

Kang & Craco, PRB (2002)



(a) $N = 100$ (b) $N = 102$ (c) $N = 101$ (d) $N = 103$
 $\varphi = 0$ (dot-dashed), $\varphi = \pi/2$ (solid) $\varphi = \pi$ (dotted)

- Mesoscopic Kondo effect can be detected by transport experiment for weak lead-dot coupling
- Phase-dependent Kondo resonance for small ring ($L < \xi_K$) with modulo 4 (For larger rings the Kondo resonance is not phase-sensitive)

Summary

- Coherent transport in quantum dots
- Kondo effect in quantum dots
- Phase coherence of the Kondo effect
- “Mesoscopic” Kondo effect and spin-charge separation