

## TITLES and ABSTRACTS

**Benoit Daniel** (l'Université Paris 12 - Val de Marne)

*Title:* Harmonic maps and minimal surfaces in the Heisenberg group

*Abstract:* We study the Gauss map of minimal surfaces in the Heisenberg group  $Nil_3$  endowed with a left-invariant Riemannian metric. We prove that the Gauss map of a nowhere vertical minimal surface is harmonic into the hyperbolic plane  $\mathbb{H}^2$ . Conversely, we obtain a Weierstrass-type representation formula that allows to recover a minimal surface from a harmonic map into  $\mathbb{H}^2$ . We apply it to construct properly embedded minimal annuli. We use these annuli to prove a half-space type theorem, and we prove that complete minimal graphs are entire. We will also review some recent results on constant mean curvature surfaces in  $\mathbb{H}^2 \times \mathbb{R}$ .

**Mohammad Ghomi** (Georgia Institute of Technology)

*Title:* Locally convex hypersurfaces with boundary

*Abstract:* We begin with a survey of recent results concerning the geometry and topology of hypersurfaces with nonnegative curvature and prescribed boundary in Euclidean space. A fundamental open problem is when a closed curve in 3-space bounds a surface of positive curvature. In joint work with Stephanie Alexander and Jeremy Wang, the author has shown that if the curve is smooth, then there exists at most finitely many distinct topological solutions. On the other hand, this is not true if the curve is only differentiable. We will also mention joint work with Marek Kossowski on the applications of the h-principle to this problem.

**Robert Gulliver** (University of Minnesota)

*Title:* Total curvature of curves and of graphs

*Abstract:* Where singularities are present, there may be a bifurcation of geometric concepts. We are interested in carrying over ideas of curvature to curved polyhedra, as a generalization of submanifolds. As a first example, we consider the integral with respect to arc length of the curvature of a piecewise-smooth curve in an ambient manifold: the total curvature. The exterior angle at a vertex (of valence two) serves as the correct contribution at the vertex to total curvature in many contexts, such as the theorems of Gauss-Bonnet and of Fary-Milnor. However, the total curvature of a smooth curve also appears as the maximum variation of length among variation vector fields having length at most one: for this application, the contribution at a vertex is twice the sine

of half the exterior angle. This bifurcation of concepts becomes more fundamental when the curve is replaced by a graph, at a vertex of valence three or greater: one notion of total curvature, called cone total curvature, is useful for bounding the density of a 2-dimensional minimal current with the given graph as boundary; and a different notion, called net total curvature, is relevant to the isotopy class of the graph. This is joint work with Sumio Yamada.

**Robert Hardt** (Rice University)

*Title:* Some remarks on  $W^{2,2}$  mappings

*Abstract:* Consider a mapping  $u$  from a smooth domain to a compact Riemannian manifold  $N$  whose Hessian energy  $\int |D^2u|^2 dx$  is finite. In joint work with T. Riviere, relations between the Hessian energy, the topological singularity of the map, and approximability by smooth maps is studied for  $W^{2,2}(B^5, S^3)$  and other topologically related cases. We also describe constructions needed for this work and for joint work with CY Wang on proving the optimal partial regularity of Hessian energy-minimizers with various target manifolds  $N$ .

**Soomin Kim** (Rice University)

*Title:* Limits of minimal surfaces with increasing genus

*Abstract:* This talk is devoted to the classification of minimal surfaces: specifically, limits to a family of minimal surfaces with increasing genus. I will show that a subsequence of generalized Enneper surfaces of genus  $n$  converges to a minimal surface of infinite genus. It is anticipated that the only possible limit surface will be the Scherk surface. This is the first nontrivial example of a limit being taken of a family of minimal surfaces of increasing topological complexity. As a classification result, this would limit the set of possible minimal surfaces, as we would see that new surfaces would not be created through the taking of limits of existing families of surfaces in this way.

**Motoko Kotani** (Tohoku University)

*Title:* Geometric aspects of random walks on a crystal lattice

*Abstract:* A crystal lattice is an infinite graph with periodicity, such as the interger lattices, the triangular lattice, and the hexagonal lattice. I will talk how geometry of a crystal lattice relates with long time behaviors, such as a central limit theorem and a large deviation property, of random walks on it. Spectra of magnetic transition operators are also discussed.

**Rob Kusner** (University of Massachusetts Amherst)

*Title:* Nondegeneracy of Coplanar Constant Mean Curvature Surfaces

*Abstract:* Let  $S$  be a complete embedded CMC surface of genus zero and a finite number of ends  $k$ . Main Theorem: If  $S$  is contained in a half-space, then the only square integrable Jacobi function is identically zero. Corollary: The classifying map from the moduli space of such coplanar surfaces to the  $(2k - 3)$ -dimensional space of spherical  $k$ -point metrics is a real analytic diffeomorphism. Remark: This space is diffeomorphic to a  $(2k - 3)$ -ball.

**Rémi Langevin** (Université de Bourgogne)

*Title:* Spheres and circles and surfaces in  $\mathbb{R}^3$  or  $\mathbb{S}^3$

*Abstract:* First we will state a Fary-Fenchel-Milnor like theorem for closed curves in  $\mathbb{R}^3$  or non-splittable curve link using the number of intersection points of the curve(s) with spheres. Then we will translate properties of a curve in the space of spheres into geometrical properties of the surface envelope of the corresponding family of spheres. Finally we will consider special foliations defined on a surface by a conformal property: the principal curvature lines and the Darboux curves.

**Yng-Ing Lee** (National Taiwan University)

*Title:* Self-similar solutions and translating solitons for Lagrangian mean curvature flow

*Abstract:* In this talk, I will report my recent joint work with D. Joyce, and M.P. Tsui. We construct many self-similar solutions and translating solitons for Lagrangian mean curvature flow, which include examples with arbitrarily small oscillation on the Lagrangian angle. Moreover, the translating solitons we obtained are gradient solitons. Our results generalize the works of H. Anciaux, D. Joyce, G. Lawlor, and Y.-I. Lee and M.T. Wang respectively. One of our results says that given two Lagrangian planes with sum of characteristic angles less than  $\pi$ , then one can find a Lagrangian expander asymptotic to this pair of planes. The Maslov class of these expanders is zero. Thus they can serve as good local models for surgeries on Lagrangian mean curvature flow.

**Conan Leung** (Chinese University of Hong Kong)

*Title:* Real, complex, quaternion and octonion geometry

*Abstract:* We give a unified description of Riemannian geometry over different normed division algebras. We show that they coincide with the Berger's classification of Riemannian holonomy groups. Then we explain several applications of this to gauge theory, (special) Lagrangian geometry and MCF and Hard Lefschetz actions. We will also explain

how conformal geometry can also be viewed in this unified setting by adapting the notion of Jordan algebra. We will also give an application of this idea to the second variation of cycles in projective spaces.

**John McCuan** (Georgia Institute of Technology)

*Title:* Lawson's conjecture for symmetric surfaces

*Abstract:* In this talk, I will describe a natural notion of symmetry for surfaces in the three sphere. The complete surfaces of constant mean curvature having this symmetry have not been classified. For compact, complete surfaces, however, we were able to obtain a classification, which I will briefly describe. It follows that among these compact CMC immersions, the only embedded minimal example is the Clifford torus, in accord with a conjecture of B. Lawson.

**Andre Neves** (Princeton University)

*Title:* Insufficient convergence of inverse mean curvature flow on asymptotically hyperbolic manifolds

*Abstract:* In 1998, Huisken and Ilmanen used a weak formulation of inverse mean curvature flow in order to prove a Penrose inequality for asymptotically flat manifolds. In this talk I will explain why, contrarily to what was expected, inverse mean curvature flow does not have the necessary behavior in order to prove a Penrose inequality in asymptotically hyperbolic manifolds.

**Frank Pacard** (l'Université Paris 12 - Val de Marne)

*Title:* Extremal metrics in Kähler geometry

*Abstract:* I will present some recent work, in collaboration with C. Arezzo and M. Singer on the existence of extremal metrics (as defined by E. Calabi) on the blow up at finitely many points of manifolds which carry an extremal metric.

**Manuel Ritoré** (Universidad de Granada)

*Title:* Area-stationary surfaces in the subriemannian Heisenberg group

*Abstract:* The subriemannian Heisenberg group is a Carnot group that appears both as Hausdorff-Gromov limit of Nil manifolds and by blowing up a pseudohermitian 3-manifold at a given point. In recent years, variational problems related to the subriemannian area have been intensively studied. In this talk we shall describe recent progress on this subject, specially the Alexandrov Theorem (characterization of critical points of area under a volume constraint), the Bernstein problem

(characterization of minimal graphs over the  $xy$ -plane), and a conjecture on the isoperimetric solutions.

**Pascal Romon** (l'Université de Marne-la-Vallée)

*Title:* The spectral curve of Hamiltonian stationary tori

*Abstract:* Hamiltonian stationary Lagrangian submanifolds are solutions of a natural and important variational problem in Kähler geometry. For surfaces in Euclidean 4-space, we use the fact that the Euler-Lagrange equation is a completely integrable system to describe all such tori by means of their spectral data: a complete algebraic curve, a rational function and a line bundle. This data yields explicit formulas for all weakly conformal HSL immersions of a 2-torus into Euclidean 4-space and describe the moduli space of those with given conformal type and Maslov class. We also show that each such torus admits a family of Hamiltonian deformations through HSL tori, the dimension of this family being related to the genus of its spectral curve.

**Richard Schoen** (Stanford University)

*Title:* Positive scalar curvature and minimal hypersurface singularities

*Abstract:* We will describe methods for handling minimal hypersurface singularities in the context of the slicing argument which is used to study manifolds of positive scalar curvature including the Positive Mass Theorem in all dimensions.

**Leon Simon** (Stanford University)

*Title:* A frequency function and singular set bounds for branched minimal immersions

*Abstract:* This is joint work with Neshan Wickramasekera. One of Wickramasekera's main results is that if  $M$  is the varifold limit of smooth stable codimension 1 minimal immersions, then, near a point where there is a multiplicity 2 tangent plane,  $M$  is a 2 valued  $C^{1,a}$  graph for some  $a > 0$ . Here we prove that  $a = 1/2$  (which is in general best possible) and that the branch set has Hausdorff dimension at most  $n - 2$ , where  $n$  is the dimension of  $M$ .

**Henry Wente** (The University of Toledo)

*Title:* Exotic capillary tubes

*Abstract:* Exotic containers are vessels containing fluid which for certain volumes support a continuum of non-congruent equilibria. The construction of such vessels was initiated in the work of R.Gulliver-S.Hildebrandt (1986) and greatly expanded in papers of P.Concus-R.Finn (1989). In this lecture we introduce families of exotic capillary

tubes. Such tubes, whose construction is not so difficult, exhibit some very surprising properties. We discuss the construction of these tubes along with their strange behavior.

**Michael Wolf** (Rice University)

*Title:* The Weil-Petersson hessian of length

*Abstract:* The length of a closed curve defines a function on Teichmuller space, and we compute a concise formula for the Hessian of that function with respect to the Weil-Petersson metric on that space. We derive some consequences for the asymptotic behavior of infinite length Weil-Petersson geodesics.

**Seong-Deog Yang** (Korea University)

*Title:* Zero mean curvature surfaces in Lorentz-Minkowski three-space

*Abstract:* In this talk, we present some of the recent examples and developments in the theory of zero mean curvature surfaces in Lorentz-Minkowski three-space  $\mathbb{L}^3$ , in comparison with those of minimal surfaces in  $\mathbb{E}^3$ . Main topics will include the family of spacelike zero mean surfaces discovered by Kim and Yang using the Weierstrass representation formula, the singularities of the spacelike zero mean curvature surfaces from the viewpoint of the singular Bjorling problem, and a relation between spacelike zero mean curvature surfaces and timelike zero mean curvature surfaces with some particular class of singularities.