TITLES and ABSTRACTS

Richard Schoen (Stanford University)
Title: Minimal surfaces as extremals for eigenvalue problems
Abstract: The minimal surface condition in special ambient manifolds expresses the condition that the embedding/coordinate functions satisfy a linear equation with respect to the induced metric. For example a minimal surface in Euclidean space is characterized by the condition that the coordinate functions are harmonic. Similarly, minimal surfaces in the sphere are characterized by the condition that the coordinate function are eigenfunctions.

There is a similar eigenfunction characterization for free boundary minimal surfaces in a Euclidean ball. It turns out that the induced metrics on such surfaces are critical metrics for the variational problem of extremizing an eigenvalue over the space of metrics with an area normalization. One can then turn this around and attempt to solve the intrinsic problem of extremizing (usually maximizing) an eigenvalue over the space of metrics. If the solution of this problem exists and is smooth it will be represented as the induced metric on a minimal immersion by eigenfunctions.

In this series of lectures we will spend the first two introducing this rather extensive subject and describing the results which have been obtained both for closed surfaces and for surfaces with boundary. We will emphasize unsolved problems both for the eigenvalue problem and for the free boundary minimal surface problem. In the second two lectures we will give outlines of proofs of some of the recent results which have been obtained.

Tristan Rivière (ETH Zürich)
Title: The variations of Yang-Mills Lagrangian
Abstract: Yang-Mills theory is growing at the interface between high energy physics and mathematics. It is well known that Yang-Mills theory and Gauge theory in general had a profound impact on the development of modern differential and algebraic geometry. One could quote Donaldson invariants in four dimensional differential topology, Hitchin Kobayashi conjecture relating holomorphic bundles over Kähler manifolds and Mumford stability in complex geometry or also Gromov Witten invariants in symplectic geometry...etc. While the influence of Gauge theory in geometry is quite notorious, one tends sometimes to forget that Yang-Mills theory has been also at the origin of fundamental progresses in the non-linear analysis of Partial Differential Equations in the last decades.

The purpose of this mini-course is to present the variations of this important Lagrangian. We shall raise analysis question such as existence and regularity of Yang-Mills minimizers or critical point of Yang-mills Lagrangian in general. *(continued)*

We will first describe during the two first hours the progresses which have been made in these directions for the critical dimension 4 and below. At this occasion we will mostly insist on the important contributions by K.Uhlenbeck from the late seventies - early eighties. The second part of the mini-course will be devoted to the study of Yang-Mills fields in dimension larger than 4. We will first describe the Lin-Tian concentration compactness method in super critical dimension as well as the epsilon-regularity results obtained by Tao-Tian and Meyer-Rivière. In the last part of the course we shall present very recent results obtained in collaboration with M. Petrache regarding the ad-hoc framework for producing Yang-Mills minimizers in dimension larger than 4.

Yng-Ing Lee (National Taiwan University)
Title: The metric properties of Lagrangian submanifolds
Abstract: In this series of talks we will focus on the interplay of Riemannian Geometry and Symplectic Geometry through studying the metric properties of Lagrangian submanifolds. The first two lectures will give an introduction to Lagrangian submanifolds, minimal Lagrangians, special Lagrangians, Hamiltonian stationary Lagrangians, and their properties.

Special Lagrangians play an important role in String theories. Minimal Lagrangians/Special Lagrangians can serve as canonical representatives in their homology classes, and are good models for higher co-dimensional minimal submanifolds. Hamiltonian stationary Lagrangians also arise naturally and are related models for incompressible elasticity theory. Although we are of great interests to these natural objects, the fundamental question on their existence is still not well understood. In the last two lectures, we will explain different approaches in studying this problem and report some recent progress.

André Neves (Imperial College London)
Title: 1. Existence of geodesics I, 2. Existence of geodesics II, 3. Existence of minimal hypersurfaces, 4. Willmore conjecture
Abstract: 1. I will sketch the proof that every non-simply connected manifolds has at least one geodesic. Then I will talk about curve shortening flow.
2. I will sketch the proof that every two sphere has three simple closed geodesics (Lusternick-Schnirelman) and that every compact manifolds has at least one closed geodesic (Lusternick-Fet).
3. I will talk about minimal hypersurfaces and explain why manifolds with positive Ricci curvature have an infinite number of minimal embedded hypersurafces. This is joint work with Fernando Marques.
4. I will explain what the conjecture is and its recent proof due to Fernando Marques and myself.