Let (M, ω, J) be a $2n$-dimensional symplectic manifold with a symplectic form ω and an almost complex structure J compatible with ω. Also, we define a Hermitian metric on M as $h(v, w) = \omega(Jv, w) + i\omega(v, w)$. Let F be a compact symplectic submanifold.

Let Q be a nondegenerate Hermitian form defined on the normal bundle N_F of F. The Hermitian form Q determines an endomorphism T of N_F by $h(T(v), w) = Q(v, w)$ for each vector $v, w \in N_F$. Let $f : M \to \mathbb{R}$ be a smooth function defined by $f(v) = Q(v, v)$ for each vector v in N_F where we think N_F as a neighborhood of F in M. Let a function $g : M \to \mathbb{R}$ be compactly supported in a sufficiently small neighborhood of F. Also, assume that g is fiberwise constant in a more smaller neighborhood of F. Then, we obtain

$$\int_M e^{itf} g \, \frac{\omega^n}{n!} \sim i^{-n} \int_F e^{i\omega_F} g \Big/ \det\Big(-\frac{T}{2\pi} t - \frac{\Omega_F}{2\pi i} \Big)$$

where Ω_F is the curvature 2-form of the normal bundle N_F and ω_F is the symplectic form ω restricted to F.

Proof is done by direct calculations, usual stationary phase method, and MacMahon's Master Theorem.