Standard Model
(4)

김 형 도 교수 (서울대)

Six-Quark Model

∘ $L_{jL} = \begin{pmatrix} \nu_j \\ l_j^- \end{pmatrix}_L \qquad j=1,2,3.$

l_{jR}^-

$\vec{T} L_{jL} = \frac{\vec{\tau}}{2} L_{jL}, \quad Y L_{jL} = -\frac{1}{2} L_{jL}$

$\vec{T} l_{jR}^- = 0, \qquad Y l_{jR}^- = -l_{jR}^-$

$\mathcal{L} \supset - y_{jk}^l \overline{l_{jR}^-} \phi^+ L_{kL} + h.c.$

$\Rightarrow - y_{jk}^l \frac{v}{\sqrt{2}} \overline{l_{jR}^-} l_{kL}^- + h.c.$

; mass term for charged leptons

∘ $\psi_{jL} = \begin{pmatrix} U_j \\ D_j \end{pmatrix}_L$

$U_{jR}, \qquad D_{jR}$

$(Q = \frac{2}{3}) \qquad (Q = -\frac{1}{3})$

$\vec{T} \psi_{jL} = \frac{\vec{\tau}}{2} \psi_{jL}, \quad Y \psi_{jL} = \frac{1}{6} \psi_{jL}$

$\vec{T} U_{jR} = \vec{T} D_{jR} = 0$

$Y U_{jR} = \frac{2}{3} U_{jR}, \qquad Y D_{jR} = -\frac{1}{3} D_{jR}$

$$\mathcal{L} \supset -y^D_{jk}\,\bar{D}_{jR}\,\phi^\dagger \psi_{kL} + h.c.$$

$$\langle \phi \rangle \neq 0$$

$$\Rightarrow \quad -M^D_{jk}\,\bar{D}_{jR} D_{kL} + h.c.$$

$$M^D_{jk} = y^D_{jk}\frac{v}{\sqrt{2}}$$

For U, we need $\tilde{\phi} = i\tau_2 \phi^*$

$$\delta\tilde{\phi} = i\epsilon_a \frac{\tau_a}{2}\tilde{\phi} - \frac{i}{2}\epsilon\tilde{\phi}$$

$$T\tilde{\phi} = \frac{\tau}{2}\tilde{\phi} \quad , \quad Y\tilde{\phi} = -\frac{1}{2}\tilde{\phi}$$

$$\mathcal{L} \supset -y^U_{jk}\,\bar{U}_{jR}\,\tilde{\phi}^\dagger \psi_{kL} + h.c.$$

$$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ \phi^- \end{pmatrix}, \qquad \phi^- = -\phi^{+*}$$

$$\Rightarrow \quad -M^U_{jk}\,\bar{U}_{jR} U_{kL} + h.c.$$

$$M^U_{jk} = y^U_{jk}\frac{v}{\sqrt{2}}$$

$$M^U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

by $\psi_L^i \to V_L \psi_L^i$

$U_R \to V_R U_R$

$$- M^D_{j\ell} V^\dagger_{\ell k} \bar{D}_{jR} D_{kL} + h.c.$$

$$M^D = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = V^\dagger D_L$$

$$D_L = V \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L}_{\text{mass eigenstates}}$$

⊗ changed current

$$(\bar{u} \ \bar{c} \ \bar{t}) \gamma^\mu (1+\gamma_5) \underline{V} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

nine → 3 real parameters (mixing angle

and 1 phase.

$$-\bar{U}_R' M^U U_L'$$

$$-\bar{D}_R' M^D D_L'$$

Suppose $\quad V_R^U M_{diag}^U V_L^U = M^U$

$$-\bar{U}_R' V_R^U M_{diag}^U V_L^U U_L'.$$

We can redefine

$$\bar{U}_R = \bar{U}_R' V_R^U$$

$$Q_L = V_L^U Q_L'$$

If $\quad V_R^D M_{diag}^D V_L^D = M^D,$

$$-\underbrace{\bar{D}_R' V_R^D}_{\bar{D}_R} M_{diag}^D \underbrace{V_L^D \cdot V_L^{U\dagger}}_{V_{CKM}^\dagger} \cdot \underbrace{V_L^U D_L'}_{D_L''}$$

$$\Rightarrow -\bar{D}_R M_{diag}^D \underbrace{V_L^D V_L^{U\dagger} D_L''}_{= D_L}$$

◎ $\bar{U}_L \gamma^\mu D_L'' = \bar{U}_L \gamma^\mu V_{CKM} D_L W^+$

Z_0^0 couples to the quark neutral current in any basis. $x = \sin^2\theta$: Weinberg angle

$$\mathcal{L} \supset \frac{e}{\sin\theta\cos\theta} \sum_j \Big\{ \left(\frac{1}{2} - \frac{2}{3}x\right) \bar{U}_{jL}\gamma^\mu U_{jL} + \left(-\frac{1}{2} + \frac{1}{3}x\right) \bar{D}_{jL}\gamma^\mu D_{jL} - \frac{2}{3}x \bar{U}_{jR}\gamma^\mu U_{jR} + \frac{1}{3}x \bar{D}_{jR}\gamma^\mu D_{jR} \Big\}$$

V appears in U-D interactions.
(charged current)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix}$$

$$(\bar{u} \;\; \bar{c} \;\; \bar{t})\, \gamma^\mu \frac{(1+\gamma_5)}{2} V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \cdot \frac{g}{\sqrt{2}} W_\mu^{a+}$$

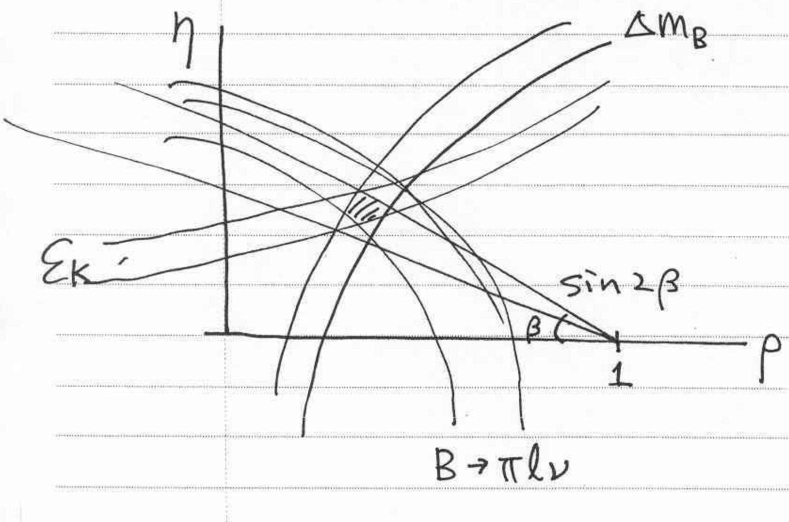
Wolfenstein parametrization

$\lambda \equiv \sin\theta_c \simeq 0.22$, $A \simeq 0.8$

$$V \simeq \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\bar{\rho}-i\bar{\eta}) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\bar{\rho}-i\bar{\eta}) & -A\lambda^2 & 1 \end{pmatrix}$$

$$+ \mathcal{O}(\lambda^4)$$

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0$$



$\bar{\rho} \simeq 0.2$ $\bar{\eta} \simeq 0.4$

CKM Matrix Elements

$|V_{us}| = 0.22$

$|V_{cb}| = 0.0042$

$|V_{ub}| = 0.00036$

$\left|\dfrac{V_{ub}}{V_{cb}}\right| = 0.086$

$\sin 2\beta = 0.74$

Mass of Quarks and Leptons

$m_u \simeq 2\,\mathrm{MeV}$

$m_d \simeq 5\,\mathrm{MeV}$

$m_s \simeq 100\,\mathrm{MeV}$

$m_c \simeq 1.5\,\mathrm{GeV}$

$m_b \simeq 4.2\,\mathrm{GeV}$

$m_t \simeq 165\,\mathrm{GeV}$

$m_e \simeq 0.5\,\mathrm{MeV}$

$m_\mu \simeq 104\,\mathrm{MeV}$

$m_\tau \simeq 1.777\,\mathrm{GeV}$

Any clue from the data?

$$\lambda = |V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \sim \sqrt{\frac{1}{20}}$$

$$\left|\frac{V_{ub}}{V_{cb}}\right| \simeq \sqrt{\frac{m_u}{m_c}} \simeq 0.05$$

relations come from symmetric matrix with zero elements.

e.g. 1) $$Y^U_{2\times 2} = \begin{pmatrix} Y^U_{11} & 0 \\ 0 & Y^U_{22} \end{pmatrix} \begin{pmatrix} u \\ c \end{pmatrix}$$

$$Y^D_{2\times 2} = \begin{pmatrix} 0 & Y^D_{12} \\ Y^D_{12} & Y^D_{22} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

$$V_{CKM} = U \cdot V^\dagger = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$$\tan\theta_c = \frac{Y^D_{12}}{Y^D_{22}} \simeq \sin\theta_c$$

$$Y^D_{2\times 2}\Big|_{diag} \simeq \begin{pmatrix} -\frac{(Y^D_{12})^2}{Y^D_{22}} & 0 \\ 0 & Y^D_{22} \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}$$

Neutrinos (with ν_R)

$$\mathcal{L}^D = -Y^{\nu}_{ij}\, L_{iL}\, \phi^\dagger\, \overline{\nu}_{Rj} + h.c.$$

$$m^{\nu}_{D_{ij}} = Y^{\nu}_{ij} \cdot \frac{v}{\sqrt{2}} \sim \mathcal{O}(100\ \mathrm{GeV})$$

$$\mathcal{L}^M_R = -\frac{1}{2} M_{ij}\, \overline{\nu}_{Ri}\, (\nu_R)^c_j + h.c.$$

For $M \simeq M_{GUT} \simeq 10^{15} \sim 10^{16}\ \mathrm{GeV}$,

$$\text{See-Saw}: \quad \begin{array}{cc} \overline{\nu_L^c} & \overline{\nu_R} \end{array} \quad \begin{array}{c} \\ \end{array}$$

$$\begin{pmatrix} 0 & m_D \\ m_D & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \nu_R^c \end{pmatrix}$$

$$\xrightarrow{\text{diagonalize}} \quad -\frac{m_D^2}{M} : \nu_L + \frac{m_D}{M}\nu_R^c$$

$$M : \nu_R - \frac{m_D}{M}\nu_L^c$$

$$\frac{m_D^2}{M} \sim \left(\frac{10^2}{10^{15}}\right) \cdot 10^2\ \mathrm{GeV} \simeq \underline{\underline{10^{-2}\ \mathrm{eV}}}$$

Correct range.

MOOKEUK

$\Delta m_{sol}^2 \simeq 7 \times 10^{-5} (eV)^2$

$\tan^2\theta_{sol} \simeq 0.4 \Rightarrow \theta_{sol} \simeq \frac{\pi}{4} - \varepsilon, \quad \varepsilon \ll 1$

$\Delta m_{atm}^2 \simeq 2 \times 10^{-3} (eV)^2$

$\sin^2 2\theta_{atm} = 1.0 \Rightarrow \theta_{atm} \simeq \frac{\pi}{4}$

◎ Solar Neutrino

; from standard solar model,

$$\frac{\Phi_{\nu_e}(\exp)}{\Phi_{\nu_e}(\text{theory})} \simeq \frac{1}{3}.$$

recently SNO experiments

$$\sum_{\ell=e,\mu,\tau} \Phi_{\nu_\ell} = \Phi_{\nu_e}(\text{theory}).$$

◎ Atmospheric Neutrino

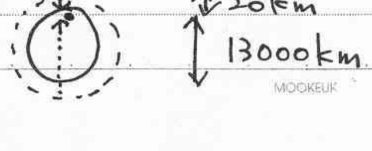
cosmic ray + nuclei $\to \pi$

$\pi \to \mu \nu_\mu$. $\quad \mu \to e \bar{\nu}_e \nu_\mu$

$$\frac{\Phi(\nu_\mu)}{\Phi(\nu_e)} \simeq 2 \text{ (theory)} \quad . \quad \frac{\Phi(\nu_\mu)}{\Phi(\nu_e)} \simeq 1 \text{ (exp)}.$$

SuperKamiokande

$\left.\frac{U}{D}\right|_\mu \simeq 0.5$



For leptons, (assuming M_R is diagonal)

$$-\bar{E}_R' M^E E_L'$$

$$-\bar{N}_R' M^\nu N_L'$$

$$V_R^E M_{diag}^E V_L^E = M^E$$

$$V_R^N M_{diag}^N V_L^N = M^N$$

$$-\underbrace{\bar{E}_R' V_R^E}_{\bar{E}_R} M_{diag}^E \underbrace{V_L^E E_L'}_{E_L}$$

$$-\underbrace{\bar{N}_R' V_R^N}_{\bar{N}_R} M_{diag}^N \underbrace{\underbrace{V_L^N V_L^{E+}}_{V_{LCKM}} \underbrace{V_L^E N_L'}_{N_L''}}_{N_L = \text{mass eigensta}}$$

$$\bar{N}_L'' \gamma^\mu E_L = \bar{N}_L V_{LCKM} \gamma^\mu E_L W^-$$

◎ $\nu_{\ell L} = \sum_{i=1}^{3} \underbrace{U_{\ell i}}_{\text{3x3 MNS matrix}} \nu_{iL}$ ← mass eigenstate

Weak eigenstate

3x3 MNS matrix

(Maki-Nakagawa-Sakata)

Quarks

		(or truth)
u (up)	c (charm)	t (top)
d (down)	s (strange)	b (beauty) (or bottom)

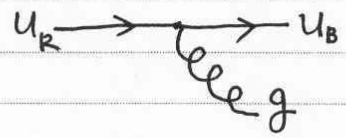
Leptons

e (electron)	μ (muon)	τ (tau)
ν_e (e neutrino)	ν_μ (μ-neutrino)	ν_τ (τ-neutrino)

	Quarks	Leptons

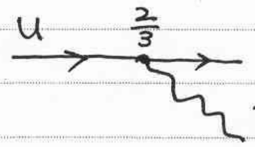
Strong

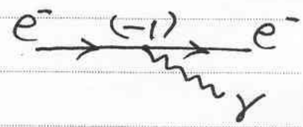
- color



EM

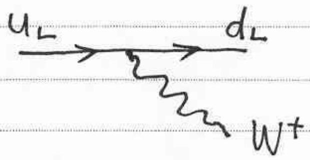
- electric charge

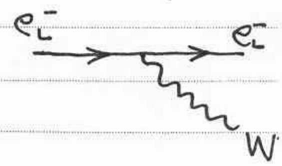




Weak

- weak charge





Possible confusions

1) θ_W vs θ_C

Weinberg angle : Cabibbo angle

gauge sector : Yukawa sector

$(W_3, X) \Downarrow (A, Z)$ $\quad$ $(d, s) \Downarrow (d', s')$

2) $SU(3)_c$ vs $SU(3)_{flavor}$

(u, d, s)

gauge symmetry vs global symmetry

exact : approximate

broken by m_u, m_d, m_s

unless $m_u = m_d = m_s$.

3) Spin vs Charge

$\cap$ $\quad$ $\cap$

Lorentz group $\quad$ Internal group

u : $s = \frac{1}{2}$ $\quad$ $Q = \frac{2}{3}$

d : $s = \frac{1}{2}$ $\quad$ $Q = -\frac{1}{3}$

$\vdots$ $\quad$ $\vdots$

History

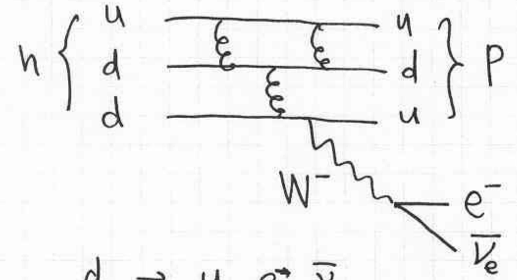
$$\begin{pmatrix} u \\ d, s \end{pmatrix} \qquad \begin{pmatrix} \nu_e \\ e \end{pmatrix} \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}$$

$\Downarrow$

$p, n, \pi,$

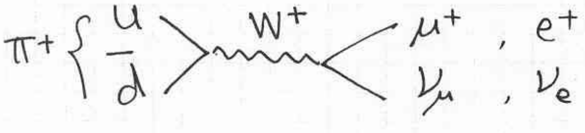
$K, \cdots$

∘ n, π decay



$d \to u \, e^- \bar{\nu}_e$

$$\Gamma(n \to p \, e^- \bar{\nu}_e) =$$

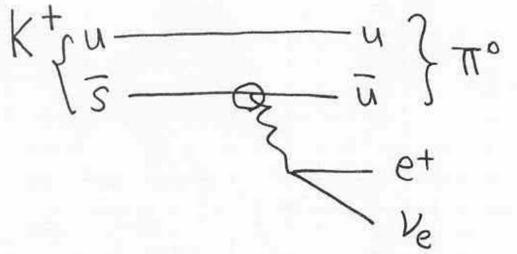


$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) =$$

$e^+ \nu_\mu$: strongly suppressed.

$$e^+ \overset{\Leftarrow}{\Longleftarrow} \pi^+ \overset{\Leftarrow}{\Longrightarrow} \nu_e$$

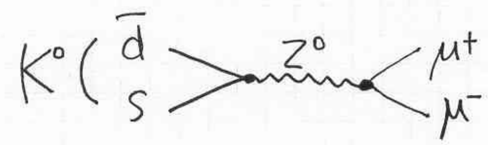
: spin = 1. angular momentum is not conserved.



$$K^+ \to \pi^0 e^+ \nu_e, \quad \pi^0 \mu^+ \nu_\mu.$$

Cabibbo : $\begin{pmatrix} u \\ d \end{pmatrix}_L \to \begin{pmatrix} u \\ d\cos\theta_c + s\sin\theta_c \end{pmatrix}_L$

$Z^0 \sin\theta_c \cos\theta_c \, \bar{d}_L s_L$ is allowed.



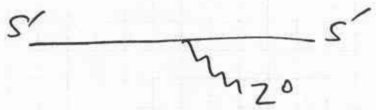
$$\Gamma(K^0 \to \mu^+\mu^-) \sim \Gamma(K^+ \to \pi^0 e^+ \nu_e)$$

$\ll$ experimentally

Glashow - Iliopoulos - Miani
GIM Mechanism

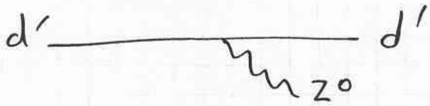
$$\begin{pmatrix} u \\ \underbrace{d\cos\theta_c + s\sin\theta_c}_{d'} \end{pmatrix}_L \qquad \begin{pmatrix} c \\ \underbrace{-d\sin\theta_c + s\cos\theta_c}_{s'} \end{pmatrix}_L$$

$\downarrow$



$$-\cos\theta_c\sin\theta_c\, Z^0\, \bar{d}_L s_L$$

cancel

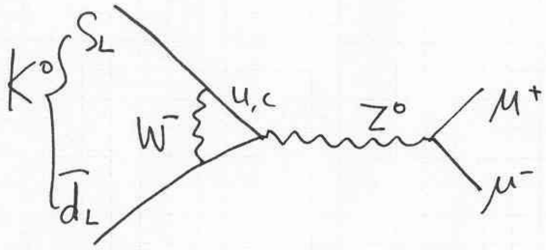


$$\cos\theta_c\sin\theta_c\, Z^0\, \bar{d}_L s_L$$

No $\bar{d}_L s_L Z^0$ coupling at tree level.

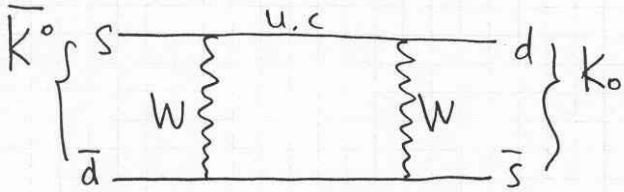
$\Gamma_{tree}(K^0 \to \mu^+\mu^-) = 0$

FCNC: loop induced phenomena in the SM



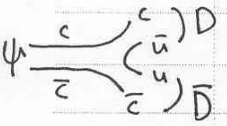
$\propto (m_c - m_u) \neq 0$

$m_c \sim 1.5$ GeV $\qquad$ J/ψ.



Discovery of Charm

$c\bar{c}$: J/ψ in 1974 . November Revolution



Ting at Brookhaven, Lederman ··· at SLAC
J ψ
(; the most bizzare name)

allowed by OZI.
not allowed energetically
$m_{J/\psi} < 2m_D$

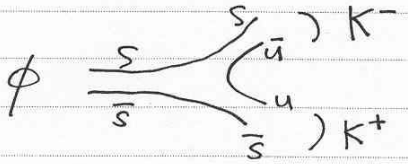
$E = 3.1\ \mathrm{GeV}$. $J^{PC} = 1^{--}$

$p + Be \to e^+e^- + X$ at $\sqrt{s} = 7.6\ \mathrm{GeV}$

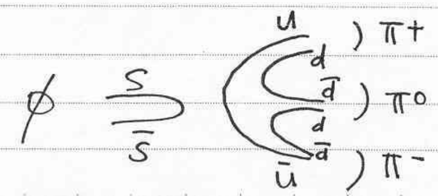
$\Gamma(J/\psi) = 87\ \mathrm{keV}$; extremely narrow decay width
(usually MeV ~ 10MeV)

◎ OZI rule (Okubo-Zweig-Iizuka)

; the decays which correspond to disconnecte
quark diagrams are forbidden.



: allowed



: forbidden

Bottom

$b\bar{b}$: Υ (upsilon meson) : $J^{PC} = 1^{--}$ in 1977 Fermilab.

$\sqrt{s} \sim 9.5\,\text{GeV} \Rightarrow m_b \simeq 4.5\,\text{GeV}$

$\Gamma \sim 50\,\text{keV}$

Top

$t \to W^+ + b$ in 1996

$m_t \simeq 175\,\text{GeV}$

$$\Gamma(t \to W^+ + b) = \frac{G_F}{8\pi\sqrt{2}} m_t^3 \left(1 - \frac{m_W^2}{m_t^2}\right)^2 \left(1 + 2\frac{m_W^2}{m_t^2}\right)$$

$\Rightarrow \quad \Gamma \simeq 1.5\,\text{GeV} \quad > \quad 0.2\,\text{GeV}$
$(\simeq 1\,\text{fm}^{-1})$

top quark decays before it forms a bound state ($t\bar{t}$ or $t\bar{q}$).

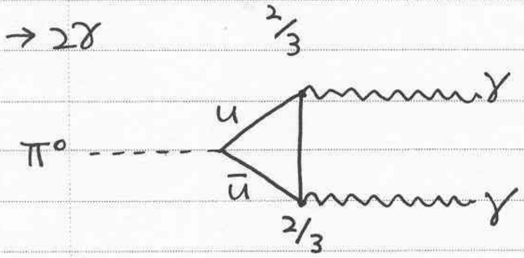
How to see color?

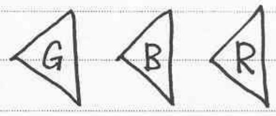
(i) π^0 : $u\bar{u} - d\bar{d}$

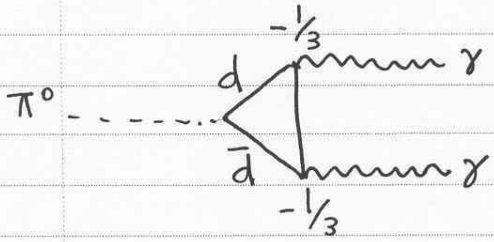
Q : $\frac{2}{3}$ for u

$-\frac{1}{3}$ for d

$\pi^0 \to 2\gamma$







no color $N_c = 1$: $M \Rightarrow \Gamma \propto |M|^2$

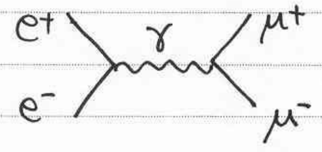
with color : $N_c M \Rightarrow \Gamma \propto N_c^2 |M|$

$\Gamma(\pi^0 \to 2\gamma) = 7.58\ \text{eV}$

(ii) e^+e^- annililation

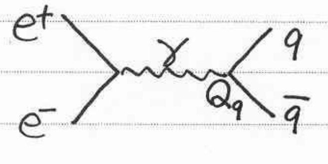
$$R = \frac{\sigma(e^+e^- \to \text{hadrons})}{\sigma(e^+e^- \to \mu^+\mu^-)}$$

$$\sigma(e^+e^- \to \mu^+\mu^-) = \frac{4\pi}{3}\alpha^2\frac{1}{s}$$



$$\sigma(e^+e^- \to \text{hadrons}) = \sum_q \sigma(e^+e^- \to q\bar{q})$$

$$\sigma(e^+e^- \to q\bar{q}) = \frac{4\pi}{3} N_c \cdot Q_q^2 \cdot \alpha^2 \cdot \frac{1}{s}$$



$$R = \underset{(N_c)}{\overset{}{3}} \sum_q Q_q^2 = 3\left(\underset{u}{\left(\tfrac{2}{3}\right)^2} + \underset{d}{\left(\tfrac{1}{3}\right)^2} + \underset{s}{\left(\tfrac{1}{3}\right)^2} + \underset{c}{\left(\tfrac{2}{3}\right)^2} + \underset{b}{\left(\tfrac{1}{3}\right)^2}\right)$$

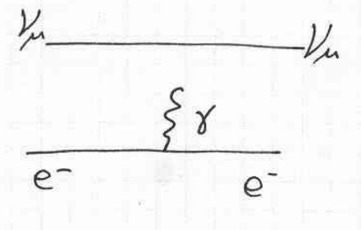
$$= \frac{11}{3} \quad : \text{correct}$$

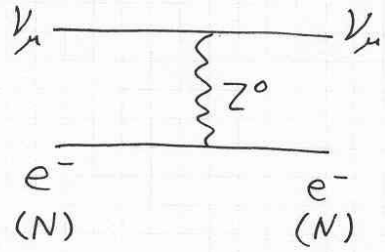
$(2m_b < E \ll M_Z)$

Neutral Current Weak Interaction

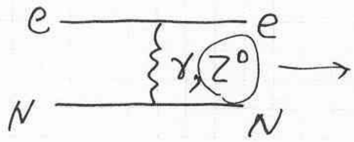
- $\nu_\mu e^- \to \nu_\mu e^-$

  not possible by charged current





- Atomic Parity Violation



$\longrightarrow$ Parity is violated