Possible confusions

1) θ_W vs θ_C

Weinberg angle : Cabibbo angle

gauge sector : Yukawa sector

(W_3, X) $\Downarrow$ (A, Z) : (d, s) $\Downarrow$ (d', s')

2) $SU(3)_c$ vs $SU(3)_{flavor}$ (u, d, s)

gauge symmetry vs global symmetry

exact : approximate broken by m_u, m_d, m_s unless $m_u = m_d = m_s$.

3) Spin vs Charge

$\cap$ Lorentz group : $\cap$ Internal group

u : $s = \frac{1}{2}$: $Q = \frac{2}{3}$

d : $s = \frac{1}{2}$: $Q = -\frac{1}{3}$

$\vdots$ $\vdots$

Isospin doublet $\begin{pmatrix} p \\ n \end{pmatrix}$

$I = \frac{1}{2}$. $I_3 = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}$ for $\begin{pmatrix} p \\ n \end{pmatrix}$

$S = 0$; $\pi, N, \Delta, \cdots$

$S = 1$. ; $K^+, \cdots$

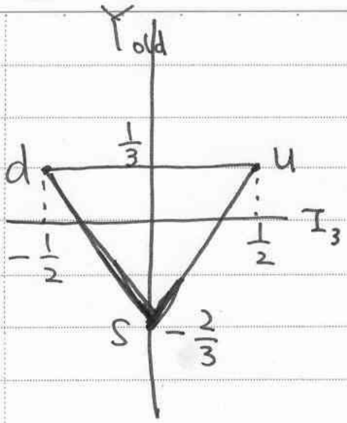
$S = -1$; $\Lambda, \Sigma, \cdots$

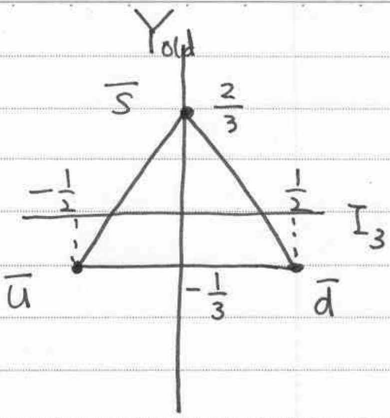
$$Y_{old} = B + S$$

$$Y = \frac{1}{2} Y_{old}$$

$$Q = I_3 + Y$$

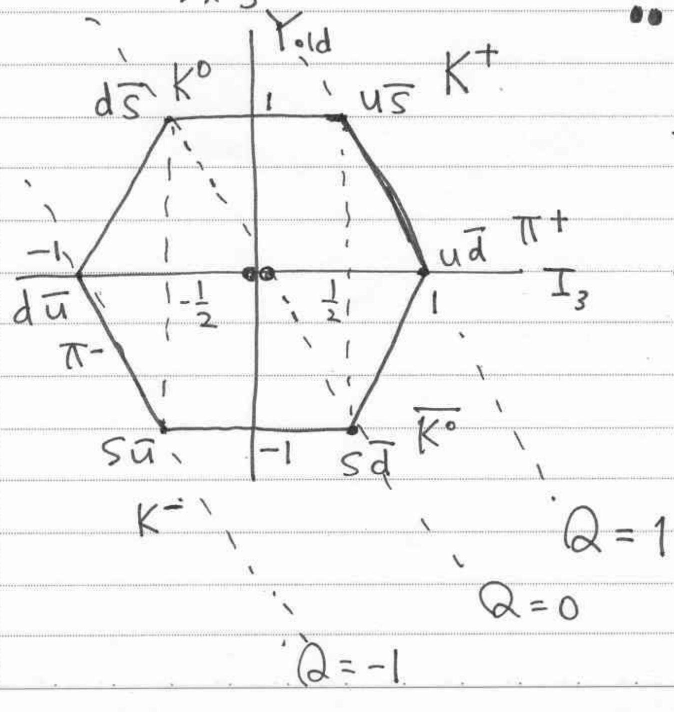
Mesons. $J^{PC} = 0^{-+}$ A2





flavor SU(3): $3 \begin{pmatrix} u \\ d \\ s \end{pmatrix}$ $\bar{3} \begin{pmatrix} \bar{u} \\ \bar{d} \\ \bar{s} \end{pmatrix}$

$3 \times \bar{3}$: $J^P = 0^-$



$$\pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} - d\bar{d})$$

$$\eta = \frac{1}{\sqrt{6}}(u\bar{u} + d\bar{d} - 2s\bar{s})$$

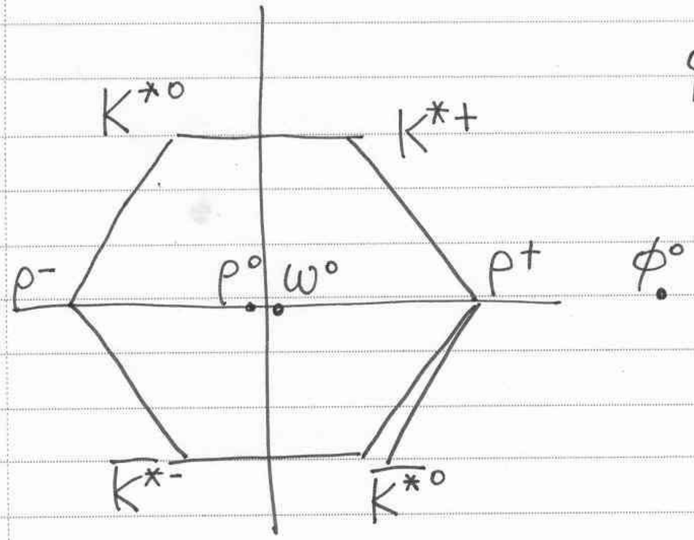
$$\eta' = \frac{1}{\sqrt{3}}(u\bar{u} + d\bar{d} + s\bar{s})$$

Vector Mesons $J^{PC} = 1^{--}$ A3

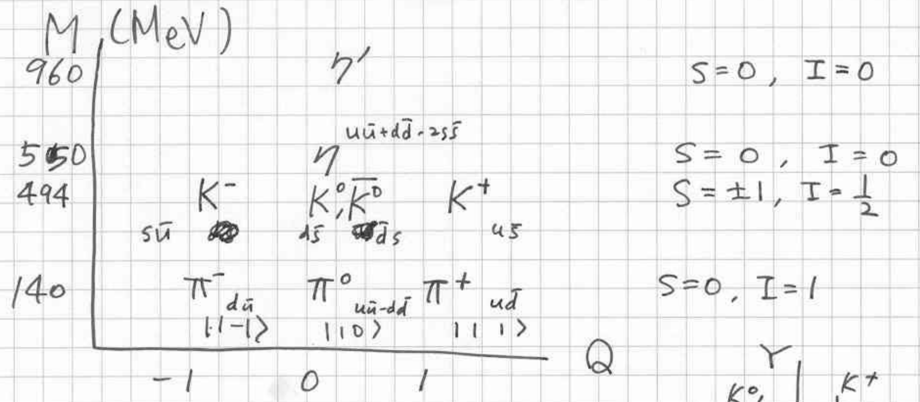
$(S=1)$

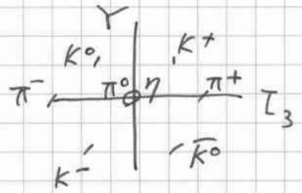
$\omega = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$

$\phi \simeq s\bar{s}$

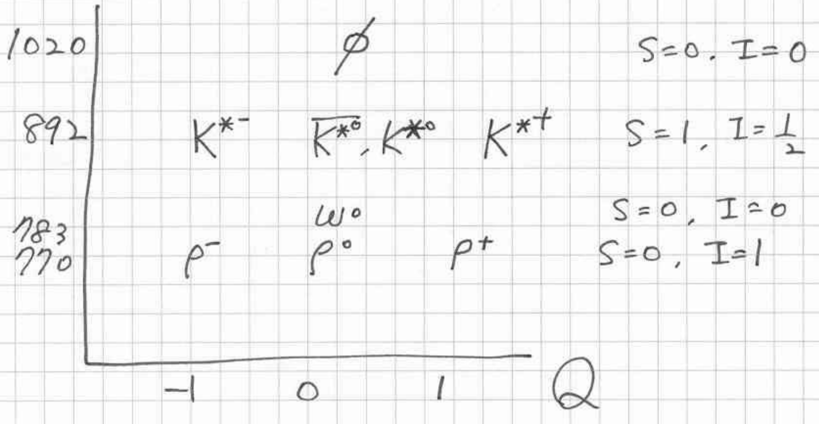


~~Lowest~~ Light Pseudoscalar Mesons ($J^P = 0^-$)





Light Vector Mesons ($J^P = 1^-$)



$$3 \times \bar{3} = \underline{\underline{8}} + 1$$

Baryons

$$3 \otimes 3 \otimes 3 = (6 \oplus \bar{3}) \otimes 3$$
$$= (6 \otimes 3) \oplus (\bar{3} \otimes 3)$$
$$= (\underline{10} \oplus 8) \oplus (8 \oplus 1)$$

Symmetric

$$2 \otimes 2 \otimes 2 = (3 \oplus 1) \otimes 2$$
$$= 4 \oplus 2 \oplus 2$$

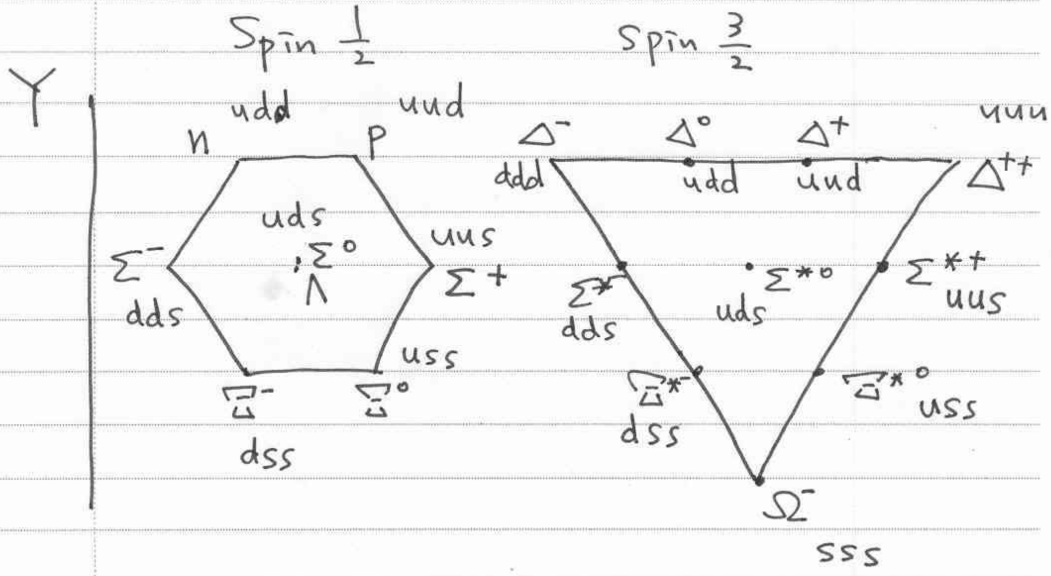
$(10 \oplus 8 \oplus 8 \oplus 1)$, $(4 \oplus 2 \oplus 2)$

$\boxed{S : (10, 4) + (8, 2)}$ *

M_S : $(10, 2) + (8, 4) + (8, 2) + (1, 2)$

M_A : $(10, 2) + (8, 4) + (8, 2) + (1, 2)$

A : $(1, 4) + (8, 2)$



prediction!

$\Delta^{++} = | u\uparrow u\uparrow u\uparrow \rangle$

; symmetric wave function of identical fermion. $\Rightarrow$ vanish.

color can explain it.

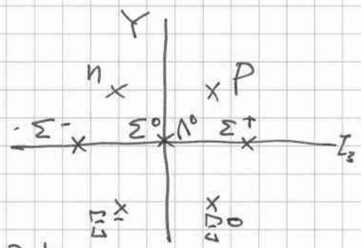
$\Delta^{++} = \epsilon^{abc} | u_a\uparrow u_b\uparrow u_c\uparrow \rangle$

$a, b, c = 1, 2, 3$ or R.G.B.

Light Baryons ($J^P = \frac{1}{2}^+$)

	-1	0	1	
1320	Ξ^-_{dss}	Ξ^0_{uss}		$S=-2,\ I=\frac{1}{2}$
1190	Σ^-_{dds}	$\Sigma^0{}^{uds}$	Σ^+_{uus}	$S=-1,\ I=1$
1115		Λ^0_{uds}		$S=-1,\ I=0$
938		n^{udd} $\lvert\frac{1}{2}\ -\frac{1}{2}\rangle$	p^{uud} $\lvert\frac{1}{2}\ \frac{1}{2}\rangle$	$S=0,\ I=\frac{1}{2}$

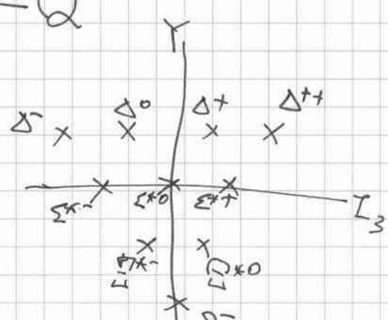
Q



Light Baryons ($J^P = \frac{3}{2}^+$)

	-1	0	1	2	
1672	Ω^-_{sss}				$S=-3,\ I=0$
1530	Ξ^{*-}_{dss}	Ξ^{*0}_{uss}			$S=-2,\ I=\frac{1}{2}$
1385	Σ^{*-}_{dds}	Σ^{*0}_{uds}	Σ^{*+}_{uus}		$S=-1,\ I=1$
1232	Δ^- ddd	Δ^0 udd	Δ^+ uud	Δ^{++} uuu	$S=0,\ I=\frac{3}{2}$

Q



$$3 \times 3 \times 3 = (6 + \bar{3}) \times 3$$

$$= 10 + 8 + 8' + 1$$

Weyl Spinor

◎ Spin $S=0$

$$(\Box + m^2)\,\phi(x) = 0 \quad : \text{Klein-Gorden eq.}$$

$$\phi \propto a(k) \quad + b^{+}(k)$$

◎ Spin $S=\frac{1}{2}$

$$(i\gamma^{\mu}\partial_{\mu} - m)\,\Psi(x) = 0 \quad : \text{Dirac eq.}$$

$$\overline{\Psi}(x) = \Psi^{+}(x)\gamma^{0}$$

$$\overline{\Psi}(x)\left[-i\gamma^{\mu}\overleftarrow{\partial}_{\mu} - m\right] = 0$$

γ^{μ} : Dirac matrices

$$\{\gamma^{\mu}, \gamma^{\nu}\} = 2g^{\mu\nu}$$

$\gamma^{0+} = \gamma^{0}$

$\gamma^{i+} = -\gamma^{i+}$

∘ $\gamma^{5} = -i\gamma^{0}\gamma^{1}\gamma^{2}\gamma^{3}$

$\gamma^{5+} = \gamma^{5}$

$\{\gamma^{5}, \gamma^{\mu}\} = 0.$

$$\gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix}$$

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Weyl representation
(or chiral representation)

$\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$: generator of Lorentz gp.

$$[\gamma^5, \sigma^{\mu\nu}] = 0$$

$$P_L = \frac{1+\gamma^5}{2}, \qquad P_L \Psi = \Psi$$

$$P_R = \frac{1-\gamma^5}{2}, \qquad P_R \Psi = -\Psi$$

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \begin{aligned} \gamma^5 \psi_L &= \psi_L \\ \gamma^5 \psi_R &= -\psi_R \end{aligned}$$

ψ_L & ψ_R do not mix if $m = 0$.

Charge conjugation

$$\left[i\gamma^\mu(\partial_\mu + ieA_\mu) - m\right]\Psi(x) = 0$$

$$\left[-i\gamma^{\mu T}(\partial_\mu - ieA_\mu) - m\right]\bar{\Psi}^T(x) = 0$$

Consider C: charge conjugation

$$\Psi(x) \to \Psi^c(x) = C\,\Psi(x)\,C^{-1}$$
$$A_\mu(x) \to A_\mu^c(x) = C\,A_\mu(x)\,C^{-1}$$

Suppose the Dirac eq. is C invariant.

$$\left[i\gamma^\mu(\partial_\mu + ieA_\mu^c) - m\right]\Psi^c(x) = 0.$$

$$\gamma^\mu = -C\gamma^{\mu T}C^{-1}$$
$$A_\mu^c = -A_\mu$$
$$\Psi^c(x) = C\bar{\Psi}^T(x)$$

$$\Psi^c(x) = C\bar{\Psi}^T(x) = -\gamma^0 C\Psi^*$$
$$\bar{\Psi}^c(x) = -\Psi^T(x)\,C^{-1}$$

$$\gamma^{\mu T} = \begin{cases} \gamma^\mu & \mu = 0, 2 \\ -\gamma^\mu & \mu = 1, 3 \end{cases} \quad \text{in Pauli rep.}$$

$C = +i\gamma^0\gamma^2, \quad C^2 = -1, \quad C^\dagger = -C = C^T$

$$\Psi^c(x) = -i\gamma^2\Psi^*$$

$$\Psi = \left(\frac{1+\gamma^5}{2}\right)\Psi + \left(\frac{1-\gamma^5}{2}\right)\Psi$$

$$= \Psi_L + \Psi_R$$

$$= \begin{pmatrix} \Psi_L \\ \psi_R \end{pmatrix}$$

$$= \begin{pmatrix} \xi \\ i\sigma_2\bar{\chi} \end{pmatrix} \quad \text{as} \quad \Psi_L = \begin{pmatrix} \xi \\ 0 \end{pmatrix}$$

$$\psi_R = \begin{pmatrix} 0 \\ i\sigma_2\bar{\chi} \end{pmatrix}$$

$$\begin{pmatrix} \xi^c \\ i\sigma_2\bar{\chi}^c \end{pmatrix} = -i\begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix}\begin{pmatrix} \xi \\ +i\sigma_2\bar{\chi} \end{pmatrix}$$

$$\xi^c = \bar{\chi}.$$

$$\Psi_D = \Psi^1_{\text{Weyl},L} + \left(\Psi^2_{\text{Weyl},L}\right)^C$$

Left , Left → Right-handed.

$$\psi = \begin{pmatrix} \chi_1 \\ i\sigma_2 \chi_2^* \end{pmatrix} \qquad \psi^* = \begin{pmatrix} \chi_1^* \\ i\sigma_2 \chi_2 \end{pmatrix}$$

$$\psi_c = C\psi^* = \begin{pmatrix} \chi_2 \\ -i\sigma_2 \chi_1^* \end{pmatrix}$$

⊙ $C = -i\gamma^2\gamma^5$ in ~~Dirac~~ (Weyl) representation (Chiral).

$$= -i \begin{pmatrix} 0 & \sigma_2 \\ -\sigma_2 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\bar{\psi}\psi = (\chi_1^\dagger \quad -\chi_2^T i\sigma_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \chi_1 \\ i\sigma_2 \chi_2^* \end{pmatrix}$$

$$= \chi_2^T(-i\sigma_2)\chi_1 + h.c.$$

$$\Rightarrow \chi_2\chi_1 + h.c.$$

$$\bar{\psi}\psi_c = (\chi_1^\dagger \quad -\chi_2^T i\sigma_2) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \chi_2 \\ -i\sigma_2 \chi_1^* \end{pmatrix}$$

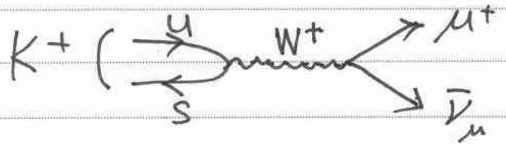
$$= \chi_2^T(-i\sigma_2)\chi_2 + \chi_1^\dagger(-i\sigma_2)\chi_1^*$$

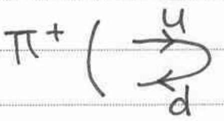
also $\bar{\psi}_c\psi \Rightarrow \chi_2\chi_2 - \chi_1\chi_1 + h.c.$

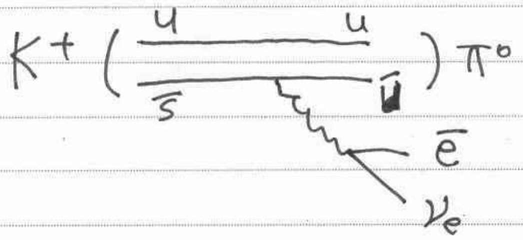
$\Delta S = 1$ vs $\Delta S = 0$

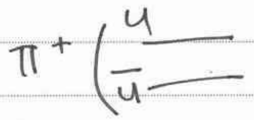
$$\frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} \sim \frac{1}{20}$$

$$\frac{\Gamma(K^+ \to \pi^0 e^+ \nu_e)}{\Gamma(\pi^+ \to \pi^0 e^+ \nu_e)} \sim \frac{1}{20}$$





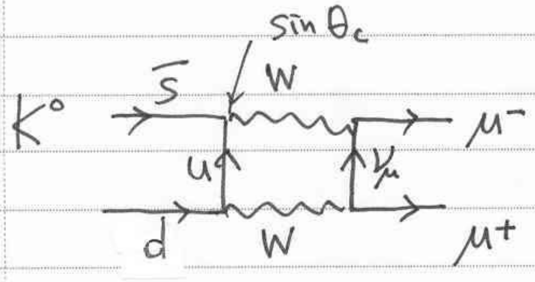




$$\frac{\Gamma(K_L^0 \to \mu^+\mu^-)}{\Gamma(K_L^0 \to \text{all modes})} \simeq 10^{-8}$$

naively $\cos^2\theta_c \cdot \sin^2\theta_c$

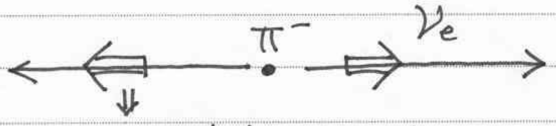
as we have $2\,\bar{d}\,s\,Z^0 \cos\theta_c \sin\theta_c$



π decay

$$\Gamma(\pi^- \xrightarrow{\text{mainly}} \mu^- \bar{\nu}_\mu)$$

$$\Gamma = \frac{1}{\tau} = \frac{G_F^2}{8\pi} f_\pi^2 m_\pi m_\mu^2 \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2$$



wrong helicity and is suppressed by m_e.

Parity violation & V−A weak current

$$\mathcal{L} \supset \frac{G_F}{\sqrt{2}} (\bar{u}_n \gamma^\mu u_p)(\bar{u}_{\nu_e} \gamma_\mu u_e)$$

~ 1930
Fermi

G_F ; Fermi constant

(charged) Current - Current interactions

$$\gamma^\mu \Rightarrow \gamma^\mu (1+\gamma^5)$$

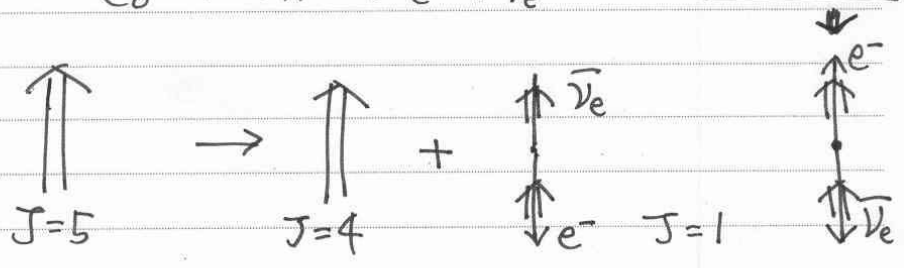
$K^+ \to 2\pi, \ 3\pi$) opposite parity

1956
Lee and Yang

- β-transition of polarized cobalt nuclei

$$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$$

no observation



$$^{60}Co \to {}^{60}Ni^* + (e^-)_L + (\bar{\nu}_e)_R$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^+ \to \mu^+ \nu_R) = 0 \qquad \cancel{P}$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) \neq \Gamma(\pi^- \to \mu^- \bar{\nu}_L) = 0 \qquad \cancel{C}$$

$$\Gamma(\pi^+ \to \mu^+ \nu_L) = \Gamma(\pi^- \to \mu^- \bar{\nu}_R) \qquad \underline{CP}$$

invariance.