Standard Model
(2)

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Spontaneous Symmetry Breaking

e.g. 1 : a real scalar field

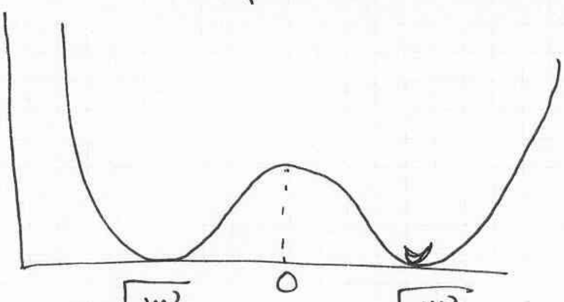
(a) $\mathcal{L}(\phi) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4}\phi^4$

◦ $\phi \to -\phi$ is a symmetry. (no ϕ^3 term)

(b) $\mathcal{L}(\phi) = \frac{1}{2}\partial^\mu\phi\partial_\mu\phi - V(\phi)$

$V(\phi) = \frac{\lambda}{4}\left(\phi^2 - \frac{m^2}{\lambda}\right)^2$

at $\phi = 0$; there is a tachyon
(a particle with imaginary mass)



$-\sqrt{\frac{m^2}{\lambda}}$ $\quad 0 \quad$ $\sqrt{\frac{m^2}{\lambda}}$

two local minimum

$$\phi' = \phi - \sqrt{\frac{m^2}{\lambda}}$$

$$\mathcal{L}(\phi') = \frac{1}{2}\partial^\mu \phi' \partial_\mu \phi' - V(\phi')$$

$$V(\phi') = \frac{\lambda}{4}\left(\phi'^2 + 2\sqrt{\frac{m^2}{\lambda}}\phi'\right)^2$$

$$= \frac{\lambda}{4}\phi'^4 + \underline{\sqrt{\frac{m^2}{\lambda}}\phi'^3} + m^2\phi'^2$$

We can not see the other vacuum in perturbation theory.

Spontaneous symmetry breaking happens only when the spacetime is infinite.

For finite space, the ground state is a linear combination of $\langle\phi_+\rangle$ and $\langle\phi_-\rangle$ vacuum.

Goldstone Theorem

$$\mathcal{L}(\phi) = \frac{1}{2} \partial_\mu \phi \, \partial^\mu \phi - V(\phi)$$

$$\delta \phi = i \epsilon_a T^a \phi$$

$\langle \phi \rangle = \lambda$ is a minimum.

$$V_{j_1 \cdots j_n}(\phi) = \frac{\partial^n}{\partial \phi_{j_1} \cdots \partial \phi_{j_n}} V(\phi)$$

$$V_j(\lambda) = 0.$$

$$V_{jk}(\lambda) \geq 0.$$

$$m^2_{jk} = \frac{1}{2} V_{jk}(\lambda)$$

$$V(\phi + \delta\phi) - V(\phi) = V_i \, \delta\phi_i = V_i \cdot i\epsilon_a (T^a)_{ij} \phi_j = 0$$

$$\frac{\partial}{\partial \phi_k} \left(\qquad\qquad\qquad \right)$$

$$\Rightarrow \quad V_{ik} (T^a)_{ij} \phi_j + V_i (T^a)_{ij} = 0$$

$$\phi = \lambda \ \Rightarrow \ V_i(\lambda) = 0$$

$$V_{ki}(\lambda)(T^a)_{ij} \lambda_j = 0 \ \Rightarrow \ M^2 T^a \lambda = 0$$

For T^a in the unbroken subgroup,

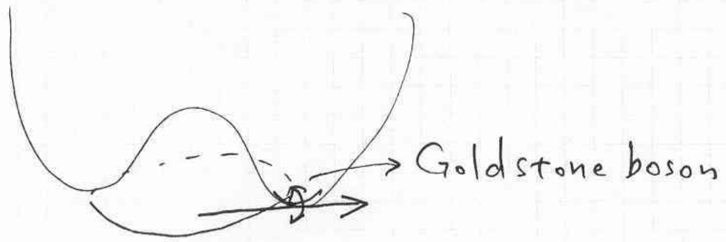
$M^2(\underbrace{T^a\lambda}_{=0}) = 0$ is trivially satisfied.

If $T^a\lambda \neq 0$. $M^2[T^a\lambda] = 0$

$\left[\begin{array}{l} T^a\lambda : \text{eigenstate} \\ M^2 = 0 : \text{eigenvalue} \end{array}\right.$

$T^a\lambda \neq 0$: Goldstone boson

If continuous global symmetry is spontaneously brok there appear massless particles (spin 0).



is related to the degeneracy of the vacuum generated by the symmetry.

Higgs Mechanism

· electron mass

$$- f \, \overline{e_R} \, \phi^{\dagger} \psi_L + h.c.$$

$$T_a \phi = \frac{\tau_a}{2} \phi . \quad S\phi = \frac{1}{2} \phi$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$$\delta \phi^{\dagger} = - i \epsilon_a \phi^{\dagger} T_a - \frac{i}{2} \epsilon \phi^{\dagger}$$

$$\delta(\phi^{\dagger} \psi_L) = - i \epsilon (\phi^{\dagger} \psi_L) \quad : \text{SU(2) singlet}$$

with $S = -1$.

$$\mathcal{L}_{KE}(\phi) = (D_\mu \phi)^{\dagger} (D^\mu \phi)$$

$$\begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{\phi_3 + i \phi_4}{\sqrt{2}} \\ \frac{\phi_1 + i \phi_2}{\sqrt{2}} \end{pmatrix}$$

$(-\sigma_2)$.

$\sigma(\ )$ $\quad \Phi = \begin{pmatrix} \phi_3 \\ \phi_4 \\ \phi_1 \\ \phi_2 \end{pmatrix}$. $\quad \sigma_j \cdot \tau_j . \quad \sigma_j \tau_k .$

$\sigma(\ )$

$$T_1 \Phi = - \frac{1}{2} \tau_1 \sigma_2 \Phi$$

$$T_2 \Phi = \frac{1}{2} \tau_2 \Phi$$

$$T_3 \Phi = - \frac{1}{2} \tau_3 \sigma_2 \Phi$$

$$S \Phi = - \frac{1}{2} \sigma_2 \Phi$$

$$\mathcal{L}_{KE}(\Phi) = \frac{1}{2} D^{*\mu}\Phi^T D_\mu \Phi$$

$$= \frac{1}{2}\left[\partial^\mu \Phi^T - i\Phi^T\left(\frac{e}{\sin\theta}\vec{T}\cdot\vec{W}_\mu + \frac{e}{\cos\theta} S X_\mu\right)\right]$$

$$\left[\partial_\mu + i\left(\frac{e}{\sin\theta}\vec{T}\cdot\vec{W}_\mu + \frac{e}{\cos\theta} S X_\mu\right)\right]\Phi$$

$$V(\phi) = \frac{\lambda}{2}\left(\phi^\dagger\phi - \frac{v^2}{2}\right)^2$$

$$\phi = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}$$

$$\langle\Phi\rangle = \begin{pmatrix} 0 \\ 0 \\ v \\ 0 \end{pmatrix} = \lambda$$

$$\Phi' = \Phi - \lambda$$

$$\mathcal{L} \supset \frac{1}{2}\lambda^T\left[\frac{e}{\sin\theta}\vec{T}\cdot\vec{W}_\mu + \frac{e}{\cos\theta} S X_\mu\right]\left[\frac{e}{\sin\theta}\vec{T}\cdot\vec{W}^\mu + \frac{e}{\cos\theta} S X^\mu\right]\lambda$$

$$\& \quad i\partial^\mu \Phi'^T\left[\frac{e}{\sin\theta}\vec{T}\cdot\vec{W}_\mu + \frac{e}{\cos\theta} S X_\mu\right]\lambda$$

Goldstone boson : $\Phi'^T T\lambda = \Phi^T T\lambda \quad (\lambda^T T\lambda = 0)$

$\phi_1(x) = \phi_2(x) = \phi_4(x) = 0.$: Unitary gauge

$$\frac{e}{\sin\theta} T_3 W_3^{\mu} + \frac{e}{\cos\theta} S X^{\mu} = e Q A^{\mu} + \frac{e}{\sin\theta\cos\theta}(T_3 - \sin^2\theta\, Q) Z^{\mu}$$

$$Q\lambda = 0.$$

Mass of Z

$$: \frac{1}{2} \frac{e^2}{\sin^2\theta\cos^2\theta} Z^{\mu} Z_{\mu} \lambda^T T_3^2 \lambda$$

$$= \frac{1}{2}\left[\frac{ev}{2\sin\theta\cos\theta}\right]^2 Z^{\mu} Z_{\mu}$$

$$M_Z = \frac{ev}{2\sin\theta\cos\theta}$$

Mass of $W^{\pm}$.

$$; \frac{1}{2}\left[\frac{ev}{2\sin\theta}\right]^2 (W_1^{\mu} W_{1\mu} + W_2^{\mu} W_{2\mu})$$

$$M_W = \frac{ev}{2\sin\theta} = M_Z\cos\theta$$

Unitary-gauge requirement: $\Phi^T T_a \lambda = 0.$

Neutral Currents

$$G_F = \frac{\sqrt{2}e^2}{8M_W^2 \sin^2\theta} = \frac{1}{\sqrt{2}v^2}$$

$$\frac{G_F}{\sqrt{2}}\left(j_1^\alpha j_{1\alpha} + j_2^\alpha j_{2\alpha} \right) \quad \cdots (A)$$

$$j_a^\alpha = \Sigma \overline{\psi}_L T_a \gamma^\alpha (1+\gamma_5) \psi_\ell$$

$$W : \frac{e}{\sin\theta} T_a$$

$$Z : \frac{e}{\sin\theta\cos\theta}\left(T_3 - \sin^2\theta\, Q\right)$$

M_Z massive by $\cos^2\theta$

$$\Rightarrow \frac{G_F}{\sqrt{2}}\left(j_3^\alpha - 2\sin^2\theta\, j_{em}^\alpha \right)\left(j_{3\alpha} - 2\sin^2\theta\, j_{em\alpha} \right) \quad \cdots (B)$$

$$\left[j_{em}^\alpha = \overline{\psi}\gamma^\alpha Q \psi \right]$$

If $\sin^2\theta = 0$ $\left(g' = 0 \right) \Rightarrow$ ~~$\mathcal{W}$~~ $(A) = (B)$.

$$SU(2)_L \times SU(2)_R \to \underline{SU(2)_D}.$$

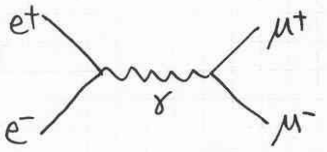
Custodial $SU(2)$ symmetry.

$G_F = 1.166 \times 10^{-5}\ \mathrm{GeV}^{-2}$

$\sigma \simeq G_F^2 s$

$s \simeq 2E e_m$

$e^+ e^- \to \mu^+ \mu^-$



Z^0

Quarks and QCD

Color SU(3)

$$q = \begin{pmatrix} q_{red} \\ q_{green} \\ q_{blue} \end{pmatrix} \qquad G^{\mu}{}_{a} \quad a = 1, \cdots, 8$$

$$\mathcal{L} = -\frac{1}{4} G^{\mu\nu}{}_{a} G_{a\mu\nu} + \sum_{\text{flavors}} \left(i \bar{q} \not{D} q - m_q \bar{q} q \right)$$

$$D^{\mu} = \partial^{\mu} + i g T_a G^{\mu}{}_{a}$$

$$i g T_a G^{\mu\nu}{}_{a} = [D^{\mu}, D^{\nu}]$$

$$\mathrm{tr}(T_a T_b) = \frac{1}{2} \delta_{ab}.$$

hadrons
- mesons $\quad \bar{q} q$
- baryons $\quad \epsilon_{ijk} q_i q_j q_k$

toy world

Varying $\dfrac{\Lambda}{m_q}$ with m_p fixed

(i) almost QED like

(ii) entire confining

Six-Quark Model

∘ $L_{jL} = \begin{pmatrix} \nu_j \\ l_j^- \end{pmatrix}_L \qquad j=1,2,3.$

l_{jR}^-

$\vec{T} L_{jL} = \frac{\vec{\tau}}{2} L_{jL}, \quad S L_{jL} = -\frac{1}{2} L_{jL}$

$\vec{T} l_{jR}^- = 0, \qquad S l_{jR}^- = -l_{jR}^-$

$$\mathcal{L} \supset - f_{jk} \overline{l_{jR}^-} \phi^\dagger L_{kL} + h.c.$$

$$\Rightarrow - f_{jk} \frac{v}{\sqrt{2}} \overline{l_{jR}^-} l_{kL}^- + h.c.$$

; mass term for charged leptons

∘ $\psi_{jL} = \begin{pmatrix} U_j \\ D_j \end{pmatrix}_L$

$U_{jR}, \quad D_{jR}$

$(Q=\frac{2}{3}) \quad (Q=-\frac{1}{3})$

$\vec{T} \psi_{jL} = \frac{\vec{\tau}}{2} \psi_{jL}, \quad S \psi_{jL} = \frac{1}{6} \psi_{jL}$

$\vec{T} U_{jR} = \vec{T} D_{jR} = 0$

$S U_{jR} = \frac{2}{3} U_{jR}, \quad S D_{jR} = -\frac{1}{3} D_{jR}$

$$\mathcal{L} \supset -g_{jk} \bar{D}_{jR} \phi^\dagger \psi_{kL} + h.c.$$

$\langle \phi \rangle \neq 0$

$$\Rightarrow \quad -M^D_{jk} \bar{D}_{jR} D_{kL} + h.c.$$

$$M^D_{jk} = g_{jk} \frac{v}{\sqrt{2}}$$

For U, we need $\tilde{\phi} = i \tau_2 \phi^*$

$$\delta \tilde{\phi} = i \epsilon_a \frac{\tau_a}{2} \tilde{\phi} - \frac{i}{2} \epsilon \tilde{\phi}$$

$$T \tilde{\phi} = \frac{\tau}{2} \tilde{\phi} \quad , \quad S \tilde{\phi} = -\frac{1}{2} \tilde{\phi}$$

$$\mathcal{L} \supset -h_{jk} \bar{U}_{jR} \tilde{\phi}^\dagger \psi_{kL} + h.c.$$

$$\tilde{\phi} = \begin{pmatrix} \phi^{0*} \\ \phi^- \end{pmatrix} , \quad \phi^- = -\phi^{+*}$$

$$\Rightarrow \quad -M^U_{jk} \bar{U}_{jR} U_{kL} + h.c.$$

$$M^U_{jk} = h_{jk} \frac{v}{\sqrt{2}}$$

$$M^U = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix}$$

by $\psi'_L \to \cancel{\ } V_L \psi'_L$

$U_R \to V_R U_R$

$$- M^D_{j\ell} V^\dagger_{\ell k} \bar{D}_{jR} D_{kL} + h.c.$$

$$M^D = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

$$\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L = V^\dagger D_L$$

$$D_L = V \underbrace{\begin{pmatrix} d \\ s \\ b \end{pmatrix}_L}_{\text{mass eigenstates}}$$

◎ charged current

$$(\bar{u}\ \bar{c}\ \bar{t})\, \gamma^\mu (1+\gamma_5)\, V \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

nine → 3 real parameters (mixing angle

and 1 phase.

Z_0^0 couples to the quark neutral current in any basis. $x = \sin^2\theta$: Weinberg angle

$$\mathcal{L} \supset \frac{e}{\sin\theta\cos\theta} \sum_j \Big\{ \left(\frac{1}{2} - \frac{2}{3}x\right) \bar{U}_{jL}\gamma^\mu U_{jL} + \left(-\frac{1}{2} + \frac{1}{3}x\right) \bar{D}_{jL}\gamma^\mu D_{jL} - \frac{2}{3}x\,\bar{U}_{jR}\gamma^\mu U_{jR} + \frac{1}{3}x\,\bar{D}_{jR}\gamma^\mu D_{jR} \Big\}$$

V appears in U-D interactions.
(charged current)

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{td} \end{pmatrix}$$

$$(\bar{u}\ \ \bar{c}\ \ \bar{t})\,\gamma^\mu \frac{(1+\gamma_5)}{2} V \begin{pmatrix} d \\ s \\ b \end{pmatrix} \cdot g W_\mu^{a+}$$

Quarks

		(or truth)
u (up)	c (charm)	t (top)
d (down)	s (strange)	b (beauty)
		(or bottom)

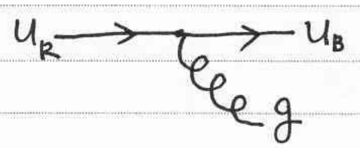
Leptons

e (electron)	μ (muon)	τ (tau)
ν_e (e neutrino)	ν_μ (μ-neutrino)	ν_τ (τ-neutrino)

Quarks Leptons

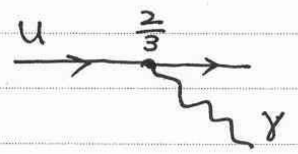
Strong

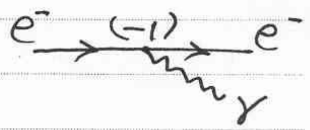
∘ color



EM

∘ electric charge





Weak

∘ weak charge