Standard Model
(1)

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References

1. Weak Interactions and Modern Particle Theory ☆☆☆☆
   Howard Georgi
   (Addison-Wesley)

2. Introduction to High Energy Physics ☆☆(☆)
   Donald H. Perkins
   (Cambridge)

3. Gauge Theories ☆☆
   E. S. Abers and Benjamin W. Lee
   (Physics Report V9C (1973) 1)

4. Particle Physics ☆☆(☆)
   B.R. Martin and G. Shaw
   (Wiley)

5. A Modern Introduction to Particle Physics ☆☆
   Fayyazuddin and Riazuddin
   (World Scientific)

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6. The God Particle ☆
   Lederman

7. Strange Beauty ☆
   G. Johnson

- Gravity, Weak, EM & Strong Force
- Quarks and Leptons
  (Baryons / Mesons) Hadrons
- Gauge Symmetry ~~(and)~~ vs Global Symmetry
- Goldstone Theorem
- Higgs Mechanism
- CKM Matrix
- Gauge and Yukawa Interactions
- Weyl Spinor

Units and Notation

$$\hbar = c = 1$$

$[\hbar] = ML^2T^{-1} = 6.6 \times 10^{-16}$ eV·s

$[c] = LT^{-1} = 3 \times 10^8$ m/s

$[\hbar c] = 0.2$ GeV · fm

1 eV = 1.6×10^{-19} J

1 keV = 10^3 eV

1 MeV = 10^6 eV : m_e

1 GeV = 10^9 eV : m_p

1 TeV = 10^{12} eV : m_W. m_h.

$$x^\mu = (t, \vec{x})$$

$$p^\mu = (E, \vec{p})$$

$$E^2 = \vec{p}^{\,2} + m^2 \quad \Rightarrow \quad p^2 = p^\mu p_\mu = m^2.$$

$$g_{\mu\nu} = \mathrm{diag}(1, -1, -1, -1)$$

History

anti-particle.

p, nuclei $\longrightarrow$ atom

n, electrically charged

neutral e.

ν.

$\bar{p}$

$\bar{n}$

$\bar{e}$

$\bar{\nu}$

~ 1932

γ : $[s=1]$ $\quad m_\gamma = 0$.

1) Gravity : Newton's Law

$$V = G_N \frac{m_p^2}{r}$$

$$G_N = \frac{1}{M_P^2} \sim \frac{1}{(10^{19}\,\mathrm{GeV})^2}.$$

$$\sqrt{G_N} = 5.3 \times 10^{-44}\ \mathrm{s}$$

$$= 1.6 \times 10^{-33}\ \mathrm{cm}$$

$$\alpha_{G_N} \simeq \left(\frac{1\,\mathrm{GeV}}{10^{19}\,\mathrm{GeV}}\right)^2 \simeq 10^{-40}.$$

Gravitational interactions are negligible in particle physics.

2) Weak Nuclear Force

$$n \to p + e^- + \bar{\nu}_e$$

$$O^{14} \to N^{14} + e^- + \bar{\nu}_e \quad : \quad \tau = 71.4\,s$$

$$G_F \simeq 10^{-5} \cdot [\mathrm{GeV}^{-2}]$$

$$\frac{1}{\sqrt{G_F}} \simeq 300\ \mathrm{GeV}$$

$$\sqrt{G_F} \simeq 7 \times 10^{-15}\ \mathrm{cm}$$

At $E \sim 1\ \mathrm{GeV}$

$$\alpha_W = \left(\frac{1\ \mathrm{GeV}}{300\ \mathrm{GeV}}\right)^2 \simeq 10^{-5}$$

3) Electromagnetic Force

$$V = \frac{1}{4\pi}\frac{e^2}{r} = \frac{\alpha}{r}$$

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137}$$

Bohr's formula

$$|E_1| = \frac{1}{2}\alpha^2 m_e = \underline{13.6\ eV}$$

4) Strong Nuclear Force

From 3), $m_e \to \frac{m_p}{2}$ for $p\bar{p}$

$$|E^{p\bar{p}}| \simeq 14\ \mathrm{keV}.$$

Nuclear binding energy $\simeq 2\,\mathrm{MeV}. \sim 100|E^{p\bar{p}}|$

$r_n \simeq 10^{-13}$ cm.

Summary

Gravity	Weak	EM	Strong
10^{-40}	10^{-7} (10^{-5})	10^{-2}	1

for proton, $m_p \sim 1\,\mathrm{GeV}$. $r \simeq 10^{-13}\,\mathrm{cm}$.

$R_W \sim 2 \times 10^{-16}\,\mathrm{cm}$ $\Rightarrow$ $M_W \sim 80\,\mathrm{GeV}$

1983

$R_s \sim 10^{-13}\,\mathrm{cm}$ $\Rightarrow$ $M_{\underset{\mu}{(\pi)}} \sim 100\,\mathrm{MeV}$

Yukawa (1935) $\Rightarrow$ 1938 muon

1947 pion

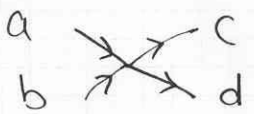
$\sim 1.4 \times 10^{-13}\,\mathrm{cm}$ $\pi^- \to \mu^- + \bar{\nu}_\mu$

$\simeq \sqrt{2}\,f$ $M_\pi \sim 140\,\mathrm{MeV}$

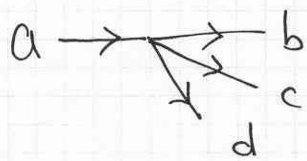
f : Fermi $= 10^{-13}\,\mathrm{cm}$

⊛ Formulas in exercise class.

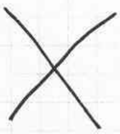
(1) Two-body scattering



(2) Three-body decay

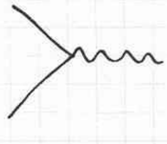


$\bar{\psi}\psi\phi \Rightarrow \bar{\psi}\psi\bar{\psi}\psi$



$\phi\phi\phi\phi$

$\bar{\psi}\gamma^{\mu}A_{\mu}\psi$



Symmetry and Charge Conservation

(1) Q : Electric charge conservation

$e^- \to \nu + \gamma$ (not seen)

$\tau_e > 4.3 \times 10^{23}$ years

(2) B : Baryon number conservation

$$\left.\begin{array}{l} p \to e^+ + \gamma \\ p \to e^+ + \pi^0 \end{array}\right) \text{ not seen}$$

$\tau_p > 10^{32}$ years

$$B = \begin{cases} 1 & : \text{baryons} \\ -1 & : \text{anti-baryons} \\ 0 & : \text{leptons and mesons} \end{cases}$$

(3) L : Lepton number conservation

$$L = \begin{cases} 1 & : \text{leptons} \\ -1 & : \text{anti-leptons} \\ 0 & : \text{others} \end{cases}$$

(4) Strangeness and Hypercharge

$S(\pi) = 0$. $\pi^{\pm}, \pi^0$. η, η'

$S(K) = 1$ K^+, K^0

$S(\bar{K}) = -1$ $\bar{K}^0, K^-$

$J^P = 0^-$ mesons.

$S(p, n) = 0$

$S(\Sigma^{\pm}, \Sigma^0, \Lambda^0) = -1$

$S(\Xi^0, \Xi^{-1}) = -2.$

S is conserved in hadronic collisons.

$\sigma_N \sim 10^{-24}\,\mathrm{cm}^2$

$\sigma_W \sim 10^{-43}\,\mathrm{cm}^2$

$$\pi^- + p \begin{cases} \to K^0 + \Lambda^0 & \Delta S = 0 \\ \not\to K^- + \Sigma^+ & \Delta S = -2 \\ \not\to K^- + p & \Delta S = -1 \\ \to n + K^+ + K^- & \Delta S = 0 \end{cases}$$

$$\left.\begin{array}{l} \Lambda \to p\pi^- \\ K^0 \to \pi^+\pi^- \end{array}\right\} \quad \tau \sim 10^{-10}\,\mathrm{s}!$$

S is not conserved in weak decays.

A Model of Leptons (Weinberg)

Why was QED so successful?

1. electron is the lightest charged particle.

2. it does not carry color.
   $\Rightarrow$ no strong interactions.

$\downarrow$

leptons : colorless, spin $\frac{1}{2}$ particles.

e^-, ν_e.

$\left.\begin{array}{ll} \mu^-, & \nu_\mu. \\ \tau^-, & \nu_\tau \end{array}\right)$ same kinds of interactions as e^-, ν_e.

but decay.

- electron is absolutely stable as it is the lightest charged particle.

A Model of Leptons

(historically strong interactions were not understood, and also weak interactions of hadrons.)

∘ $\boxed{SU(2) \times U(1)}$

$$
\begin{array}{ccc}
\nu_{eL} & e^-_L & e^-_R \\
\nu_{\mu L} & \mu^-_L & \mu^-_R \\
\nu_{\tau L} & \tau^-_L & \tau^-_R
\end{array}
$$

(no need for ν_R (or $\bar{\nu}_L$))

- neutrinos are very light.
- if neutrinos are massless, electron number, μ number and τ number are conserved separately.

$$L_e = 1 : \nu_{eL} \to e^{i\theta_e} \nu_{eL}, \quad e^-_L \to e^{i\theta_e} e^-_L, \quad e^-_R \to e^{i\theta_e} e^-_R$$

$$L_\mu = 1 : \nu_{\mu L} \to e^{i\theta_\mu} \nu_{\mu L}, \quad \mu^-_L \to e^{i\theta_\mu} \mu^-_L, \quad \mu^-_R \to e^{i\theta_\mu} \mu^-_R$$

$$L_\tau = 1 : \nu_{\tau L} \to e^{i\theta_\tau} \nu_{\tau L}, \quad \tau^-_L \to e^{i\theta_\tau} \tau^-_L, \quad \tau^-_R \to e^{i\theta_\tau} \tau^-_R.$$

- lepton number violationg effects can appear if neutrino masses are non-zero.

Assume $m_{\nu_i} = 0$. $(i = e, \mu, \tau)$

Then there is no mixing between families and electron family is enough to study interactions. (μ, τ families are copies of e-)

◎ Gauge group: $\underset{\substack{\uparrow \\ W_a^\mu}}{\underline{SU(2)}} \times \underset{\substack{\uparrow \\ X^\mu}}{\underline{U(1)}}$

4 gauge bosons: W_a^μ $\quad X^\mu$
(vector fields) $a = 1, 2, 3$

◎ Covariant derivative: [A1] QED example.

$$D^\mu = \partial^\mu + ig W_a^\mu T_a + ig' X^\mu S$$

T_a, S) matrices acting on the fields.

; generators of SU(2) and U(1), respectively.

g, g': couplings of 〃 .

Gauge Principle

$$\Psi(x) \to \Psi'(x) = e^{ig\Lambda}\,\Psi(x)$$

Lagrangian is invariant for constant Λ.

$$\mathcal{L} = \bar{\Psi}\, i\gamma^\mu \partial_\mu \Psi \left(- m\bar{\Psi}\Psi\right)$$

Suppose $\Lambda = \Lambda(x)$.

$$\mathcal{L}' = \bar{\Psi}\, e^{-ig\Lambda}\left[i\gamma^\mu \partial_\mu\right] e^{ig\Lambda}\,\Psi$$

$$= \bar{\Psi}\, i\gamma^\mu\left[\partial_\mu - ig\partial_\mu\Lambda\right]\Psi$$

We can introduce $A_\mu(x)$ s.t.

$$\partial_\mu \Psi(x) \to D_\mu \Psi(x) \equiv (\partial_\mu + igA_\mu)\Psi(x)$$

$$A_\mu \to A_\mu + \partial_\mu \Lambda.$$

$\mathcal{L}$ is invariant.

$$\mathcal{L} = \bar{\Psi}\, i\gamma^\mu D_\mu \Psi - m\bar{\Psi}\Psi$$

$\Rightarrow$ QED

NO.

Quantum ChromoDynamics

$SU(3)_C$

$$\begin{pmatrix} u_r' \\ u_g' \\ u_b' \end{pmatrix} = U \begin{pmatrix} u_r \\ u_g \\ u_b \end{pmatrix}$$

$$U = e^{\frac{i}{2}\lambda_A \Lambda_A(x)}$$

$\underbrace{\lambda_A}_{3\times3}$: Gell-Mann matrix, $A = 1, \cdots, 8$

$$D_\mu = \partial_\mu - \frac{i}{2} g_s \lambda_A \underbrace{G_{A\mu}(x)}_{\text{gluons}}$$

Exercise : Gauge Transformation Rule for $G_{A\mu}(x)$?

$$\psi_L = \begin{pmatrix} \nu_{eL} \\ e^-_L \end{pmatrix}$$: weak (electron) doublet

$$T_a \psi_L = \frac{\tau_a}{2} \psi_L, \qquad T_a e^-_R = 0.$$

τ_a : Pauli matrices

$$\left[\quad \tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \ \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \ \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \right]$$

$$[T_a, T_b] = i \epsilon_{abc} T_c$$

QED can be incorporated if we define

$$Q = \underset{\substack{\downarrow}}{T_3} + Y$$

distinguish charges of ν_{eL} and e^-_L.

$$Q \, \nu_{eL} = 0$$

$$Q \, e^-_L = - e^-_L$$

$$Y: \ \psi_L = -\frac{1}{2} \psi_L$$

$$Q \, e^-_R = Y e^-_R = - e^-_R$$

$$[T_a, Y] = 0$$

$$\boxed{\Psi_L = \begin{pmatrix} \nu_L \\ e^-_L \end{pmatrix}_{-\frac{1}{2}} , \quad e^-_{R\,(-1)}}$$

◎ Interactions

W^1, W^2 : change particle identity

(T_1, T_2 : off-diagonal)

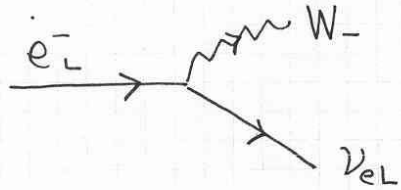
W^3. X : do not change particle identity

(T_3 : diagonal , $Y \propto$ identity)

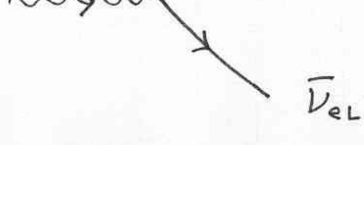
$$i \, \overline{\Psi_L} \not{D} \Psi_L$$

$$\Rightarrow \quad -\frac{g}{2}\left[\overline{\nu_{eL}} \overbrace{(\not{W}_1 - i \not{W}_2)}^{W_+} e^-_L + \overline{e^-_L} \overbrace{(\not{W}_1 + i \not{W}_2)}^{W_-} \nu_{eL} \right]$$

$$W^\mu_\pm = \frac{W_1 \mp i W_2}{\sqrt{2}}$$



or



$i\bar{\Psi}_L \not{\partial} \Psi_L \quad + \quad i\,\bar{e}_R^- \not{\partial} e_R^-$

gauge theory

$\Rightarrow \quad \not{\partial} = \gamma^\mu \partial_\mu , \qquad \partial_\mu \Rightarrow D_\mu$

$$D_\mu = \mathbb{1}_{2\times 2}\cdot\partial_\mu + i g T_a W^a_\mu + i g' Y X \cdot \mathbb{1}_{2\times 2}$$

$$\mathcal{L} \supset \underset{(i)}{i\bar{\Psi}_L \not{D} \Psi_L} \quad + \quad \underset{(ii)}{i\,\bar{e}_R \not{D} e_R}$$

$$(i) \quad i\left(\bar{\nu}_L \quad \bar{e}_L^-\right)\gamma^\mu\left[\begin{pmatrix}\partial_\mu & 0\\ 0 & \partial_\mu\end{pmatrix} + i\frac{g}{2}\begin{pmatrix}W_3 & W_1 - iW_2\\ W_1 + iW_2 & -W_3\end{pmatrix}\right.$$

$$\left. + i\frac{g'}{2}\begin{pmatrix}-X & 0\\ 0 & -X\end{pmatrix}\right]\begin{pmatrix}\nu_L\\ e_L^-\end{pmatrix}$$

$\Rightarrow$ charged currents given in p. 15

$$+ \; i\left(\bar{\nu}_L \quad \bar{e}_L^-\right)\gamma^\mu\begin{bmatrix}\partial_\mu + \frac{i}{2}[gW_3 - g'X] & 0\\ 0 & \partial_\mu + \frac{i}{2}[-gW_3 - g'X]\end{bmatrix}$$

$$\begin{pmatrix}\nu_L\\ e_L^-\end{pmatrix}$$

$$\begin{pmatrix} W_3^\mu \\ X^\mu \end{pmatrix} \to \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix}$$

$$Z \propto g W_3 - g' X$$

$$Z = \frac{1}{\sqrt{g^2+g'^2}} \left[g W_3 - g' X \right]$$

$$A \propto g' W_3 + g X$$

$$\left(\begin{array}{l} W^3_{\mu\nu} W_3^{\mu\nu} + X_{\mu\nu} X^{\mu\nu} \\ \Rightarrow Z_{\mu\nu} Z^{\mu\nu} + A_{\mu\nu} A^{\mu\nu} \end{array} \right) \text{Kinetic terms unchanged.}$$

$$A = \frac{1}{\sqrt{g^2+g'^2}} \left[g' W_3 + g X \right]$$

$$\sin \theta_W = \frac{g'}{\sqrt{g^2+g'^2}}$$

$$\sin^2 \theta_W = \frac{g'^2}{g^2+g'^2} \quad ; \theta_W : \text{Weinberg angle}$$

$$\simeq 0.23$$

Neutral Currents

$$i\begin{pmatrix}\bar{\nu}_L & \bar{e}_L^-\end{pmatrix}\gamma^\mu\begin{bmatrix}\partial_\mu+\frac{i}{2}[gW_3-g'X] & 0 \\ 0 & \partial_\mu+\frac{i}{2}[-gW_3-g'X]\end{bmatrix}\begin{pmatrix}\nu_L \\ e_L^-\end{pmatrix}$$

$$\frac{i}{\cancel{2}}\, g W_3 T_3 + i g' X Y$$

$$\begin{cases} A = \sin\theta_W W_3 + \cos\theta_W X \\ Z = \cos\theta_W W_3 - \sin\theta_W Y \end{cases}$$

$$\begin{cases} W_3 = \sin\theta_W A + \cos\theta_W Z \\ X = \cos\theta_W A - \sin\theta_W Z \end{cases}$$

$$i[g\sin\theta_W A T_3 + g'\cos\theta_W A Y$$
$$+ g\cos\theta_W Z T_3 - g'\sin\theta_W Z Y]$$

for e_L^- ; $T_3 = -\frac{1}{2}$, $Y = -\frac{1}{2}$

$$-\frac{gg'}{\sqrt{g^2+g'^2}}A = -\underset{\overset{\shortparallel}{Q}}{e}A$$

$$e = \frac{gg'}{\sqrt{g^2+g'^2}}.$$

$$e = g\sin\theta_W = g'\cos\theta_W$$

$$\Rightarrow \quad g = \frac{e}{\sin\theta_W}.$$

$$g' = \frac{e}{\cos\theta_W}.$$

$$i\Big[eAT_3 + eAY$$

$$+ \left\{ e\cdot\frac{\cos\theta_W}{\sin\theta_W} T_3 - \frac{e\cdot\sin\theta_W}{\cos\theta_W} Y \right\} Z \Big]$$

$$\Rightarrow \quad i\Big[e(T_3+Y)A$$

$$+ \left(e\frac{\cos\theta_W}{\sin\theta_W} T_3 - e\frac{\sin\theta_W}{\cos\theta_W} Y \right) Z \Big]$$

$$Q = T_3 + Y. \Rightarrow Y = Q - T_3, \quad -Y = T_3 - Q$$

$$\Rightarrow \quad i\left[eQA + \frac{e}{\sin\theta_W\cos\theta_W}\left[T_3 - \sin^2\theta_W Q\right] Z \right]$$

$$[T_3, T_1 \pm i T_2] = i T_2 \pm i(-i) T_1$$

$$= \pm T_1 + i T_2$$

$$= \pm (T_1 \pm i T_2)$$

$T_1 \pm i T_2$ are Q eigenstates.

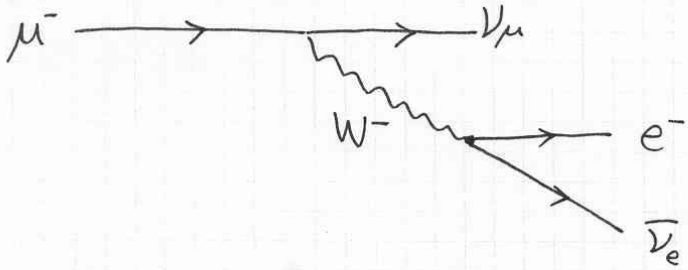
$$\begin{cases} T^+ = \dfrac{T_1 + i T_2}{2} \\ T^- = \dfrac{T_1 - i T_2}{2} \end{cases}$$

$$\begin{cases} T_1 = T^+ + T^- \\ T_2 = \dfrac{T^+ - T^-}{i} \end{cases}$$

$$T_1 W_1 + T_2 W_2$$

$$= T^+ \underbrace{(W_1 - i W_2)}_{W^+} + T^- \underbrace{(W_1 + i W_2)}_{W^-}$$

◎ μ decay



in order to make it realistic,

We should give mass to μ^-, e^- and $W^\pm$.

(If $W^\pm$ are massless,

We will have new long-range force $\propto \frac{1}{r}$.)

μ-decay

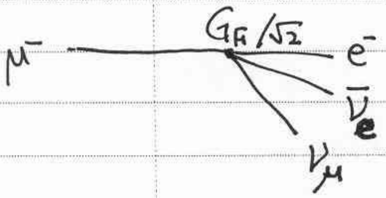
$$\mu^- \to e^- + \nu_\mu + \bar{\nu}_e$$

Lorentz vector

$$\mathcal{L}_{eff} = \frac{G_F}{\sqrt{2}} \left[\bar{\nu}_\mu \gamma^\mu (1+\gamma_5) \mu \right] \left[\bar{e} \gamma_\mu (1+\gamma_5) \nu_e \right]$$

Lorentz vector

$$\Gamma(\mu \to e \nu_\mu \bar{\nu}_e) = \frac{G_F^2}{2} \cdot m_\mu^5 \cdot \frac{1}{(4\pi)^2} \cdot \frac{1}{6\pi}$$



$(m_e \ll m_\mu)$

$$= \frac{G_F^2 m_\mu^5}{192\pi^3} = \tau_\mu^{-1}$$

$$G_F m_p^2 \simeq 10^{-5}$$

$$\Rightarrow \underbrace{\frac{10^{-10}}{192\pi^3}}_{10^3} \cdot \overbrace{\left(\frac{m_\mu^4}{m_p^4}\right)}^{10^{-4}} \cdot m_\mu \simeq \frac{1}{6} 10^{-17} \times 0.1\,\text{GeV}$$

$m_\mu \sim 100\,\text{MeV}$. $m_p \sim 1\,\text{GeV}$

$$\tau_\mu = 6 \times 10^{18} \cdot 0.2 \times 10^{-15} \cdot \frac{1}{3 \times 10^8} = 4 \times 10^{-6}\,s$$

GeV $\to$ m $\to$ s $\boxed{(2.2 \times 10^{-6}\,s)}$

∘ Neutral sector

$$A^\mu = W_3^\mu \sin\theta_w + X^\mu \cos\theta_w \quad (X^\mu \cos\theta_w + W_3^\mu \sin\theta_w)$$

$$Z^\mu = W_3^\mu \cos\theta_w - X^\mu \sin\theta_w$$

$$g \not{W}_3 T_3 + g' \not{X} Y$$

$$= g(\not{A}\sin\theta + \not{Z}\cos\theta) T_3$$

$$+ g'(\not{A}\cos\theta - \not{Z}\sin\theta) Y$$

$$= \underline{\not{A}(g\sin\theta T_3 + g'\cos\theta Y)} \quad \cdots (a)$$

$$+ \not{Z}(g\cos\theta T_3 - g'\sin\theta Y) \quad \cdots (b)$$

$$(a) = \not{A} e Q = \not{A} e (T_3 + Y)$$

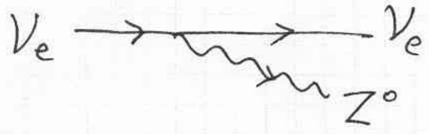
$$\Rightarrow \quad g = \frac{e}{\sin\theta}, \quad g' = \frac{e}{\cos\theta}.$$

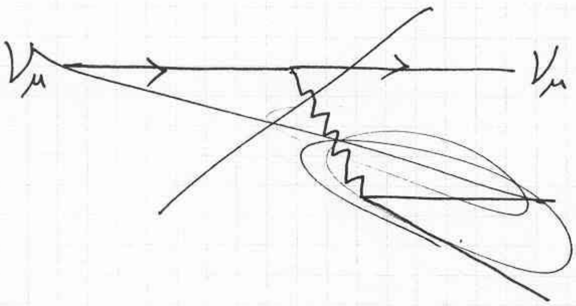
$$c^2\theta T_3 - s^2\theta Y$$

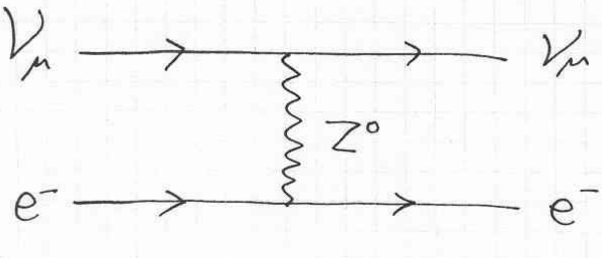
$$(b) = \not{Z}(e\cot\theta T_3 - e\tan\theta Y)$$

$$= \not{Z} \frac{e}{\sin\theta\cos\theta} \left[T_3 - \sin^2\theta Q \right]$$

neutral current weak interactions







⊙ $\nu_\mu \, e^-$ elastic scattering

(gives parity-violating long-range force
if Z^0 is massless.)

- Renormalizability

Four-Fermi theory.

; phenomenological theory of the charged-current weak interactions

$$\frac{G_F}{\sqrt{2}} J_\mu^\alpha J_{e\alpha}^*$$

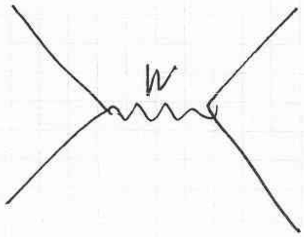
$$J_\mu^\alpha = \overline{\nu_\mu} \gamma^\alpha (1+\gamma_5) \mu^-$$

$$J_e^\alpha = \overline{\nu_e} \gamma^\alpha (1+\gamma_5) e^-$$

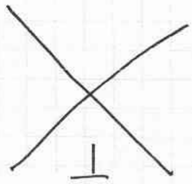
$[G_F] = -2$: $\frac{1}{(\text{mass})^2}$.

$m_p^2 G_F \simeq 10^{-5}$. m_p : proton mass.

discussions on massive gauge fields.



$\Rightarrow$



$\frac{1}{M_W^2}$