Quantum Computation and Simulation with Trapped Ions Part III

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OPEN KIAS SCHOOL ON QUANTUM INFORMATION SCIENCE

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Outline

Review of Ion Laser Interaction

- Carrier, Red-Sideband, Blue-Sideband Transition
- Schrödinger Cat State

Geometric Phase Gate and Spin Dependent Force

- Geometric Phase
- Spin Dependent Force

Scale Up

Carrier, Red sideband, Blue sideband

$$H_{\text{int}} = (\hbar/2)\Omega\left[\hat{\sigma}_+ e^{i\eta\left(ae^{-i\omega_m t}+a^+e^{i\omega_m t}\right)}e^{-i(\delta t+\varphi)} + \hat{\sigma}_- e^{-i\eta\left(ae^{-i\omega_m t}+a^+e^{i\omega_m t}\right)}e^{i(\delta t+\varphi)}\right]$$

$$\eta\sqrt{n+1} = kx_0\sqrt{n+1} << 1 : \text{Lamb-Dicke limit}$$

Stationary terms of H_{int} at particular values of δ

"CARRIER" $\delta = 0$

$$H_{carr} = (\hbar/2)\Omega\left[\hat{\sigma}_+ e^{i\varphi} + \hat{\sigma}_- e^{-i\varphi}\right]$$

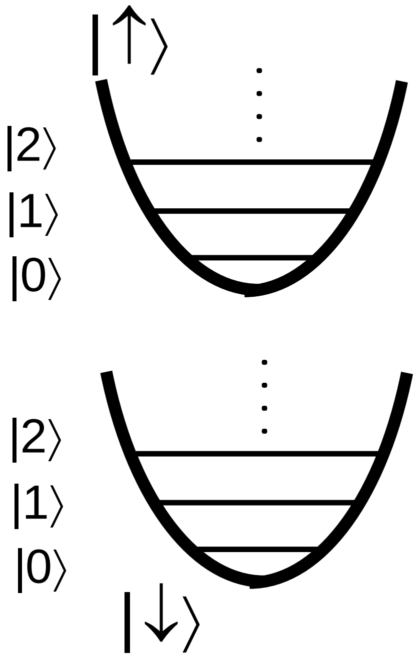
"Red Sideband" $\delta = -\omega_m$

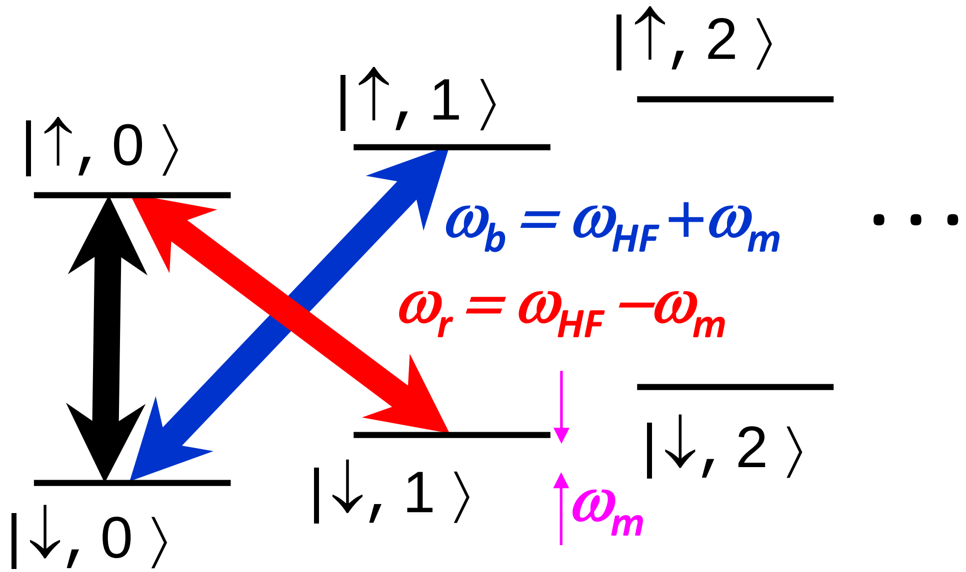
$$H_{rsb} = (\hbar/2)\eta\Omega\left[\hat{\sigma}_+ a e^{i\varphi} + \hat{\sigma}_- a^+ e^{-i\varphi}\right]$$

"Blue Sideband" $\delta = +\omega_m$

$$H_{rsb} = (\hbar/2)\eta\Omega\left[\hat{\sigma}_+ a^+ e^{i\varphi} + \hat{\sigma}_- a e^{-i\varphi}\right]$$

Carrier, Red sideband, Blue sideband





$$H^{(e)} \otimes H^{(m)}$$

$$H_{carr} = \hbar\Omega\hat{\sigma}_{+} + h.c.$$

$$H_{bsb} = -i\hbar\eta\Omega\sigma^{+}a^{\dagger} + h.c.$$

$$H_{rsb} = -i\hbar\eta\Omega\sigma^{-}a^{\dagger} + h.c.$$

Ion-Motion Coupling: Schrödinger Cat state

Applying both red-sideband and blue-sideband transitions

$$H_{rb} = H_{rsb} + H_{bsb} = (\hbar/2)\eta\Omega\hat{\sigma}_x\left[a^+ + a\right]$$

$$U_{rb}\left|\downarrow\right\rangle\left|0\right\rangle = \exp\left[-(i/\hbar)H_{rb}t\right]\left|\downarrow\right\rangle\left|0\right\rangle$$

$$= \exp\left[-i\alpha\hat{\sigma}_x\left(a^+ + a\right)\right]\left|\downarrow\right\rangle\left|0\right\rangle$$

$$= \left(\left|\downarrow_x\right\rangle\left|\alpha\right\rangle + \left|\uparrow_x\right\rangle\left|-\alpha\right\rangle\right)/\sqrt{2}$$

Schrödinger Cat state

$$\hat{D}(\alpha)\hat{D}(\beta)\left|0\right\rangle = \hat{D}(\alpha+\beta)e^{1/2\left(\alpha\beta^* - \alpha^*\beta\right)}$$

C. Monroe, et al., Science **272**, 1131 (1996)

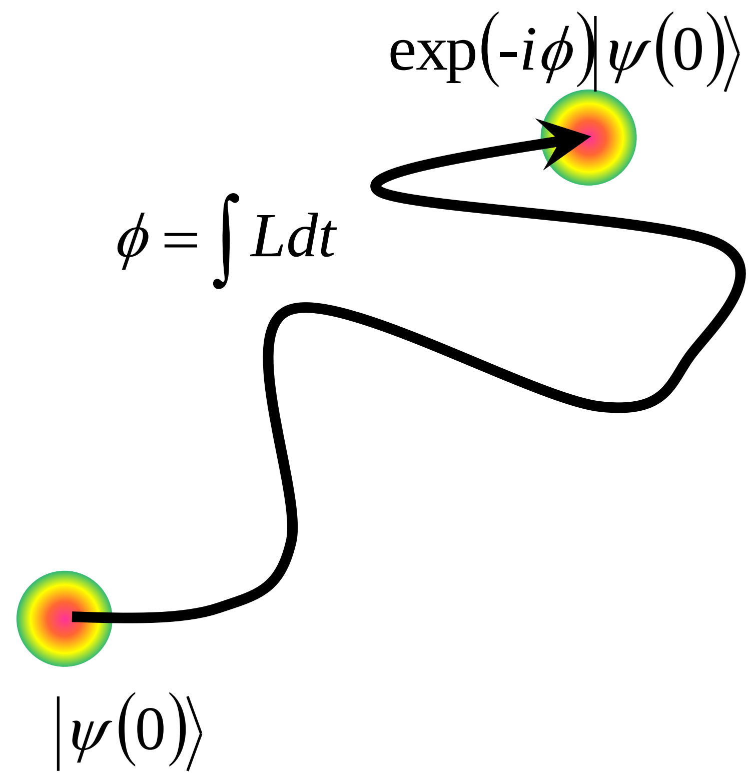
Classical Motional and Quantum Phase

Quantum State Evolution

$$|\psi(t)\rangle = \exp\left(-i\int H dt\right)|\psi(0)\rangle$$

$$= \exp\left(-i\int E dt\right)|\psi(0)\rangle,$$

if $|\psi\rangle$ is the eigenstate of H



Classical Correspondence

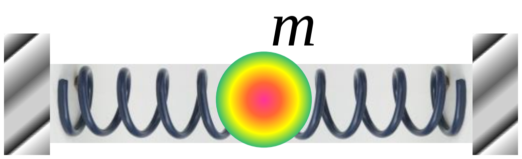
$$|\psi(t)\rangle = \exp\left(-i\int L dt\right)|\psi(0)\rangle,$$

where $L = T - V$, Lagrangian

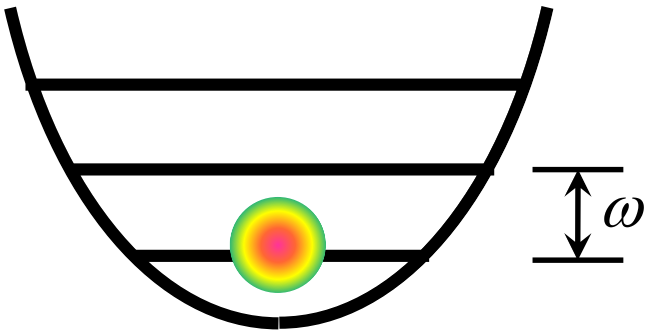
Forced Harmonic Osicllator

Harmonic Oscillator

Classical

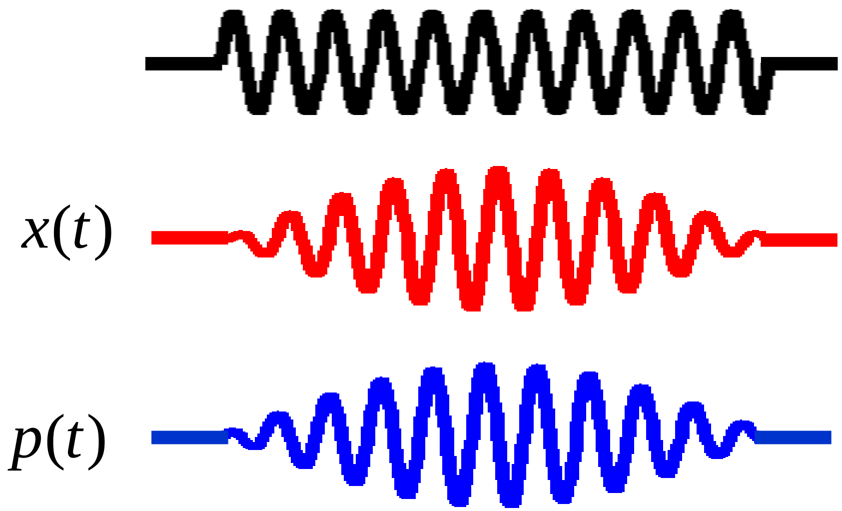


Quantum Mechanical



Force Motion

$$F(t) = F_0 \cos(\omega + \delta)t$$



Forced Harmonic Osicllator and Geometric Phase

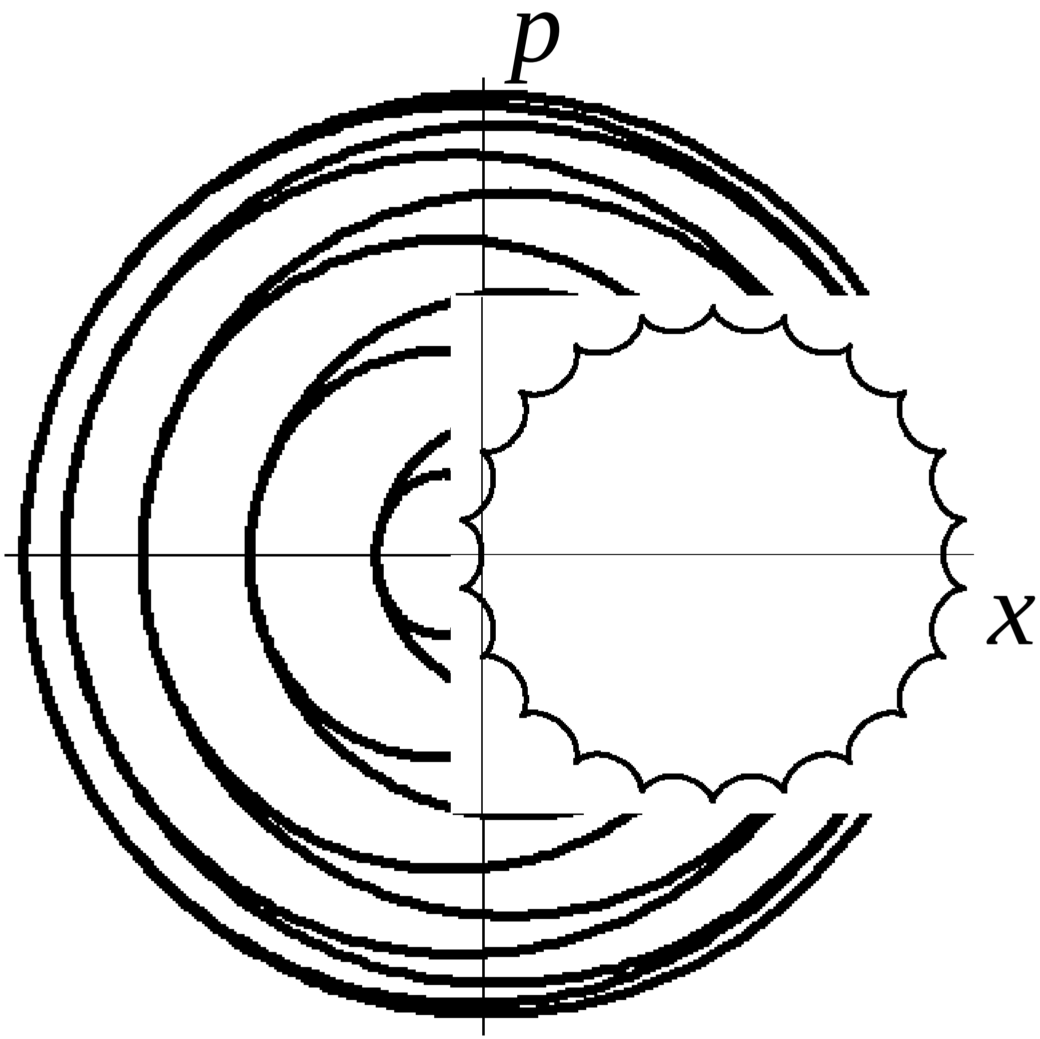
Phase Space

$$\phi = \int L dt$$

$$\phi \propto Area$$

$$\delta > 0, \phi > 0$$

$$\delta < 0, \phi < 0$$

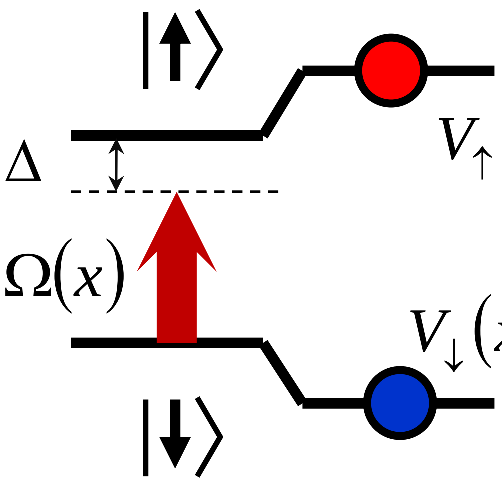


Lagrangian

$$L = \frac{p(t)^2}{2m} - \frac{1}{2} m\omega^2 x(t)^2$$

Spin-Dependent Force

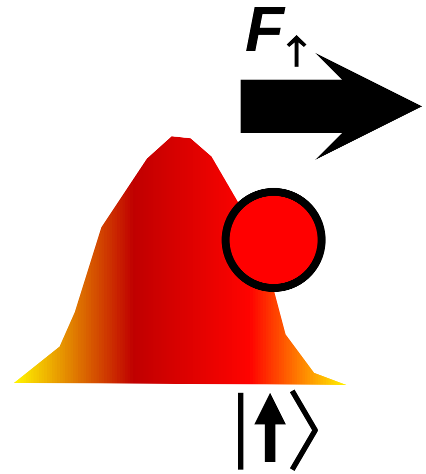
AC Stark Shift



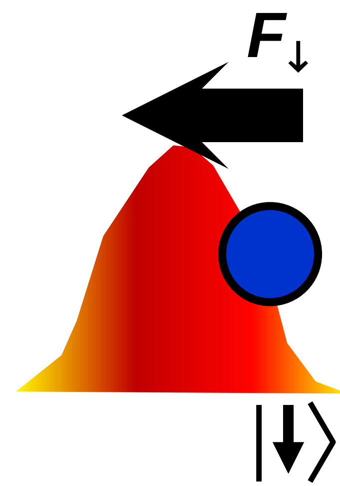
$$V_{\uparrow}(x)=\frac{\Omega(x)^2}{2\Delta}$$

$$V_{\downarrow}(x)=-\frac{\Omega(x)^2}{2\Delta}$$

Spin-Dependent Force

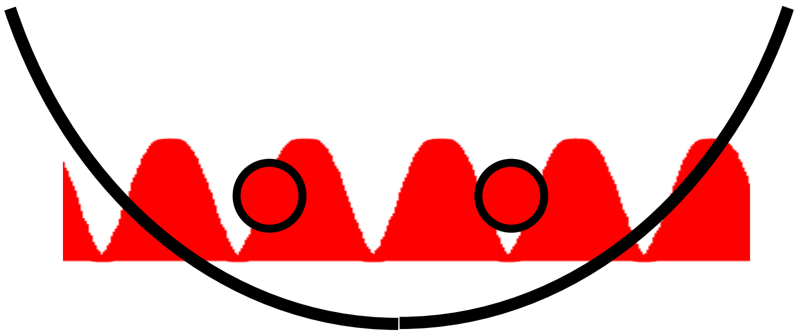


$$F_{\uparrow}=-\frac{\partial V_{\uparrow}(x)}{\partial x}=-\frac{\Omega(x)}{\Delta}\frac{\partial\Omega(x)}{\partial x}$$

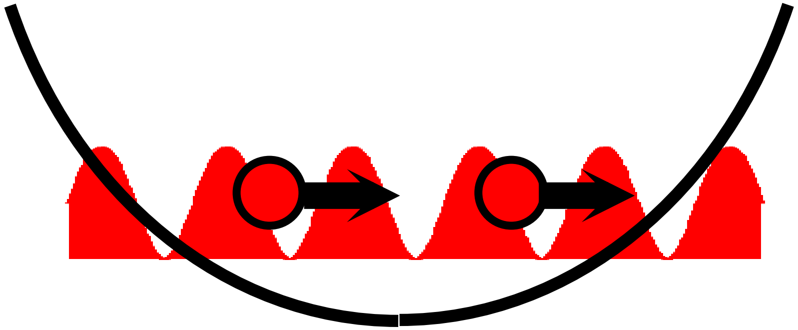


$$F_{\downarrow}=-F_{\uparrow}$$

Spin-Dependent Force and Effective Ising Interaction

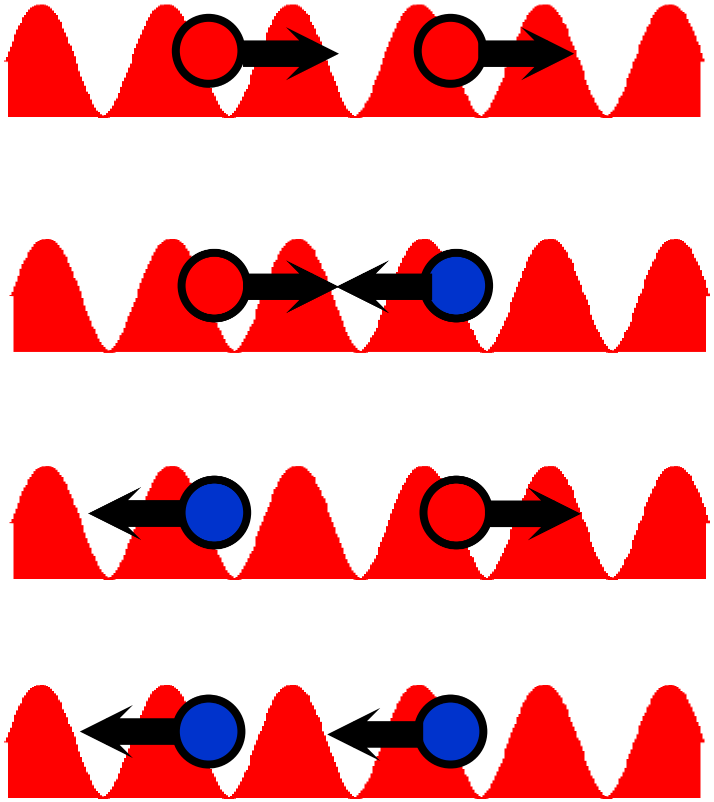


Spin-Dependent Force and Effective Ising Interaction

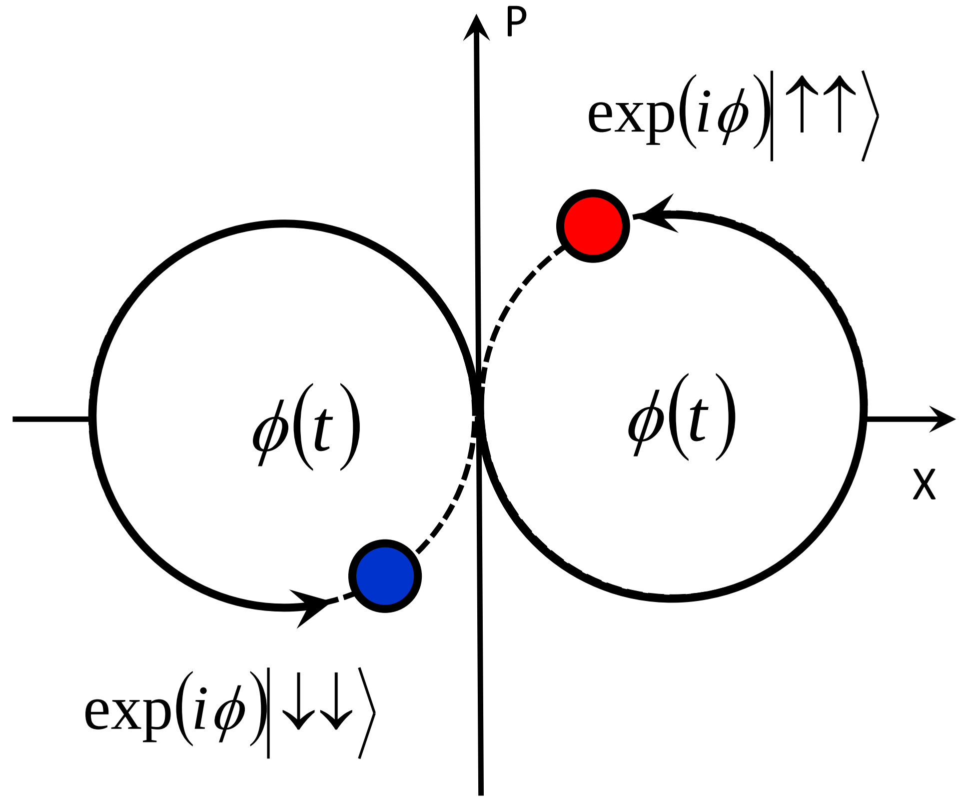


$$|\uparrow\uparrow\rangle \Rightarrow \exp(i\phi)|\uparrow\uparrow\rangle$$

Effective Ising Interaction



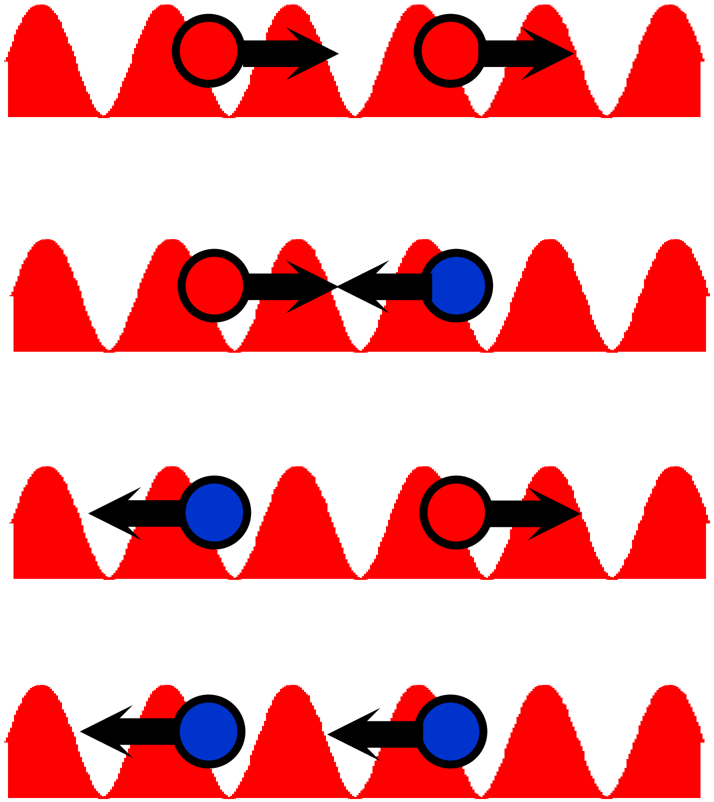
Near CM mode



$$|\uparrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\downarrow\rangle$$

$$|\downarrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\uparrow\rangle$$

Effective Ising Interaction



$$|\uparrow\uparrow\rangle \Rightarrow \exp(i\phi)|\uparrow\uparrow\rangle$$

$$|\uparrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\downarrow\rangle$$

$$|\downarrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\uparrow\rangle$$

$$|\downarrow\downarrow\rangle \Rightarrow \exp(i\phi)|\downarrow\downarrow\rangle$$

Near CM mode

$$H_J = E/2\left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)^2$$

$$\equiv E\sigma_z^{(1)}\sigma_z^{(2)}, \phi = Et$$

Effective Ising Interaction

$$H_J = E/2\left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)^2$$
$$\equiv E\sigma_z^{(1)}\sigma_z^{(2)}, \phi = Et$$

$$\left|\uparrow\uparrow\right\rangle \Rightarrow \exp(i\phi)\left|\uparrow\uparrow\right\rangle$$

$$\left|\uparrow\downarrow\right\rangle \Rightarrow \exp(i0)\left|\uparrow\downarrow\right\rangle$$

$$\left|\downarrow\uparrow\right\rangle \Rightarrow \exp(i0)\left|\downarrow\uparrow\right\rangle$$

$$\left|\downarrow\downarrow\right\rangle \Rightarrow \exp(i\phi)\left|\downarrow\downarrow\right\rangle$$

$$\left(\sigma_z^{(1)} + \sigma_z^{(2)}\right)^2$$
$$= \left(\begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 1 & \\ & & & -1 \end{pmatrix} + \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}\right)^2$$
$$= \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}$$

Addition of Blue and Red Sideband Transition

Applying red- and blue-sideband transitions for an Ion

$$H_{rb} = H_{rsb} + H_{bsb} = (\hbar/2)\eta\Omega\hat{\sigma}_x\left[a^+ + a\right]$$

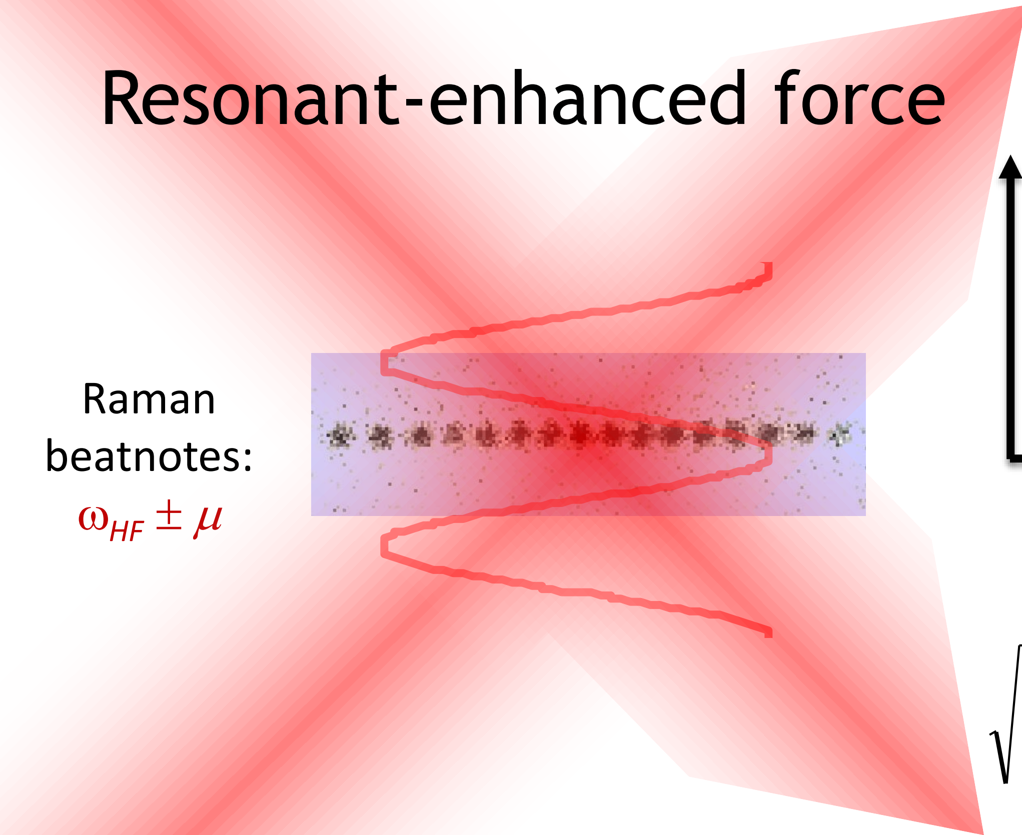
Applying red- and blue-sideband transitions for two Ion through center of mass mode

$$H_{rb} = H_{rsb} + H_{bsb} = (\hbar/2)\eta\Omega\left(\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}\right)\left[a_M^+ + a_M\right]$$

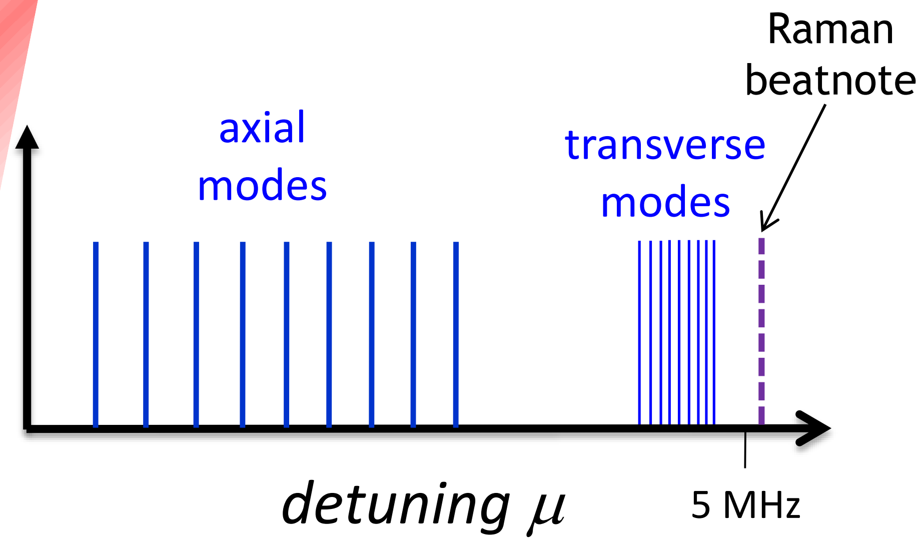
Applying red- and blue-sideband transitions for two Ion through stretch mode

$$H_{rb} = H_{rsb} + H_{bsb} = (\hbar/2)\eta\Omega\left(\hat{\sigma}_x^{(1)} - \hat{\sigma}_x^{(2)}\right)\left[a_S^+ + a_S\right]$$

Resonant-enhanced force



Raman beatnotes:
$\omega_{HF} \pm \mu$



$\sqrt{\frac{\hbar}{2m\omega_k}}$ → x_0^k

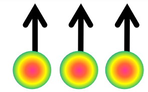
normal mode matrix: ion i, mode k → b_i^k

$$H = \Omega(\Delta k) \sum_{i,k} \hat{\sigma}_x^{(i)} x_0^k b_i^k [a_k e^{i(\mu-\omega_k)t} + a_k e^{-i(\mu-\omega_k)t}]$$

Adiabatic elimination of phonons: $|\mu - \omega| >> \Omega_0$

$$H_{eff} = \sum_{i \neq j} J_{i,j} \hat{\sigma}_x^{(i)} \hat{\sigma}_x^{(j)} \qquad J_{i,j} = \frac{\hbar \Omega^2 (\Delta k)^2}{2m} \sum_k \frac{b_i^k b_j^k}{\mu^2 - \omega_k^2}$$

Ising Spin Interaction and Vibrational Normal Modes

CM  (1,1,1)

$$|\uparrow\uparrow\uparrow\rangle \Rightarrow \exp(i\phi)|\uparrow\uparrow\uparrow\rangle$$

$$|\uparrow\uparrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\uparrow\downarrow\rangle$$

$$|\uparrow\downarrow\uparrow\rangle \Rightarrow \exp(i0)|\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\downarrow\downarrow\rangle$$

$$|\downarrow\uparrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\uparrow\uparrow\rangle$$

$$|\downarrow\uparrow\downarrow\rangle \Rightarrow \exp(i0)|\downarrow\uparrow\downarrow\rangle$$

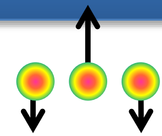
$$|\downarrow\downarrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\downarrow\uparrow\rangle$$

$$|\downarrow\downarrow\downarrow\rangle \Rightarrow \exp(i\phi)|\downarrow\downarrow\downarrow\rangle$$

$$H_J \propto \left(\sigma_z^{(1)} + \sigma_z^{(2)} + \sigma_z^{(3)}\right)^2$$

$$\propto \sigma_z^{(1)}\sigma_z^{(2)} + \sigma_z^{(2)}\sigma_z^{(3)} + \sigma_z^{(3)}\sigma_z^{(1)}$$

Ising Spin Interaction and Vibrational Normal Modes

ZZ  **(1,-2,1)**

$$|\uparrow\uparrow\uparrow\rangle \Rightarrow \exp(i0)|\uparrow\uparrow\uparrow\rangle$$

$$|\uparrow\uparrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\uparrow\downarrow\rangle$$

$$|\uparrow\downarrow\uparrow\rangle \Rightarrow \exp(i\phi)|\uparrow\downarrow\uparrow\rangle$$

$$|\uparrow\downarrow\downarrow\rangle \Rightarrow \exp(i0)|\uparrow\downarrow\downarrow\rangle$$

$$|\downarrow\uparrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\uparrow\uparrow\rangle$$

$$|\downarrow\uparrow\downarrow\rangle \Rightarrow \exp(i\phi)|\downarrow\uparrow\downarrow\rangle$$

$$|\downarrow\downarrow\uparrow\rangle \Rightarrow \exp(i0)|\downarrow\downarrow\uparrow\rangle$$

$$|\downarrow\downarrow\downarrow\rangle \Rightarrow \exp(i0)|\downarrow\downarrow\downarrow\rangle$$

$$H_J \propto \left(\sigma_z^{(1)} - 2\sigma_z^{(2)} + \sigma_z^{(3)}\right)^2$$

$$\propto -2\sigma_z^{(1)}\sigma_z^{(2)} - 2\sigma_z^{(2)}\sigma_z^{(3)} + \sigma_z^{(3)}\sigma_z^{(1)}$$

Time Evolution of the Operation

Applying red- and blue-sideband transitions with some detuning

$$H_I = H_{rsb} + H_{bsb} = (\hbar/2)\eta\Omega\hat{\sigma}_x\left[a^+ e^{i\delta t} + a e^{-i\delta t}\right]$$

$$\hat{U}(t) = \exp\left\{-\frac{i}{\hbar}\left(\int_0^t \hat{H}_I(t')dt' + \frac{1}{2}\int_0^t dt' \int_0^{t'} dt''[\hat{H}_I(t'), \hat{H}_I(t'')] + \ldots\right)\right\}$$

$$\hat{U}(t) = \exp\left[\left(\alpha a^+ - \alpha^* a\right)\hat{\sigma}_x\right]\exp\left[i\Phi(t)\right] = D(\alpha\hat{\sigma}_x)\exp\left[i\Phi(t)\hat{\sigma}_x^2\right]$$

$$\text{where, } \alpha(t) = -i\int_0^t \frac{\eta\Omega}{2} e^{i\delta t'} dt'$$

$$\Phi(t) = -\frac{1}{2}\int_0^t \left(\frac{\eta\Omega}{2}\right)^2 dt' \int_0^{t''} dt'' \left\{e^{i\delta(t''-t')} - e^{-i\delta(t''-t')}\right\}$$

$$\left(= \mathrm{Im}\left[-\int_0^t \alpha(t')^* d\alpha(t')\right]\right)$$

Time Evolution of the Operation

$$\hat{U}(t) = D(\alpha\hat{\sigma}_x)\exp\left[i\Phi(t)\hat{\sigma}_x^2\right]$$

where, $\alpha(t) = -i\int_0^t \frac{\eta\Omega}{2} e^{i\delta t'} dt' = \frac{\eta\Omega}{2\delta}\left(1 - e^{i\delta t}\right)$

$$\Phi(t) = -\frac{1}{2}\int_0^t \left(\frac{\eta\Omega}{2}\right)^2 dt' \int_0^{t''} dt'' \left\{e^{i\delta(t''-t')} - e^{-i\delta(t''-t')}\right\}$$

When $t = \frac{2\pi}{\delta}, \alpha(t) = 0,$

$$\Phi(t) = \frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2$$

Time Evolution of the Operation – Two Qubit Case

$$\hat{U}(t) = D(\alpha\hat{\sigma}_x)\exp\left[i\Phi(t)\hat{\sigma}_x^2\right]$$

$$\Rightarrow \hat{U}(t) = D\left(\alpha[\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}]\right)\exp\left[i\Phi(t)[\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}]^2\right]$$

$$\text{When } t = \frac{2\pi}{\delta}, \alpha(t) = 0,$$

$$\Phi(t) = \frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2$$

Time Evolution of the Operation – Two Qubit Case

$$\hat{U}(t) = \exp\left[i\frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2 [\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}]^2 \right]$$

$$[\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}]^2 = \left[\begin{pmatrix} & 1 & & \\ 1 & & & \\ & & & 1 \\ & & 1 & \end{pmatrix} + \begin{pmatrix} & & 1 & \\ & & & 1 \\ 1 & & & \\ & 1 & & \end{pmatrix}\right]^2$$

$$= \begin{pmatrix} & 1 & 1 & \\ 1 & & & 1 \\ 1 & & & 1 \\ & 1 & 1 & \end{pmatrix}^2 = 2\begin{pmatrix} 1 & & & 1 \\ & 1 & 1 & \\ & 1 & 1 & \\ 1 & & & 1 \end{pmatrix}$$

Time Evolution of the Operation – Two Qubit Case

$$\hat{U}(t) = \exp\left[i\frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2 [\hat{\sigma}_x^{(1)} + \hat{\sigma}_x^{(2)}]^2 \right]$$

$$\text{when} \quad \frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2 = \frac{\pi}{8}$$

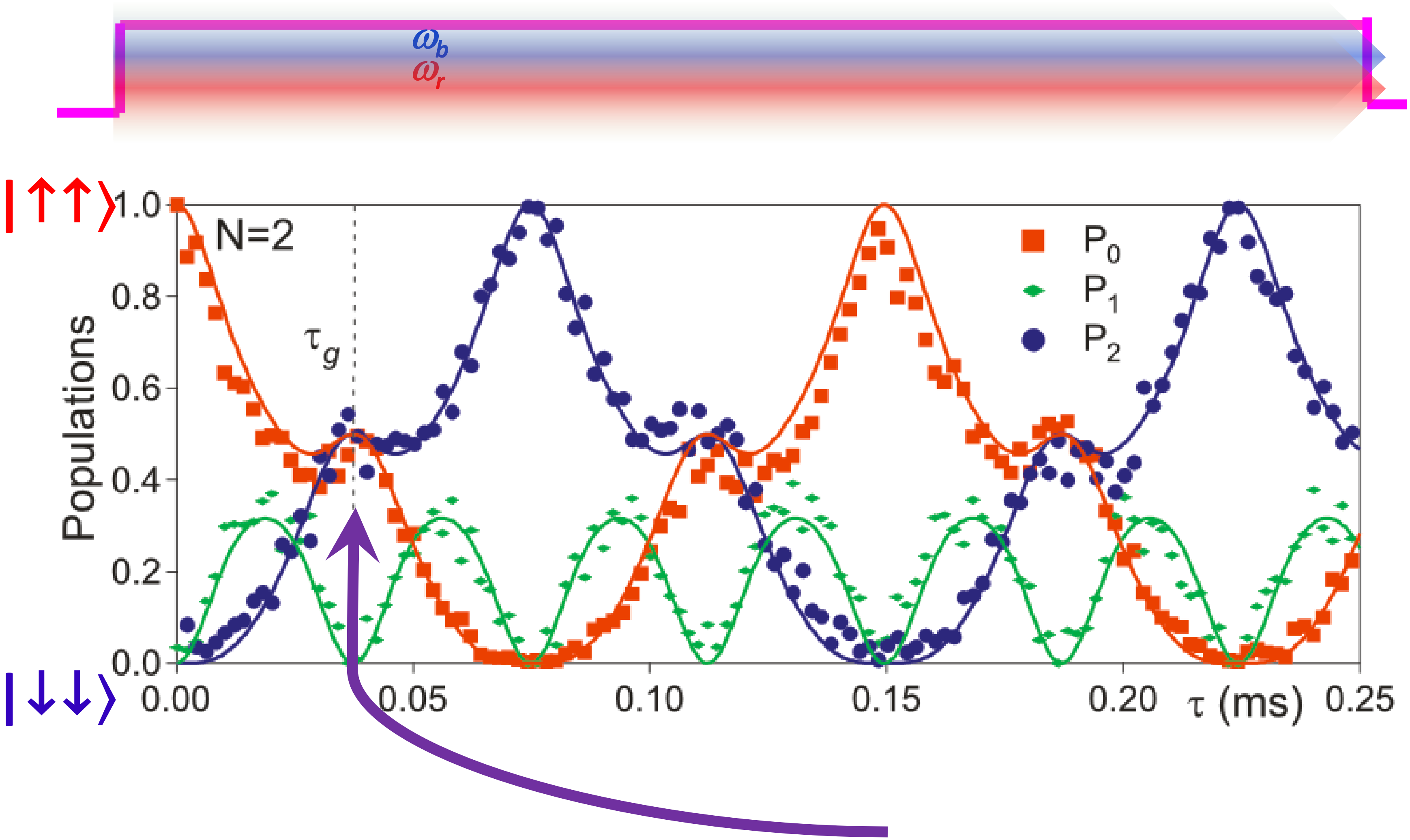
$$|\uparrow\uparrow\rangle \Rightarrow (|\uparrow\uparrow\rangle + i|\downarrow\downarrow\rangle)/\sqrt{2}$$

$$|\uparrow\downarrow\rangle \Rightarrow (|\uparrow\downarrow\rangle + i|\uparrow\downarrow\rangle)/\sqrt{2}$$

$$|\downarrow\uparrow\rangle \Rightarrow (i|\downarrow\uparrow\rangle + |\uparrow\downarrow\rangle)/\sqrt{2}$$

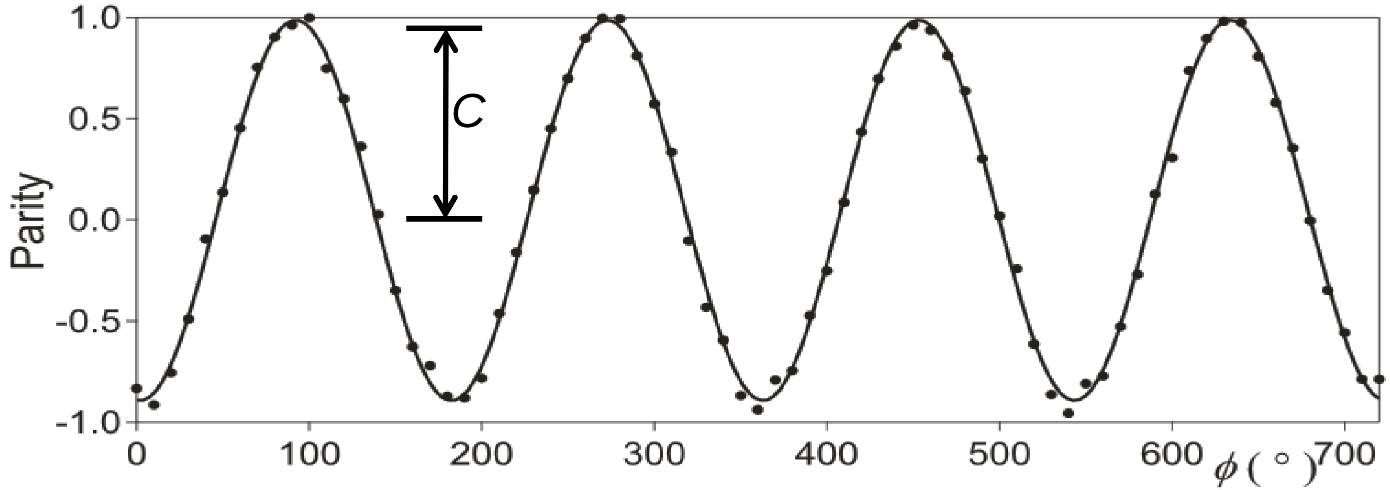
$$|\downarrow\downarrow\rangle \Rightarrow (i|\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle)/\sqrt{2}$$

Two Qubit Gate- Experiments



$|\downarrow\downarrow\rangle \rightarrow |\downarrow\downarrow\rangle + |\uparrow\uparrow\rangle$

Two Qubit Gate – Fidelity



$$\langle F \rangle = \langle \downarrow\downarrow + \uparrow\uparrow | \rho_{\text{exp}} | \downarrow\downarrow + \uparrow\uparrow \rangle$$

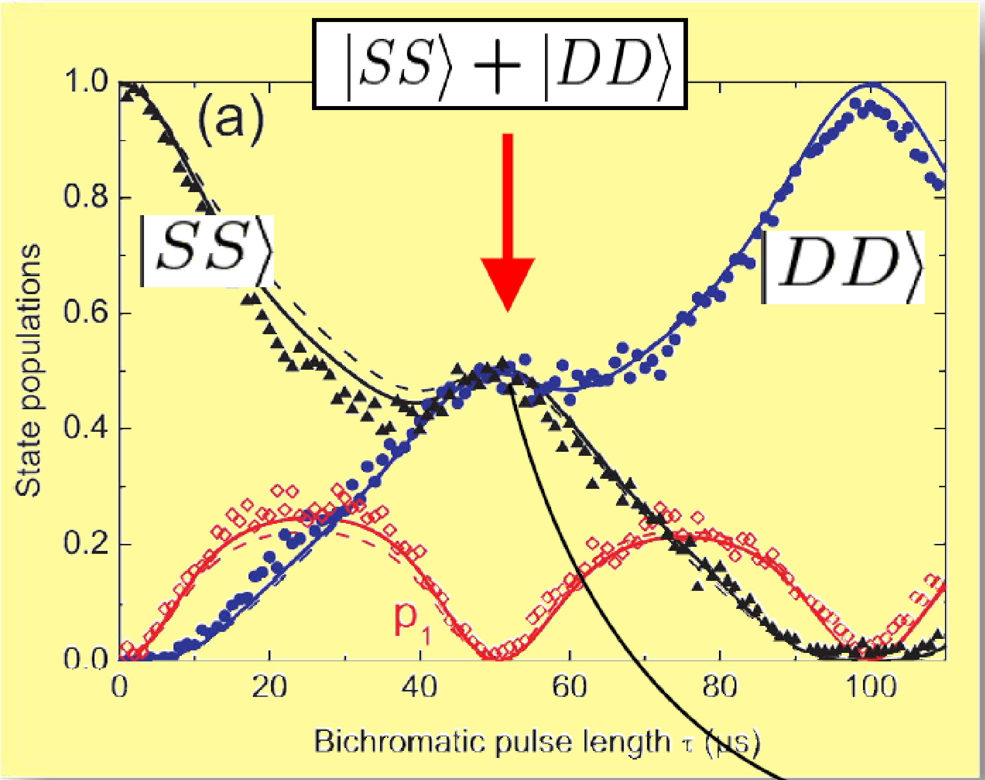
$$= 1/2\left(P_{\downarrow\downarrow} + P_{\uparrow\uparrow}\right) + C/2$$ >96(2)%

C. A. Sackett, et al., Nature **404**, 256 (2000)
K. Kim, et al., Phys. Rev. Lett. **103**, 120502 (2009)

Or Tomography

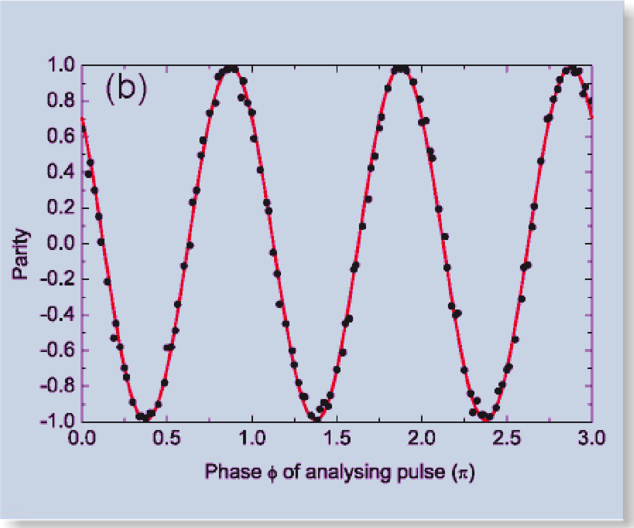
M. Riebe, et al., Phys. Rev. Lett. **97**, 220407 (2006).

Two Qubit Gate – Fidelity



J. Benhelm, G. Kirchmair, C. Roos
Theory: C. Roos,
New J. Phys. **10,** 013002 (2008)

measure entanglement via parity oscillations



gate duration $51\mu s$

average fidelity

$F_{\mathrm{MS}} = 99.3(0.2)\%$

Time Evolution of the Operation – N Qubit Case

$$\hat{U}(t) = D(\alpha\hat{\sigma}_x)\exp[i\Phi(t)]$$

$$\Rightarrow \hat{U}(t) = D\left(\alpha[\sum_j \hat{\sigma}_x^{(j)}]\right)\exp\left[i\Phi(t)[\sum_j \hat{\sigma}_x^{(j)}]^2\right]$$

$$\text{When } t = \frac{2\pi}{\delta}, \alpha(t) = 0, \Phi(t) = \frac{\pi}{2}\left(\frac{\eta\Omega}{\delta}\right)^2 = \frac{\pi}{8}$$

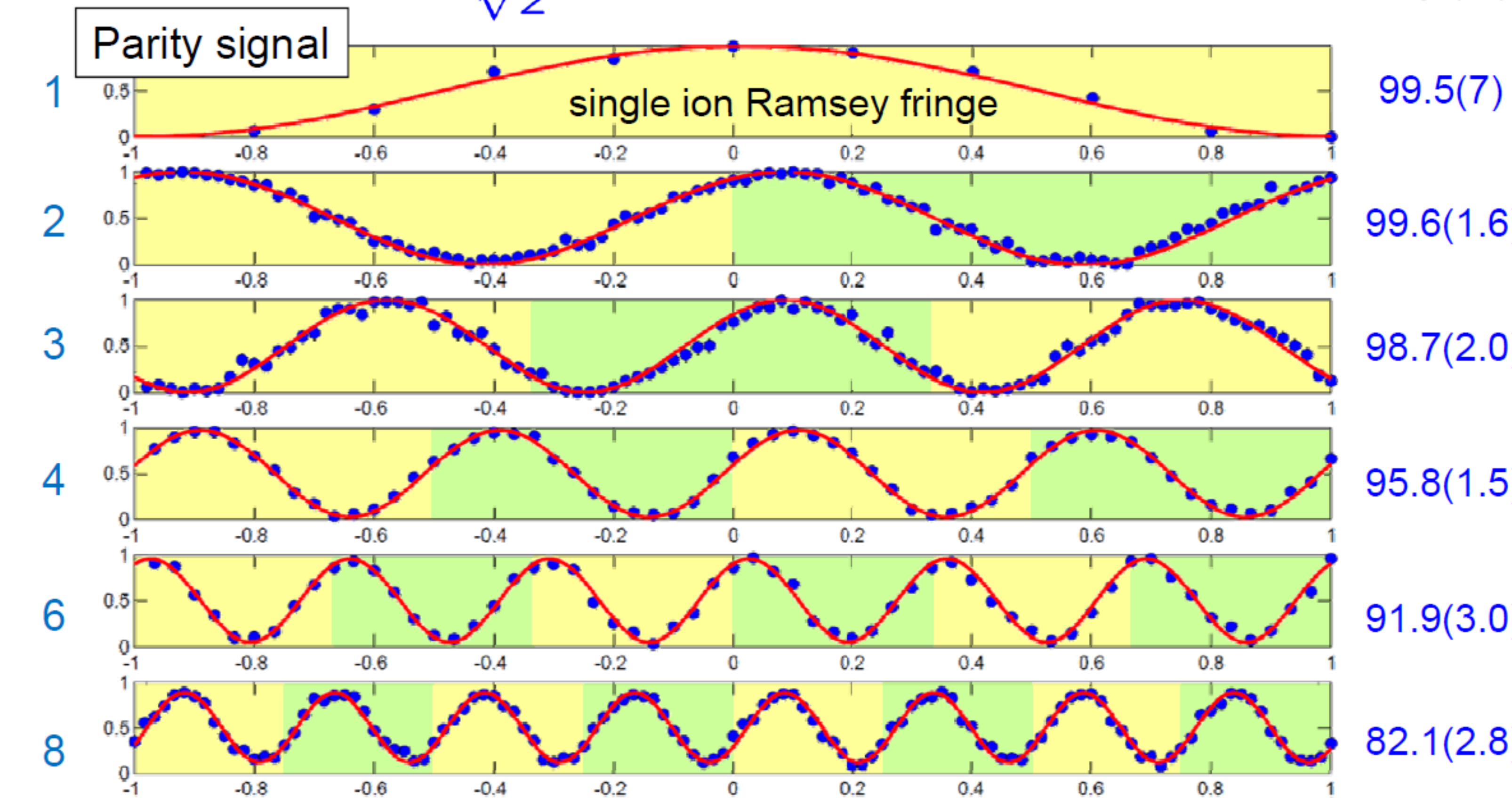
$$\left|\downarrow\downarrow\downarrow\cdots\downarrow\right\rangle \Rightarrow \left[\left|\downarrow\downarrow\downarrow\cdots\downarrow\right\rangle + \left|\uparrow\uparrow\uparrow\cdots\uparrow\right\rangle\right]/\sqrt{2}$$

Time Evolution of the Operation – N Qubit Case

N - qubit GHZ state generation

$$\Psi = \frac{1}{\sqrt{2}}(|SS\ldots S\rangle + |DD\ldots D\rangle)$$

$N =$ Fidelity (%)



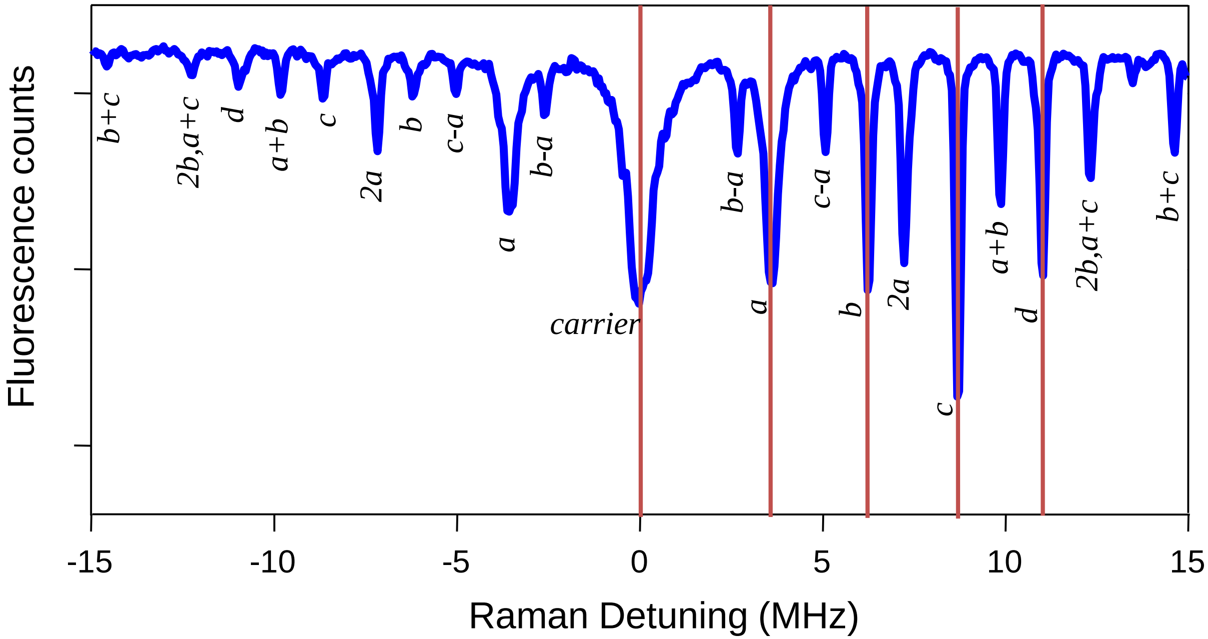
T. Monz et al., PRL 106, 130506 (2011).

Challenges of Trapped Ion System

The Main problem to scale up the system

More ions: difficult to isolate single mode of motion

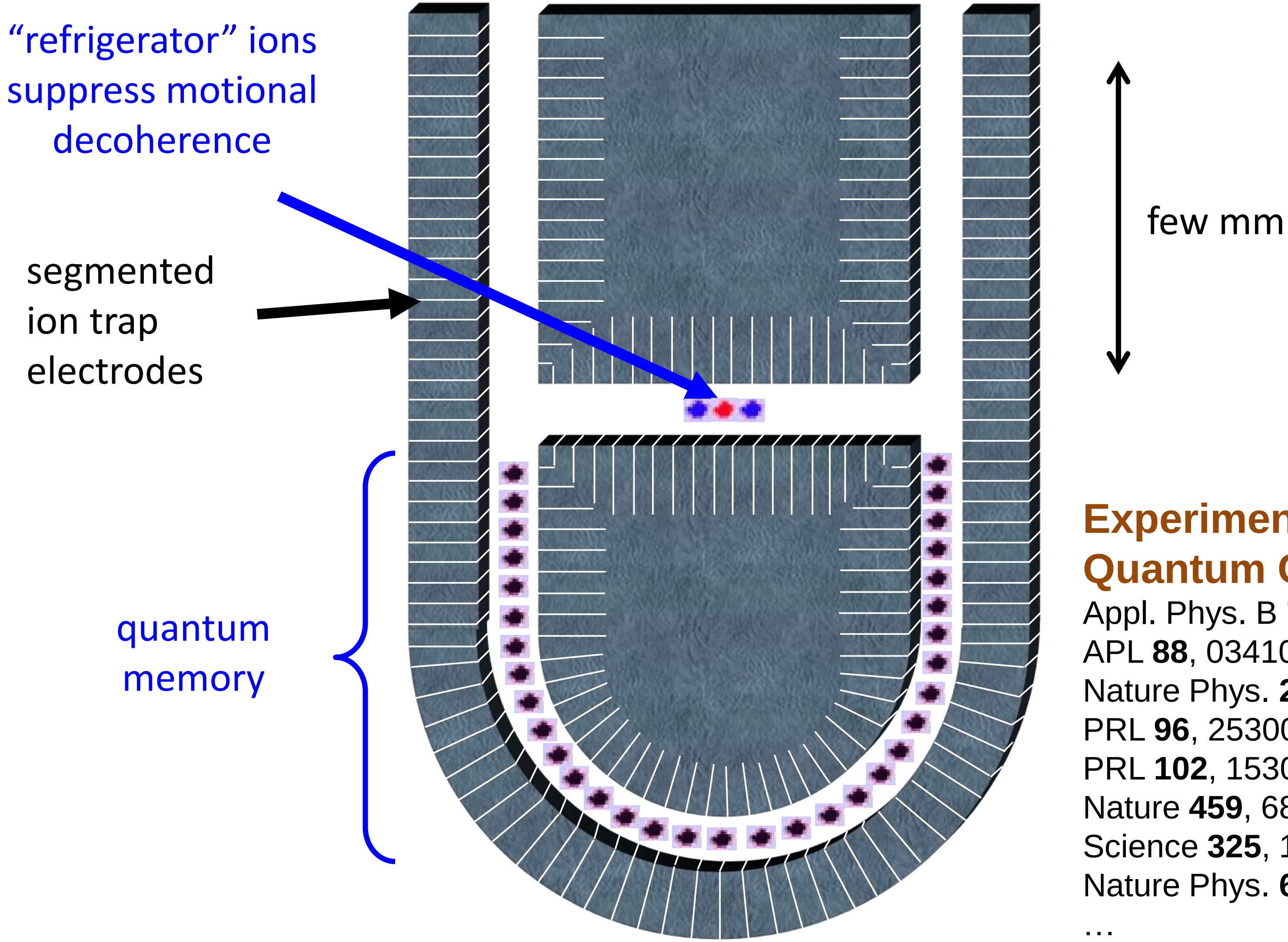
eg) 4-ion axial mode spectrum



Courtesy from Dr. David Wineland

Proposal: Quantum CCD

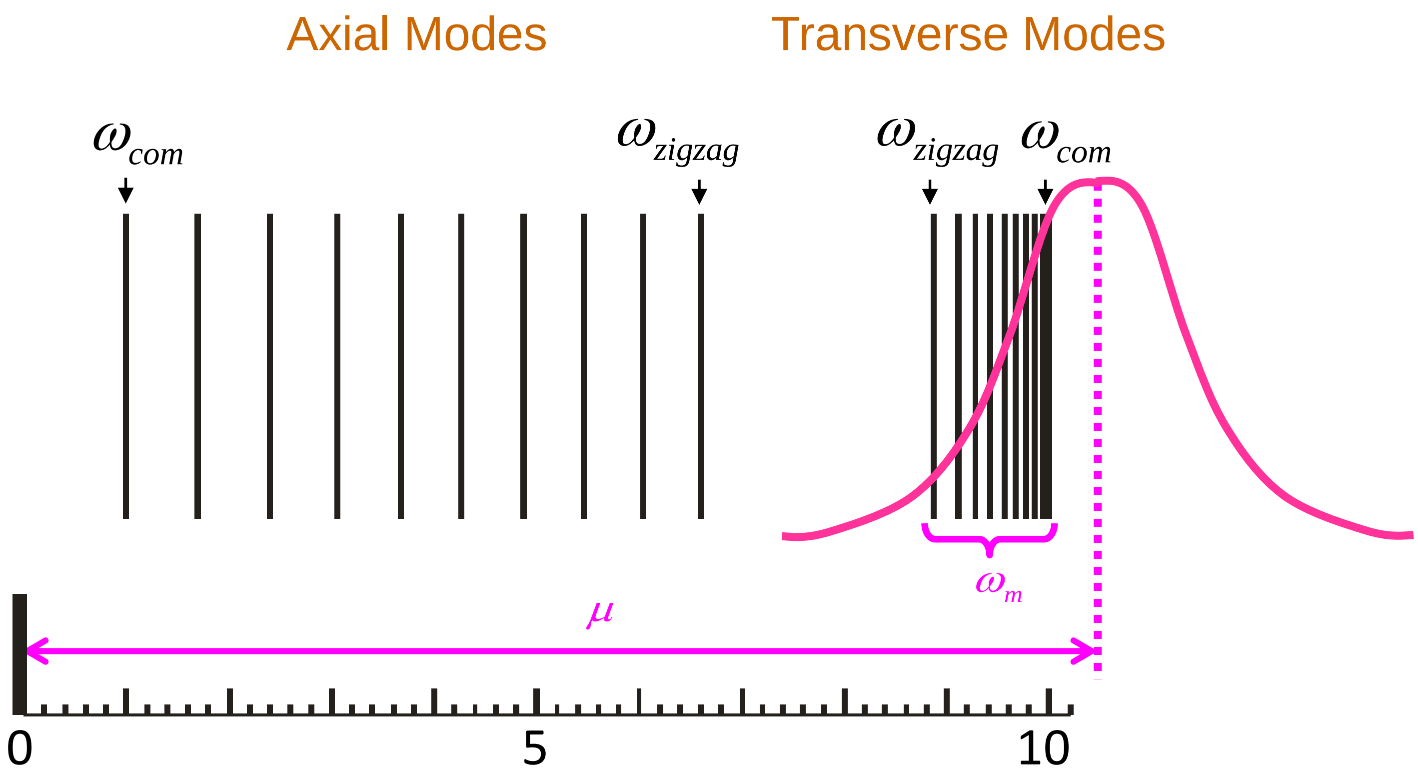
D. Kielpinski, et al. Nature 417, 709 (2002)

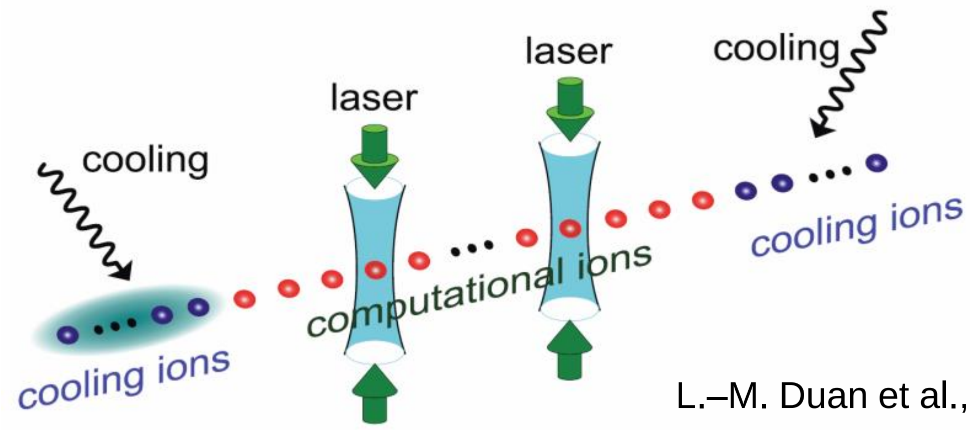


Experiments Toward Quantum CCD

Appl. Phys. B **78**, 639 (2004)
APL **88**, 034101 (2006)
Nature Phys. **2**, 36 (2006)
PRL **96**, 253003 (2006)
PRL **102**, 153002 (2009)
Nature **459**, 683 (2009)
Science **325**, 1227 (2009)
Nature Phys. **6**, 13 (2010)
…

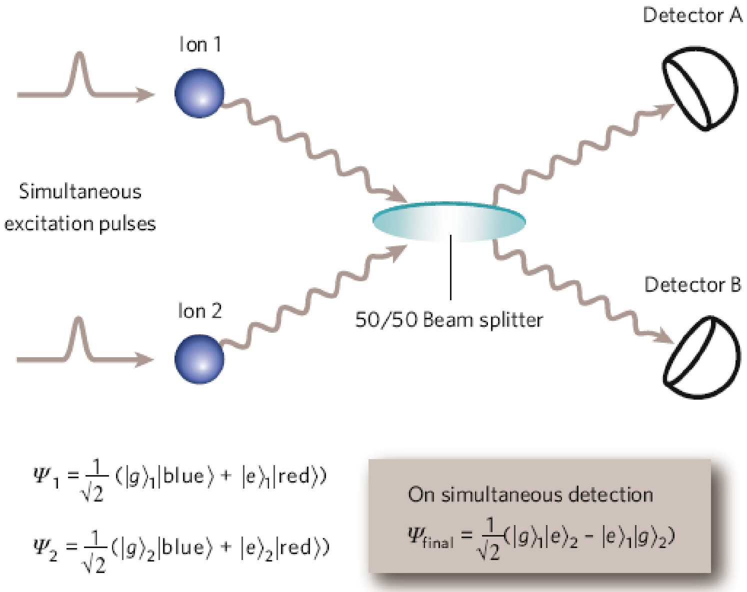
How to Scale up





L.–M. Duan et al., EPL **86**, 60004 (200

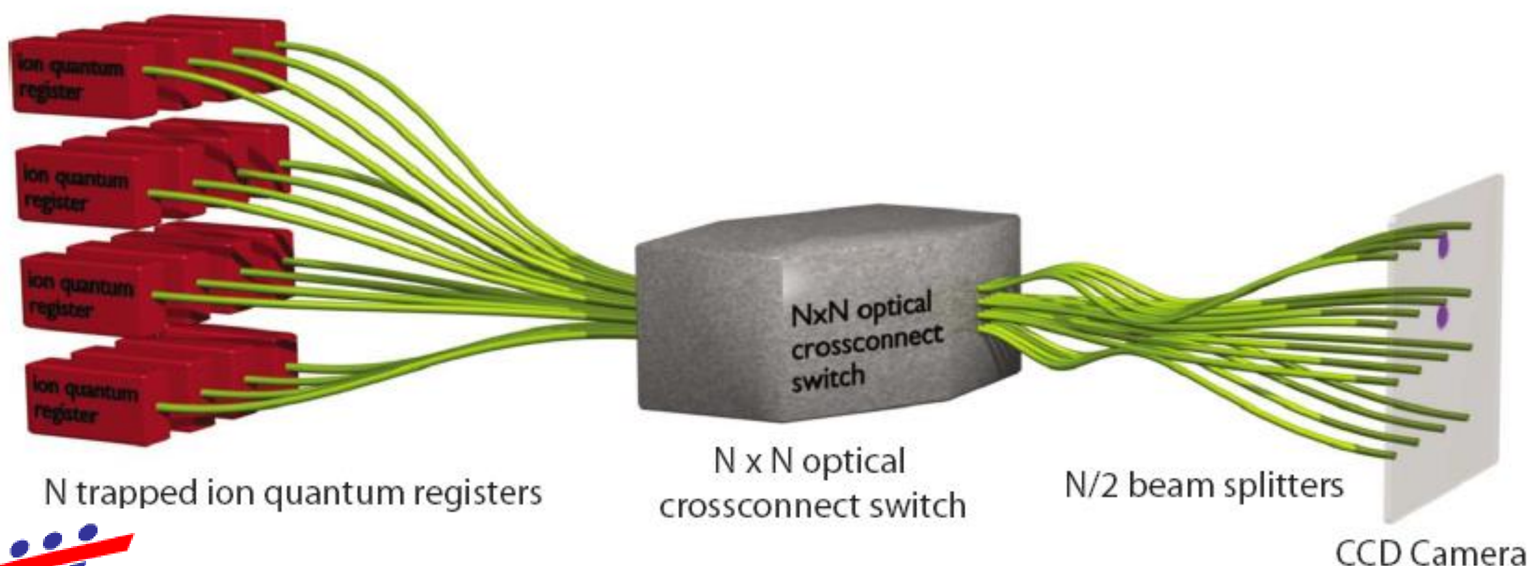
Proposal: Quantum Network



L.-M. Duan, et al., *Quant. Inf. Comp.* **4**, 165 (2004).

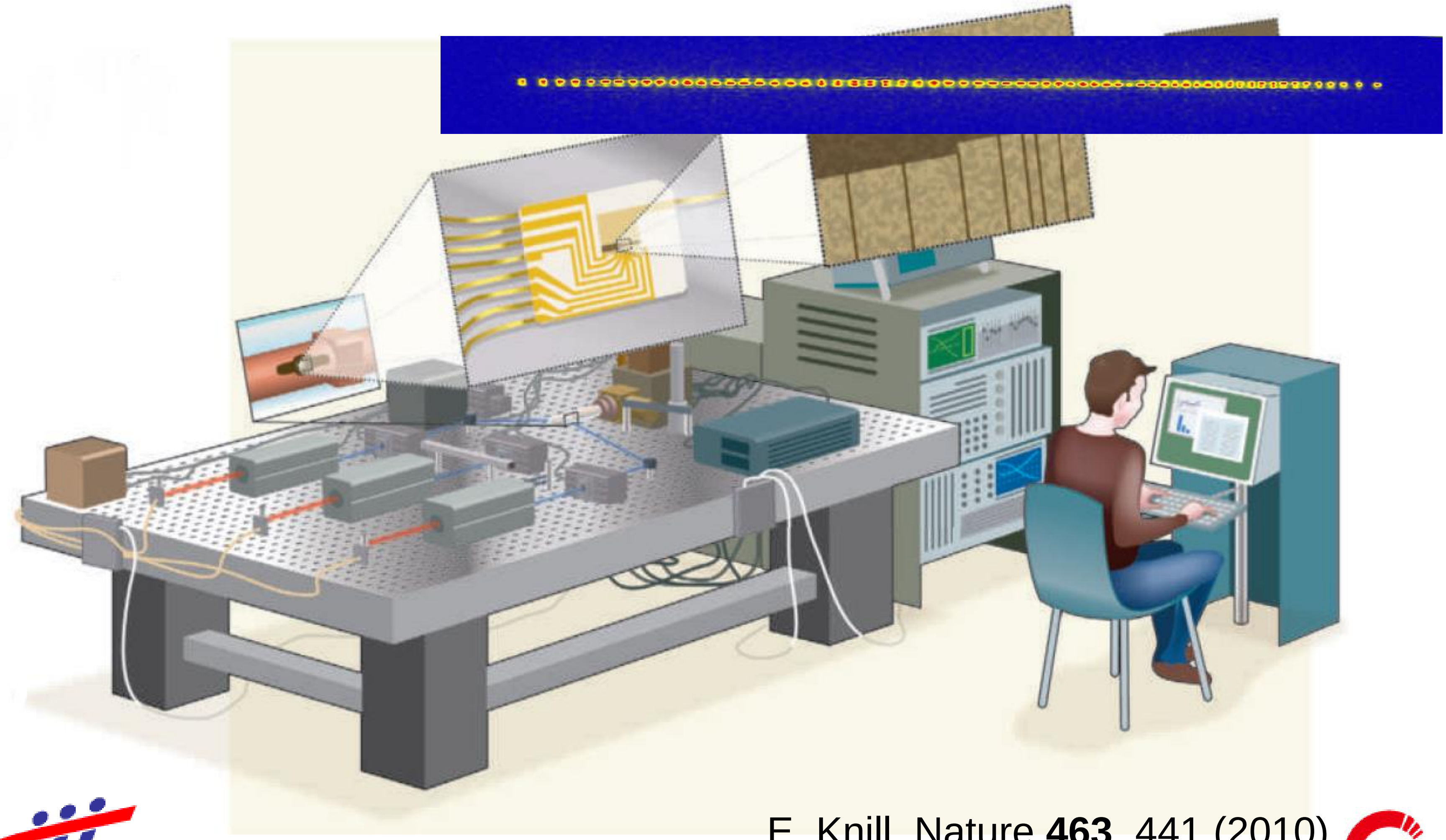
Experiments Toward Quantum Network

Nature **428**, 153 (2004)
PRL **93**, 090410 (2004)
Nature Physics **3,** 538 (2007)
Nature **449**, 68 (2007)
PRL **100**, 150404 (2008)
Science **323**, 486 (2009)
Nature **464**, 45 (2010)…



Conclusion and Outlook

To realize a well-controlled, large-scale quantum system (Quantum computer/simulator) with trapped ions



E. Knill, Nature **463**, 441 (2010).