Quantum Computation and Simulation with Trapped Ions Part I

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OPEN KIAS SCHOOL ON QUANTUM INFORMATION SCIENCE

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Outline

Introduction

- Why Quantum Computation?
- Current Status of Quantum Computation

Basic Requirements for the Quantum Computation

- Qubit, Initialization, Operation, Detection

Qubit, Initialization, Detection in Ion Trap System

Operation in Trapped Ion System Part I

- Ion Laser Interaction
- Carrier, Red Sideband, Blue Sideband Transition

Number cruncher turns 5

1st electronic computer fet

By Michael Raphael
The Associated Press

PHILADELPHIA — It had no could remember only 20 numb time, and filled a room with 50 electricity-sucking gear.

But it could crunch numbers w seemed like blinding speed.

Fifty years ago this week, the E ic Numerical Integrator and C was demonstrated for the first tin University of Pennsylvania.

ENIAC counted to 5,000 in one- second, shocking the world out of chanical age and into the world ning-quick digital processing.

ENIAC's collection of 8-foot-high gray cabinets made up the first general-purpose, large-scale, electronic computer.

Until then, "computers" were people using mechanical calculators who needed 12 hours to do what ENIAC did in half a minute. Other electronic machines had been narrower in purpose.

ENIAC will count from 46 to 96.

The Postal Service will unveil a stamp commemorating "The Birth of Computing." And Garry Kasparov, the World Chess Federation champion, this week is playing against IBM's "Deep Blue" computer.

The original assemblage of wires, vacuum tubes, resistors and switches was constructed in about a year and a half at the university's Moore School of Electrical Engineering.

When fully operational, ENIAC filled up a 30-by-50-foot room. Every second it was on, it used enough electricity — 174 kilowatts — to power a typical Philadelphia home for 1½ weeks.

Costing more than $486,000, ENIAC might never been attempted were it not for World War II.

"A lot of people said we were dreaming," said Herman Goldstine, who served as liaison between the Army and ENIAC team.

"The electronics people said there were too many vacuum tubes and it would never run. The mathematics people said there were no problems complex enough that computers were needed."

The Army provided both the complex problems and the money.

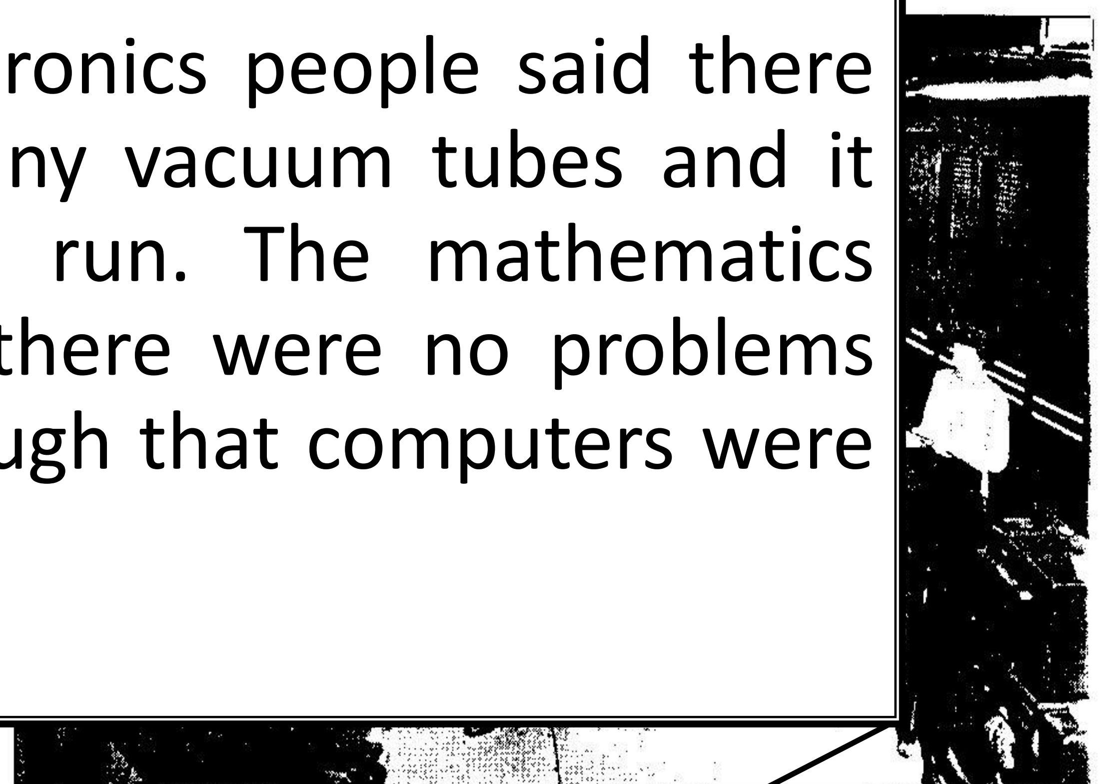
John Mauchly, one of two masterminds behind ENIAC, knew the Army was having a terrible time working out the complicated firing tables to help gun crews aim new artillery being used against German forces.

Each firing table had to list numbers for hundreds of potential trajectories. Calculating a single trajectory could take 40 hours using a mechanical desktop calculator, and 30 minutes using a sophisticated machine called a differential analyzer.

Mauchly, then 32, bravely told Ar my officials his machine could do th job in a matter of minutes.

ENIAC was completed just as th war was ending, too late for those ar tillery tables.

However, it fulfilled another mili tary purpose. During test runs in 194 it did millions of calculations on ther monuclear chain reactions, predict ing the destruction that could b caused by the hydrogen bomb.

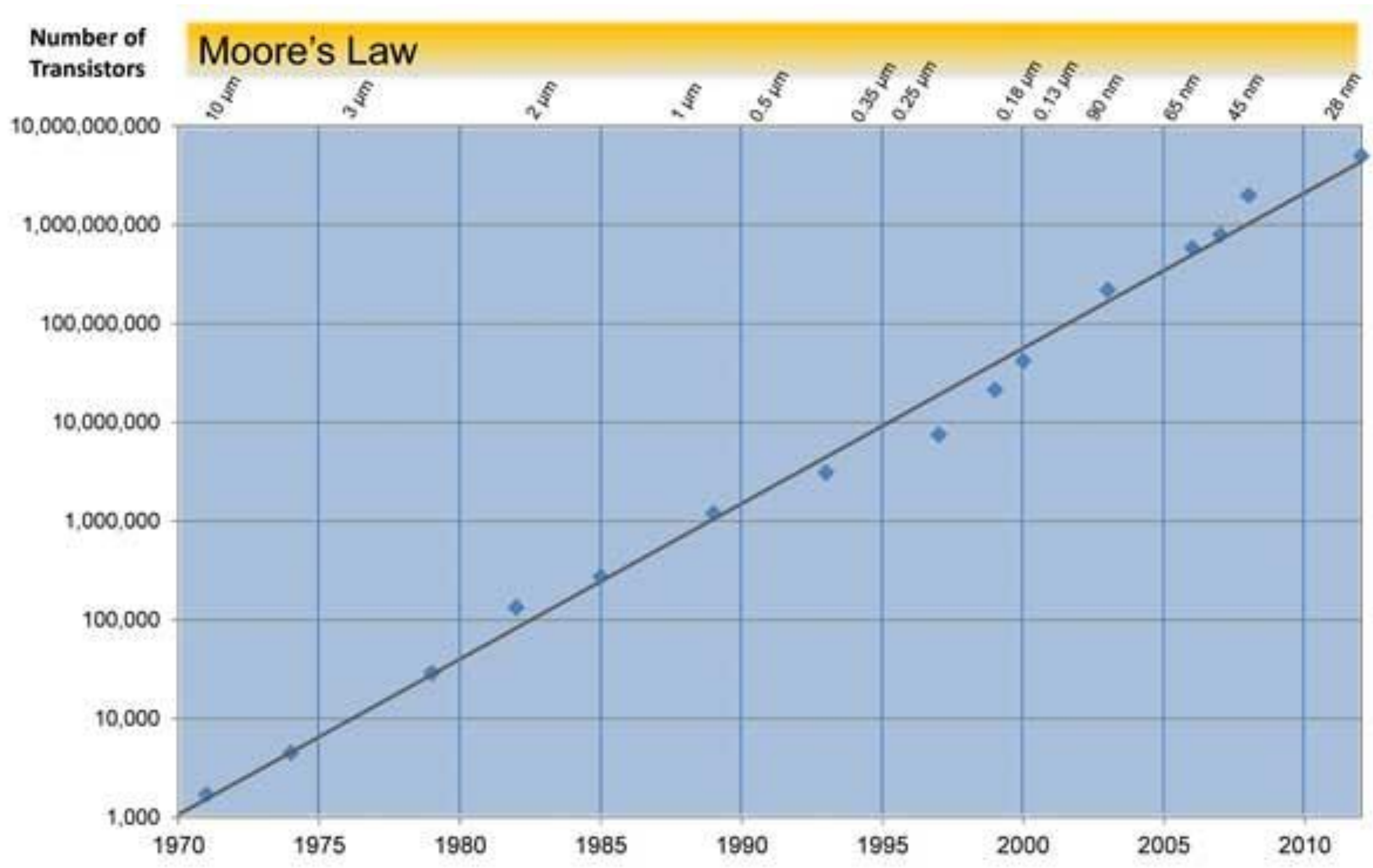


Associated Pres

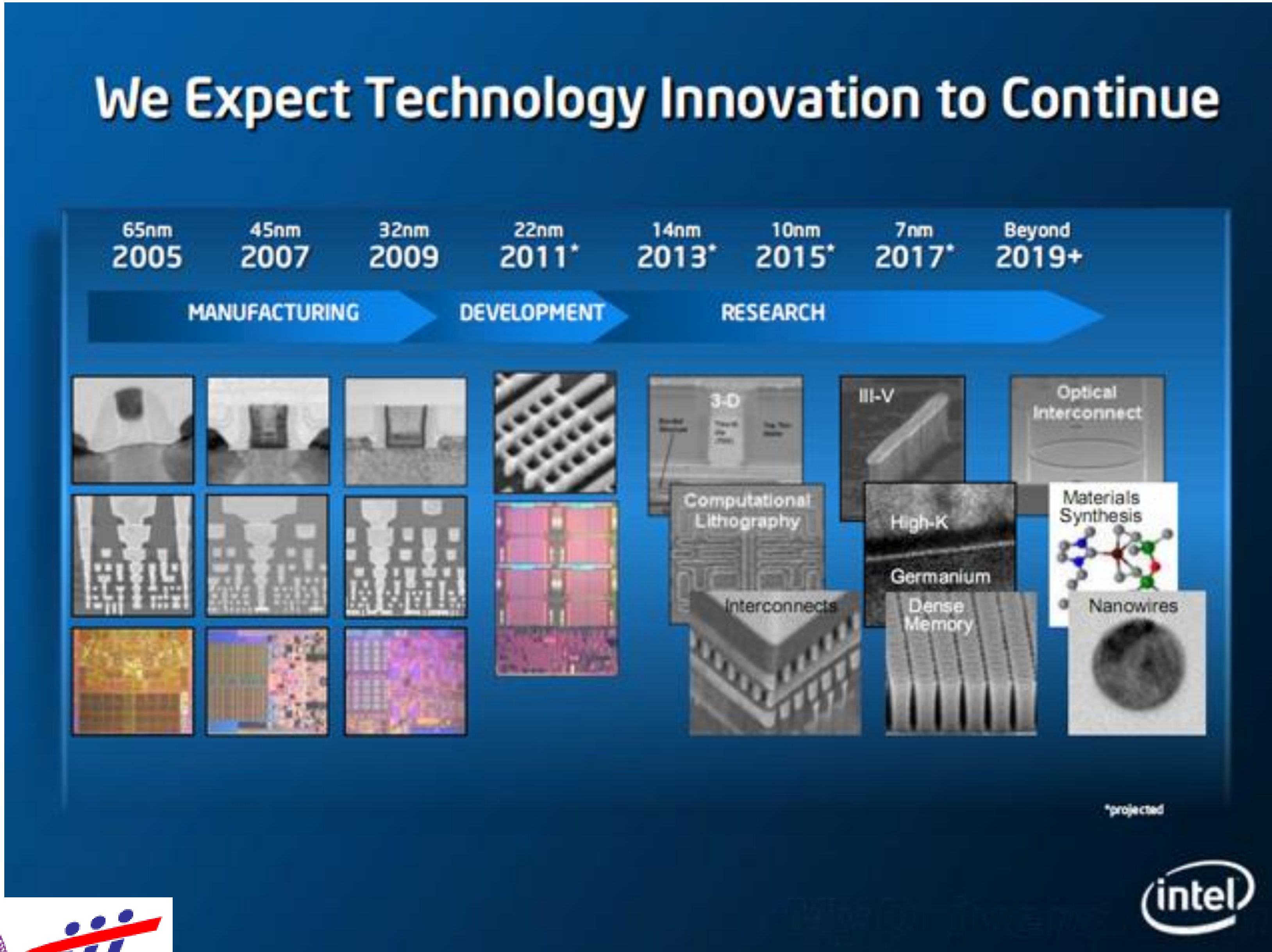
CALCULATING MINDS: J. Presper Eckert, left foreground, and John Mauchly, near pole, masterminded ENIAC, pictured in undated photo from the University of Pennsylvania Archives.

"The electronics people said there were too many vacuum tubes and it would never run. The mathematics people said there were no problems complex enough that computers were needed."

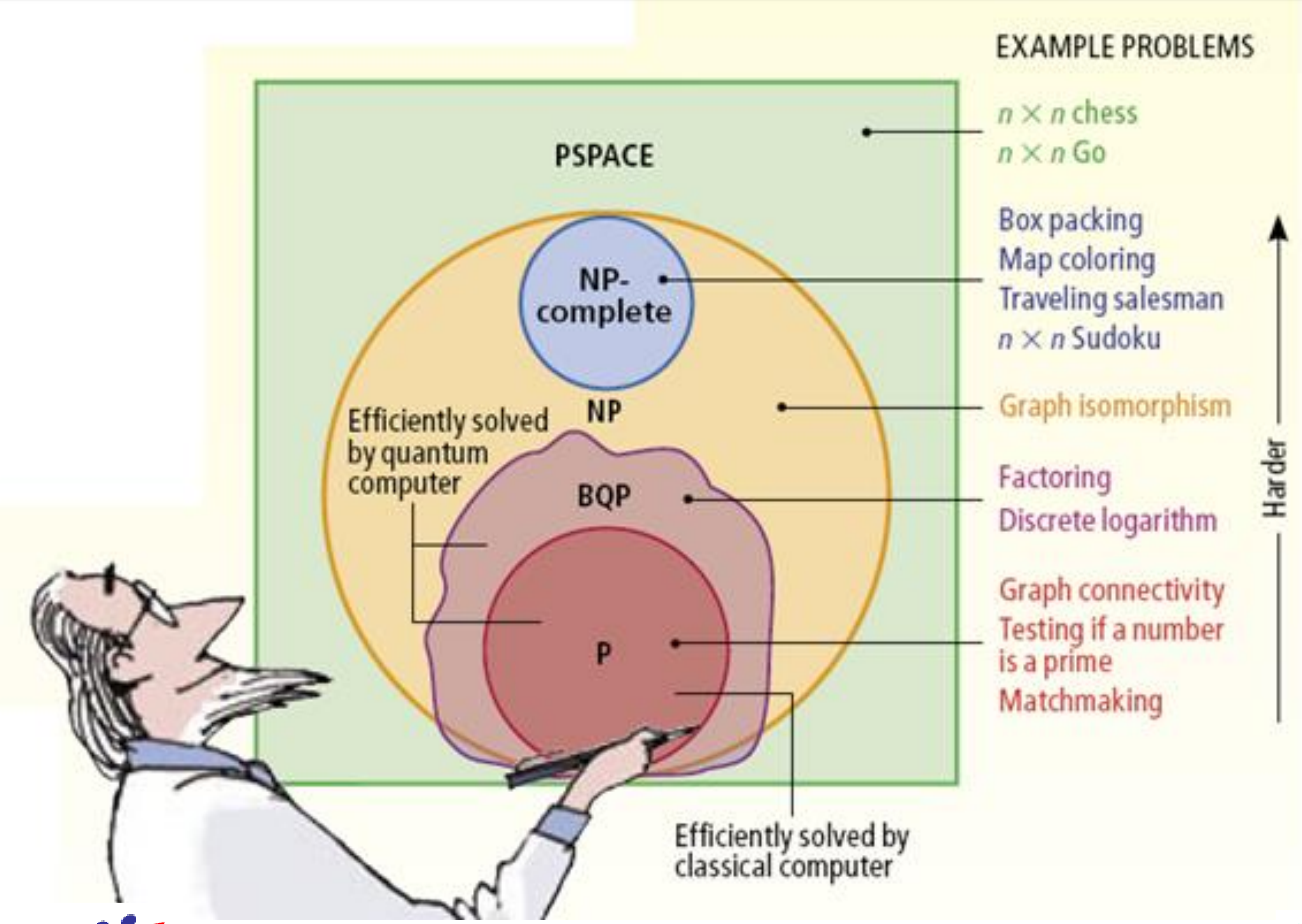
Development of Computation – Moore's Law



Development of Computation – Moore's Law



Why Quantum Computer?



by S. Aaronson, Scientific America (03/2008).

The Amount of Information in Quantum System

$$
\begin{aligned}
|\Psi\rangle_{\text{reg}} \;=\;& \alpha_0\,|000\rangle + \alpha_1\,|001\rangle + \alpha_2\,|010\rangle + \alpha_3\,|011\rangle + \\
& \alpha_4\,|100\rangle + \alpha_5\,|101\rangle + \alpha_6\,|110\rangle + \alpha_7\,|111\rangle
\end{aligned}
$$

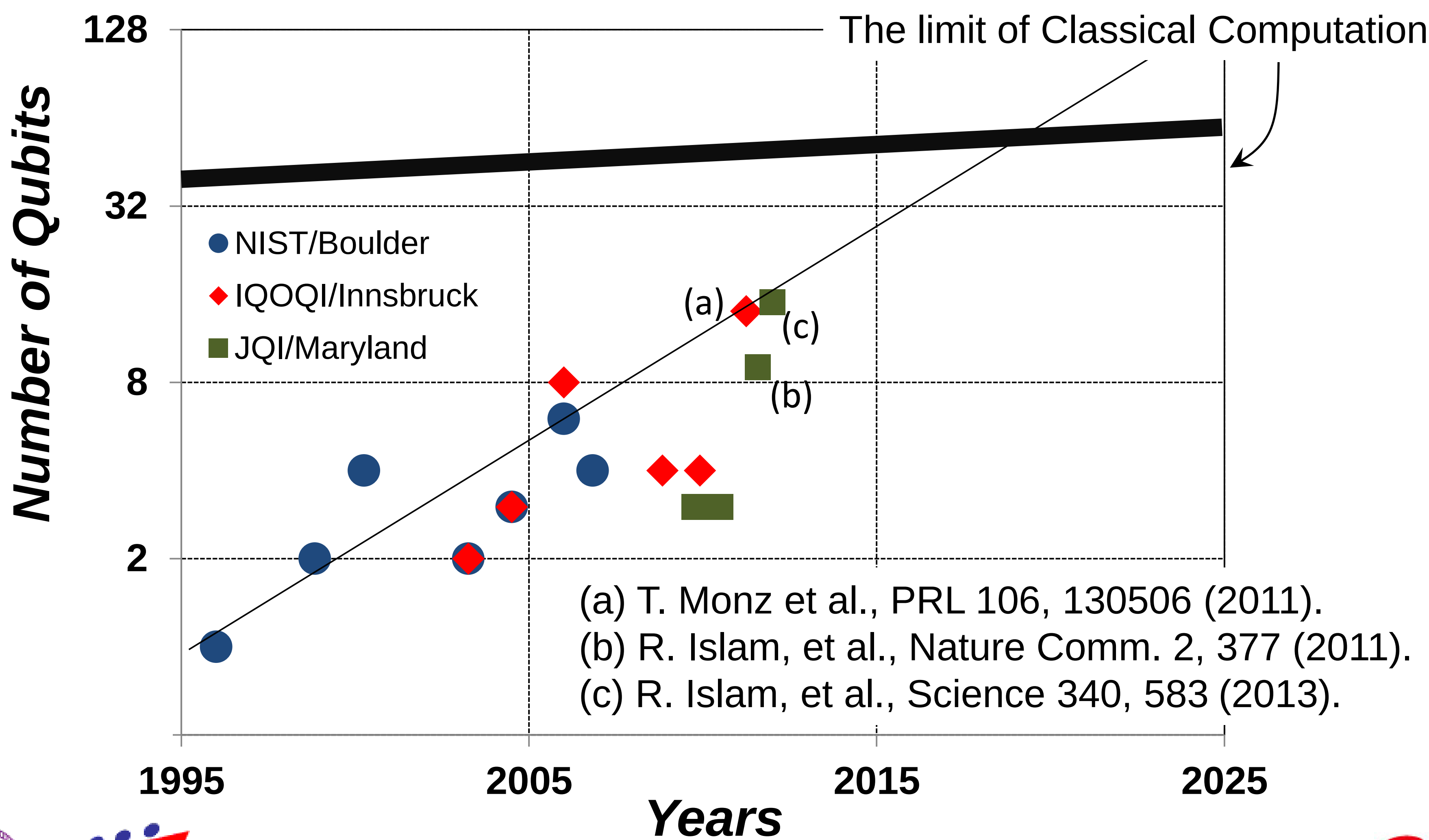
# bits	classical	quantum mechanical
1	1	0.5208 + 0.7059i, 0.3014 + 0.3736i
2	01	0.2044 + 0.4911i, 0.1732 + 0.3855i, 0.2040 + 0.4890i, 0.3193 + 0.3947i
3	001	0.2583 + 0.2704i, 0.2310 + 0.1150i, 0.2956 + 0.3118i, 0.3558 + 0.2113i, 0.1943 + 0.1377i 0.3273 + 0.2613i, 0.0643 + 0.2033i, 0.3643 + 0.1654i
4	1010	0.1691 + 0.0891i 0.1096 + 0.0828i 0.1420 + 0.2873i 0.0741 + 0.2419i 0.1902 + 0.0448i 0.2495 + 0.0039i 0.1738 + 0.2933i 0.2102 + 0.0653i 0.0686 + 0.0980i 0.1246 + 0.2170i 0.2570 + 0.0933i 0.2234 + 0.1540i 0.1513 + 0.0213i 0.1863 + 0.3243i 0.2606 + 0.1912i 0.0194 + 0.1390i
5	10001	0.1060 + 0.1416i 0.0103 + 0.0118i 0.0064 + 0.0976i 0.0734 + 0.0716i 0.0030 + 0.2054i 0.0902 + 0.0035i 0.1605 + 0.1804i 0.0218 + 0.2280i 0.0083 + 0.2326i 0.1438 + 0.1853i 0.1429 + 0.1090i 0.0037 + 0.1171i 0.0038 + 0.0503i 0.0446 + 0.1512i 0.1379 + 0.0752i 0.0135 + 0.2255i 0.0863 + 0.1707i 0.1483 + 0.0968i 0.1686 + 0.1749i 0.1627 + 0.0628i 0.0197 + 0.1033i 0.1067 + 0.2192i 0.1038 + 0.1605i 0.0830 + 0.0489i 0.0361 + 0.1971i 0.1587 + 0.1477i 0.1642 + 0.0314i 0.1709 + 0.0487i 0.1124 + 0.1426i 0.1303 + 0.1480i 0.0284 + 0.0870i 0.1059 + 0.1351i
6	110101	0.0585 + 0.1064i 0.0295 + 0.1327i 0.0929 + 0.0406i 0.1090 + 0.0379i 0.0559 + 0.1286i 0.0015 + 0.0345i 0.0624 + 0.1196i 0.1120 + 0.1350i 0.1180 + 0.0345i 0.1367 + 0.0356i 0.1255 + 0.0074i 0.0547 + 0.0116i 0.0923 + 0.0952i 0.1087 + 0.0284i 0.0288 + 0.1254i 0.1345 + 0.0258i 0.0846 + 0.0254i 0.0939 + 0.1478i 0.0348 + 0.0654i 0.0816 + 0.0505i 0.1384 + 0.0467i 0.0498 + 0.0543i 0.0874 + 0.0584i 0.0582 + 0.0879i 0.0932 + 0.0178i 0.1039 + 0.0057i 0.0590 + 0.0682i 0.0615 + 0.1293i 0.0974 + 0.1388i 0.1245 + 0.0989i 0.0552 + 0.0238i 0.0632 + 0.1297i 0.0884 + 0.0354i 0.0841 + 0.0960i 0.1065 + 0.1437i 0.0760 + 0.0988i 0.1154 + 0.1283i 0.0727 + 0.0015i 0.0276 + 0.0204i 0.1041 + 0.1217i 0.1460 + 0.0638i 0.1199 + 0.1323i 0.1046 + 0.1092i 0.0721 + 0.1021i 0.0170 + 0.0514i 0.0988 + 0.0247i 0.0543 + 0.0231i 0.0208 + 0.0284i 0.0842 + 0.0628i 0.1223 + 0.1272i 0.1002 + 0.0729i 0.1485 + 0.1213i 0.1429 + 0.0685i 0.0087 + 0.0680i 0.0535 + 0.0670i 0.0815 + 0.0613i 0.0389 + 0.1340i 0.0888 + 0.0006i 0.0073 + 0.0442i 0.0849 + 0.0073i 0.1042 + 0.1030i 0.1430 + 0.0866i 0.1115 + 0.1461i 0.1100 + 0.0821i
7	1001010	0.0880 + 0.0466i 0.1054 + 0.0684i 0.0239 + 0.0866i 0.0759 + 0.0090i 0.0563 + 0.1020i 0.1006 + 0.0888i 0.0769 + 0.0649i 0.0246 + 0.0273i 0.0485 + 0.0842i 0.0186 + 0.0554i 0.1045 + 0.0790i 0.0384 + 0.0455i 0.0053 + 0.1037i 0.0815 + 0.0078i 0.0965 + 0.0597i 0.0309 + 0.0315i 0.0271 + 0.0825i 0.1006 + 0.0362i 0.0141 + 0.0734i 0.1015 + 0.0058i 0.0757 + 0.0385i 0.0914 + 0.0537i 0.0226 + 0.0468i 0.0491 + 0.0607i 0.0087 + 0.0665i 0.0918 + 0.0122i 0.0606 + 0.0869i 0.0344 + 0.0814i 0.0404 + 0.0853i 0.0936 + 0.0879i 0.0401 + 0.0723i 0.0079 + 0.0217i 0.0216 + 0.0294i 0.0053 + 0.0675i 0.0611 + 0.0579i 0.0131 + 0.0064i 0.0563 + 0.0086i 0.0126 + 0.0293i 0.0830 + 0.0441i 0.0404 + 0.0511i 0.0888 + 0.0980i 0.0050 + 0.0643i 0.0645 + 0.0355i 0.1024 + 0.0516i 0.0311 + 0.0644i 0.0959 + 0.0174i 0.0110 + 0.0894i 0.0070 + 0.1091i 0.0253 + 0.0642i 0.1006 + 0.0031i 0.0068 + 0.0876i 0.0285 + 0.0658i 0.1078 + 0.0756i 0.0229 + 0.0099i 0.0537 + 0.0458i 0.0313 + 0.0405i 0.0725 + 0.0179i 0.1033 + 0.0898i 0.0827 + 0.0804i 0.0718 + 0.0487i 0.0141 + 0.1032i 0.0103 + 0.0158i 0.0016 + 0.0938i 0.0311 + 0.0830i 0.0881 + 0.0479i 0.1063 + 0.0669i 0.0019 + 0.1026i 0.0884 + 0.0690i 0.0670 + 0.0267i 0.0604 + 0.0380i 0.0263 + 0.0203i 0.0886 + 0.0529i 0.0284 + 0.0441i 0.0813 + 0.0500i 0.0711 + 0.0659i 0.0231 + 0.0077i 0.0649 + 0.0338i 0.0652 + 0.0656i 0.0711 + 0.0189i 0.0188 + 0.0670i 0.0686 + 0.0265i 0.0184 + 0.0633i 0.0582 + 0.0546i 0.0672 + 0.0501i 0.0740 + 0.0584i 0.0730 + 0.1016i 0.0946 + 0.0369i 0.0014 + 0.0433i 0.0335 + 0.0332i 0.0840 + 0.0444i 0.0331 + 0.0308i 0.0999 + 0.0425i 0.0732 + 0.0542i 0.0080 + 0.0779i 0.0076 + 0.0330i 0.0013 + 0.0121i 0.0245 + 0.0478i 0.0557 + 0.0503i 0.0494 + 0.0016i 0.0758 + 0.0716i 0.0628 + 0.0781i 0.0549 + 0.0304i 0.0080 + 0.0282i 0.0208 + 0.0764i 0.0409 + 0.0845i 0.0893 + 0.0425i 0.0989 + 0.0562i 0.0122 + 0.0774i 0.0876 + 0.0614i 0.0979 + 0.0497i 0.0169 + 0.0480i 0.0132 + 0.0085i 0.0822 + 0.0478i 0.0778 + 0.0395i 0.0703 + 0.0326i 0.0813 + 0.0919i 0.0715 + 0.0819i 0.0953 + 0.1024i 0.0293 + 0.0602i 0.0452 + 0.0015i 0.0230 + 0.0643i
8	10101011	0.0199 + 0.0027i 0.0033 + 0.0063i 0.0005 + 0.0656i 0.0443 + 0.0262i 0.0573 + 0.0359i 0.0622 + 0.0704i 0.0491 + 0.0176i 0.0194 + 0.0664i 0.0111 + 0.0506i 0.0502 + 0.0687i 0.0729 + 0.0376i 0.0629 + 0.0765i 0.0717 + 0.0288i 0.0239 + 0.0410i 0.0207 + 0.0140i 0.0413 + 0.0387i 0.0126 + 0.0325i 0.0163 + 0.0509i 0.0167 + 0.0519i 0.0502 + 0.0738i 0.0041 + 0.0148i 0.0177 + 0.0086i 0.0514 + 0.0436i 0.0240 + 0.0747i 0.0236 + 0.0018i 0.0555 + 0.0671i 0.0736 + 0.0021i 0.0101 + 0.0400i 0.0053 + 0.0148i 0.0097 + 0.0552i 0.0128 + 0.0193i 0.0702 + 0.0720i 0.0105 + 0.0106i 0.0476 + 0.0402i 0.0207 + 0.0690i 0.0170 + 0.0726i 0.0549 + 0.0258i 0.0423 + 0.0337i 0.0726 + 0.0363i 0.0254 + 0.0115i 0.0543 + 0.0105i 0.0727 + 0.0410i 0.0448 + 0.0559i 0.0678 + 0.0307i 0.0578 + 0.0276i 0.0293 + 0.0220i 0.0559 + 0.0670i 0.0125 + 0.0483i 0.0737 + 0.0186i 0.0151 + 0.0754i 0.0598 + 0.0494i 0.0473 + 0.0177i 0.0125 + 0.0525i 0.0024 + 0.0513i 0.0222 + 0.0104i 0.0748 + 0.0017i 0.0733 + 0.0202i 0.0176 + 0.0090i 0.0739 + 0.0053i 0.0524 + 0.0657i 0.0042 + 0.0139i 0.0462 + 0.0025i 0.0303 + 0.0566i 0.0166 + 0.0414i 0.0141 + 0.0213i 0.0059 + 0.0284i 0.0006 + 0.0010i 0.0608 + 0.0685i 0.0014 + 0.0667i 0.0677 + 0.0196i 0.0272 + 0.0439i 0.0557 + 0.0123i 0.0746 + 0.0458i 0.0120 + 0.0255i 0.0126 + 0.0508i 0.0242 + 0.0666i 0.0023 + 0.0437i 0.0276 + 0.0756i 0.0021 + 0.0610i 0.0612 + 0.0118i 0.0770 + 0.0642i 0.0085 + 0.0148i 0.0480 + 0.0493i 0.0102 + 0.0516i 0.0239 + 0.0595i 0.0104 + 0.0293i 0.0172 + 0.0340i 0.0306 + 0.0372i 0.0104 + 0.0469i 0.0186 + 0.0136i 0.0715 + 0.0002i 0.0301 + 0.0609i 0.0394 + 0.0396i 0.0072 + 0.0164i 0.0017 + 0.0080i 0.0123 + 0.0121i 0.0651 + 0.0314i 0.0678 + 0.0314i 0.0144 + 0.0041i 0.0764 + 0.0726i 0.0549 + 0.0116i 0.0672 + 0.0286i 0.0370 + 0.0240i 0.0382 + 0.0130i 0.0222 + 0.0691i 0.0047 + 0.0249i 0.0202 + 0.0566i 0.0144 + 0.0317i 0.0707 + 0.0308i 0.0095 + 0.0390i 0.0010 + 0.0130i 0.0285 + 0.0404i 0.0538 + 0.0484i 0.0585 + 0.0012i 0.0456 + 0.0545i 0.0121 + 0.0618i 0.0244 + 0.0538i 0.0180 + 0.0356i 0.0006 + 0.0064i 0.0306 + 0.0633i 0.0501 + 0.0148i 0.0066 + 0.0343i 0.0583 + 0.0010i 0.0747 + 0.0238i 0.0551 + 0.0575i

by H. Häffner, UC Berkeley

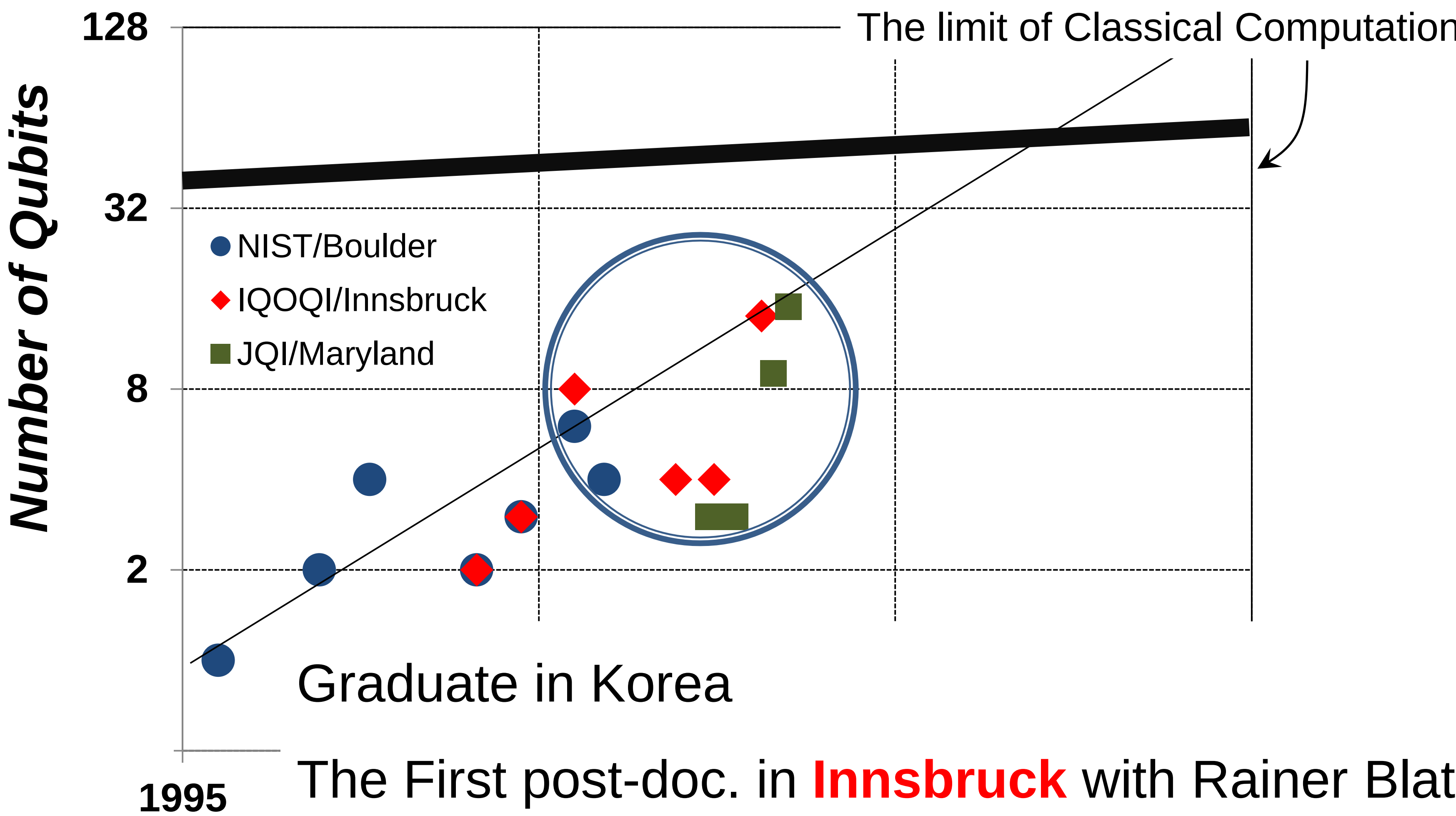
- 40 spins ~ 10 Tera Bytes
- 300 spins ~ Total # of atoms in the Universe

Status for the Realization of a Quantum Computer

Moore's Law in Quantum Computation?



Introduction of Kihwan Kim



Graduate in Korea

The First post-doc. in **Innsbruck** with Rainer Blatt

The Second post-doc. in **JQI** with Chris Monroe

Now at Tsinghua University in China

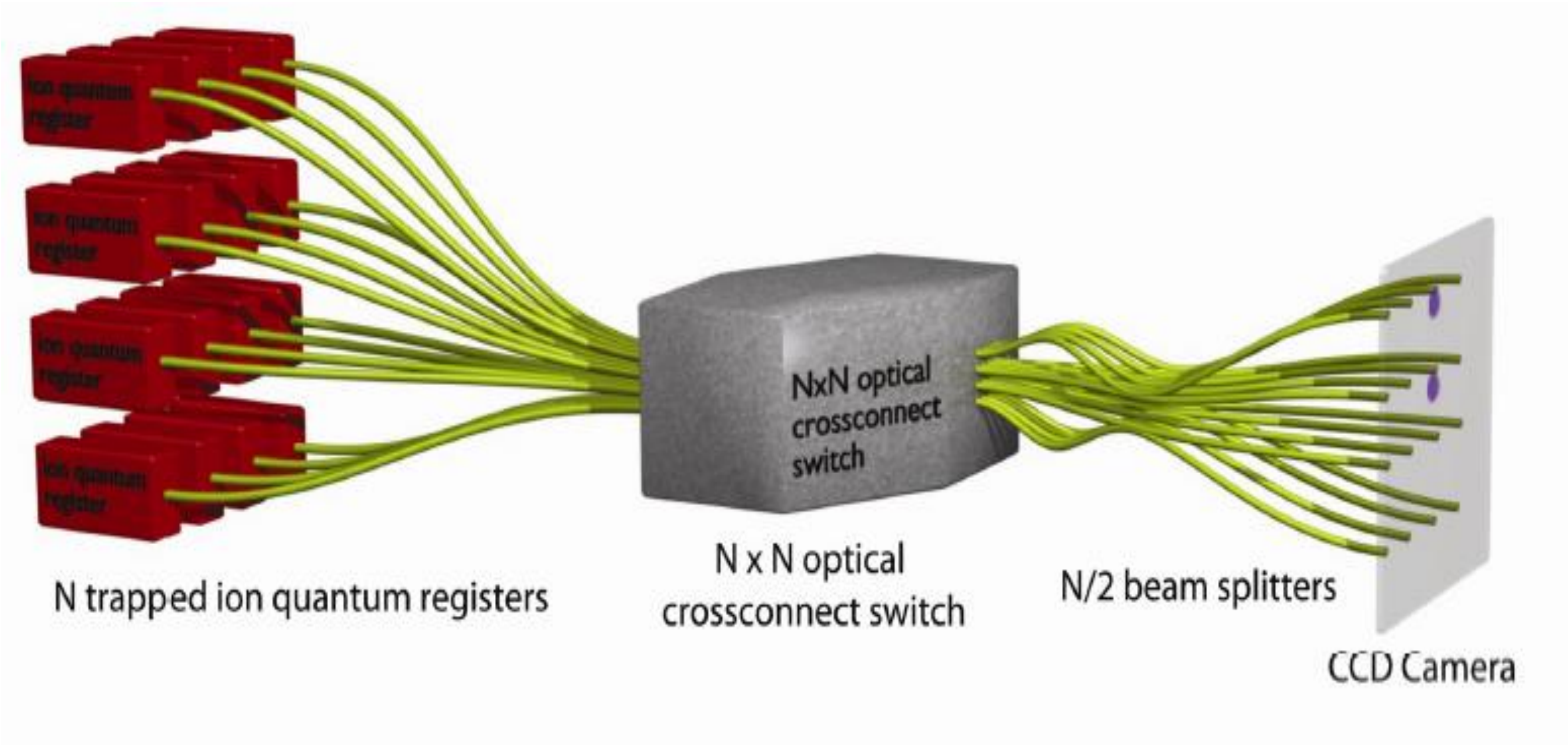
The First Practical Quantum Computer?

A quantum computer/simulator up to ~ a few tens of qubits

R. Blatt

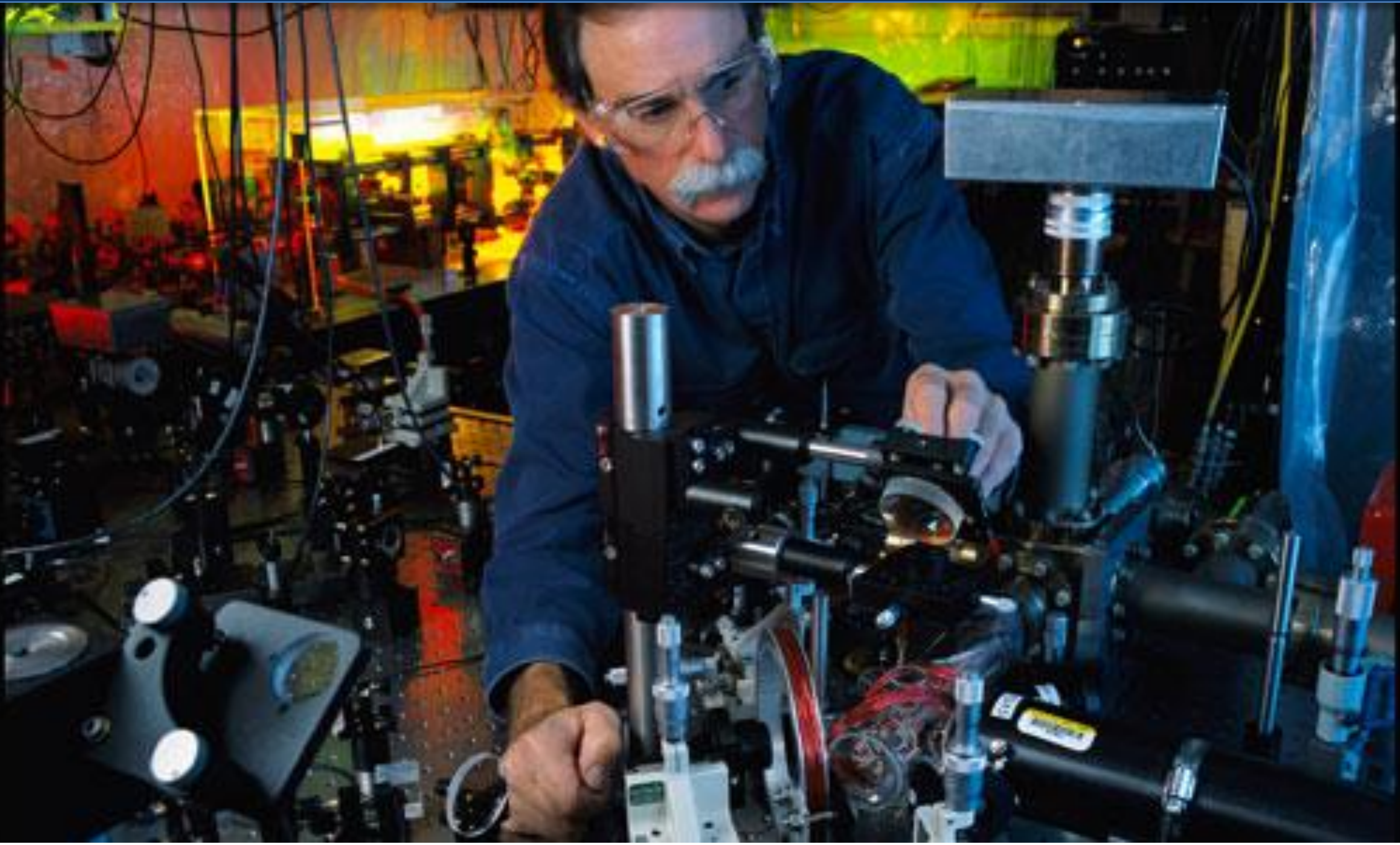
E. Knill, Nature **463**, 441 (2010).

Scalable Structure of Trapped Ion Quantum Computer



Scaling the Ion Trap Quantum Processor, Science 339, 1164 (2013)

2012 Noble Prize: Dr. David Wineland



http://www.nist.gov/pml/div688/grp10/index.cfm
물리학과 첨단기술 2012년 12월호

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- Carrier, Red Sideband, Blue Sideband Transition

Classical and Quantum Computation

Classical Computation

- Bits

  0 or 1

- Initialization
- Operations
  - 1-bit operations (NOT)
  - 2-bit operations (NAND)
- Detections

→ Results

Classical and Quantum Computation

Classical Computation

- Bits

  0 or 1

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  - 1-bit operations (NOT)
  - 2-bit operations (NAND)
- Detections

  → Results

Quantum Computation

- Qubits

  $|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$, 2^N Hilbert Space

- Initialization
- Operations
  - 1-qubit operations (rotations)

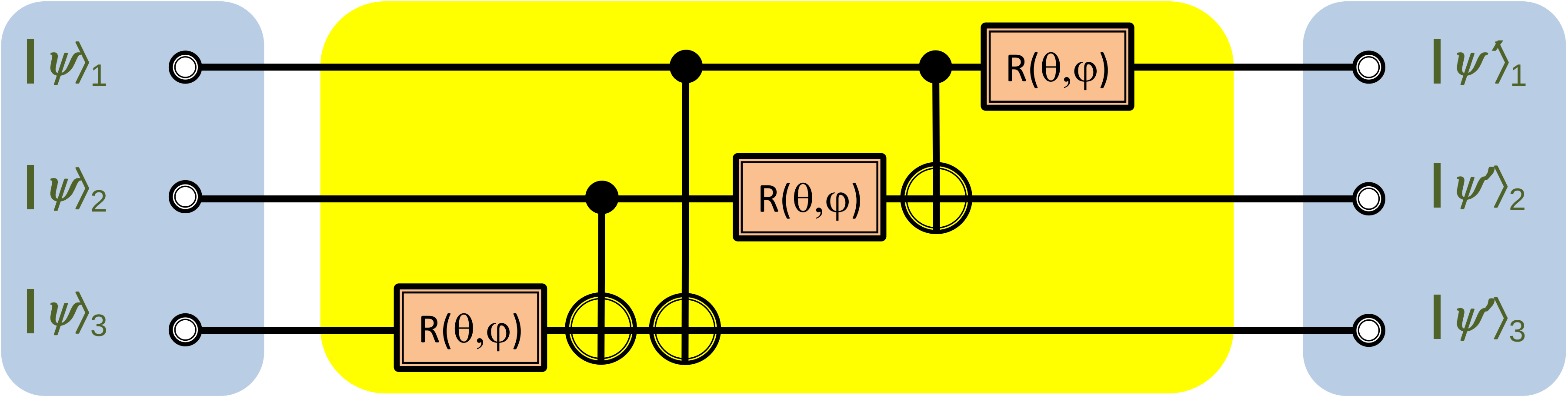
    → Superposition
  - 2-qubit operations (CNOT gates)

    → Entanglements
- Detections of Qubits

  → Gain of Classical Information

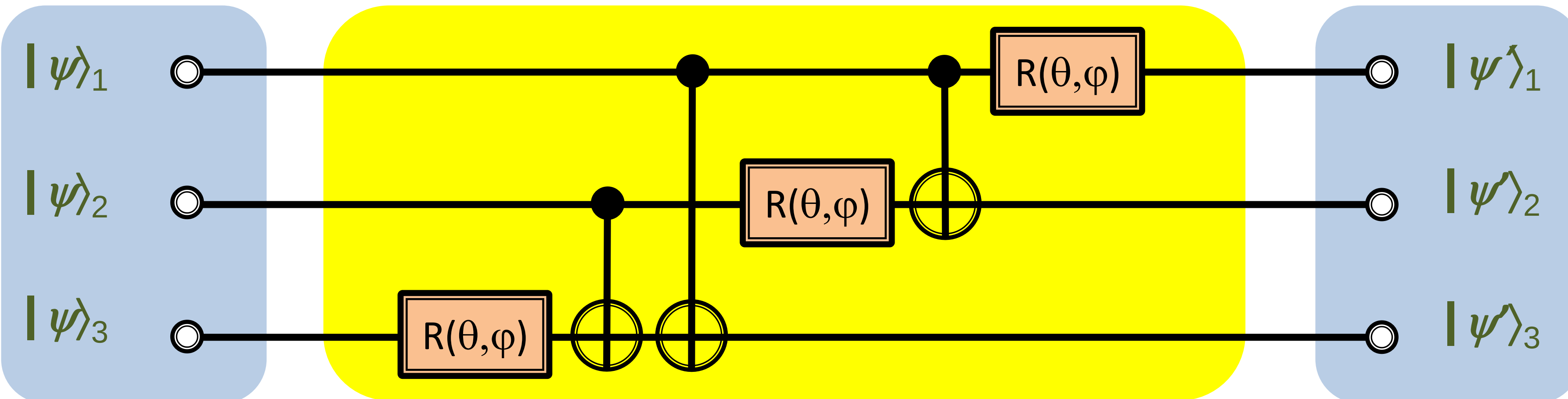
Quantum Computations

Circuit Quantum Computations



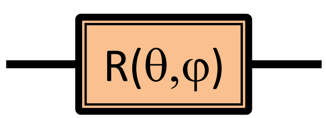
Initialization **Operations** **Detections**

Circuit Quantum Operations



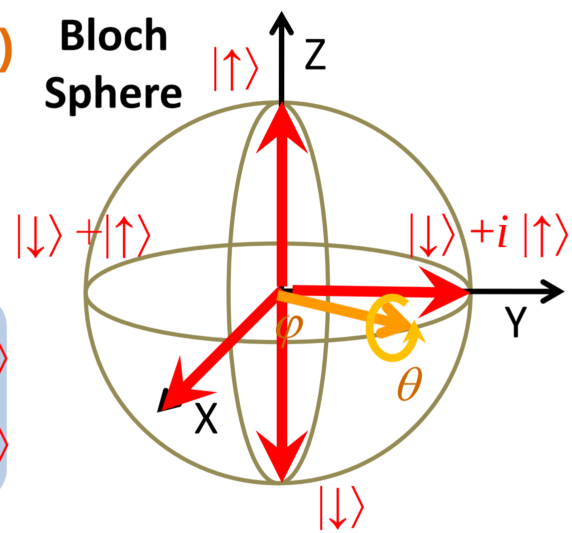
1-qubit operation

(Rotation in the Bloch sphere)

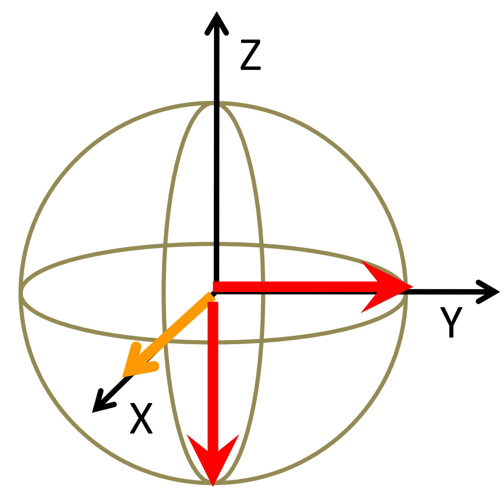


$$|\downarrow\rangle \rightarrow \cos\theta/2\,|\downarrow\rangle + ie^{i\varphi}\sin\theta/2\,|\uparrow\rangle$$

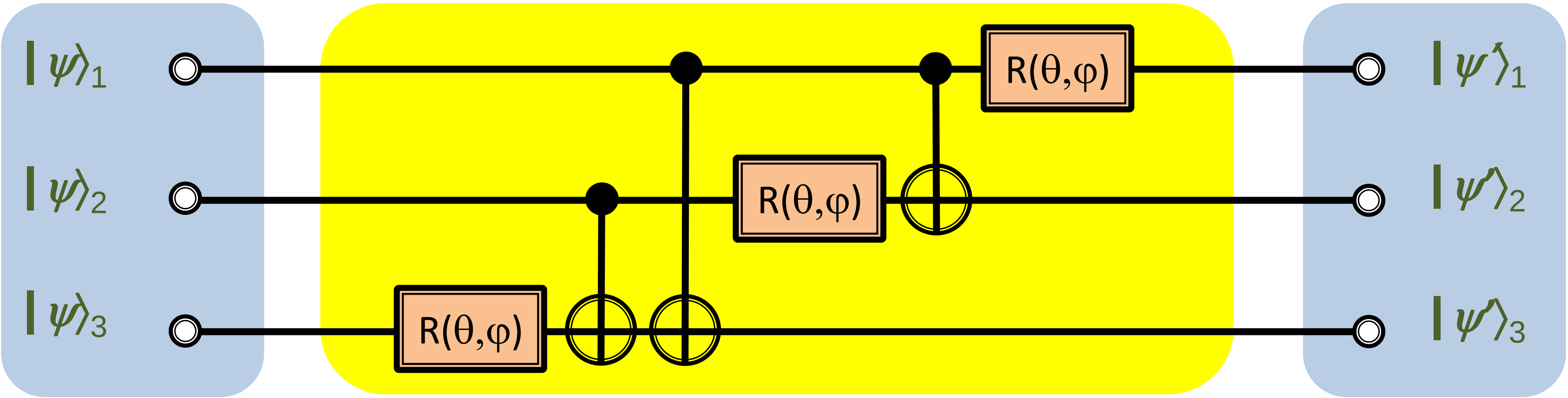
$$|\uparrow\rangle \rightarrow \cos\theta/2\,|\uparrow\rangle - ie^{-i\varphi}\sin\theta/2\,|\downarrow\rangle$$



ex) R(π/2,0)



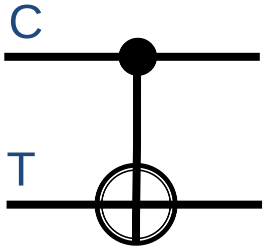
Circuit Quantum Operations



2-qubit operation

(Controlled-NOT

Or Entangling Gate)



C T

$|\downarrow\rangle\,|\downarrow\rangle \rightarrow |\downarrow\rangle\,|\downarrow\rangle$

$|\downarrow\rangle\,|\uparrow\rangle \rightarrow |\downarrow\rangle\,|\uparrow\rangle$

$|\uparrow\rangle\,|\downarrow\rangle \rightarrow |\uparrow\rangle\,|\uparrow\rangle$

$|\uparrow\rangle\,|\uparrow\rangle \rightarrow |\uparrow\rangle\,|\downarrow\rangle$

superposition → entanglement

$[|\downarrow\rangle + |\uparrow\rangle]|\downarrow\rangle \rightarrow |\downarrow\rangle\,|\downarrow\rangle + |\uparrow\rangle\,|\uparrow\rangle$

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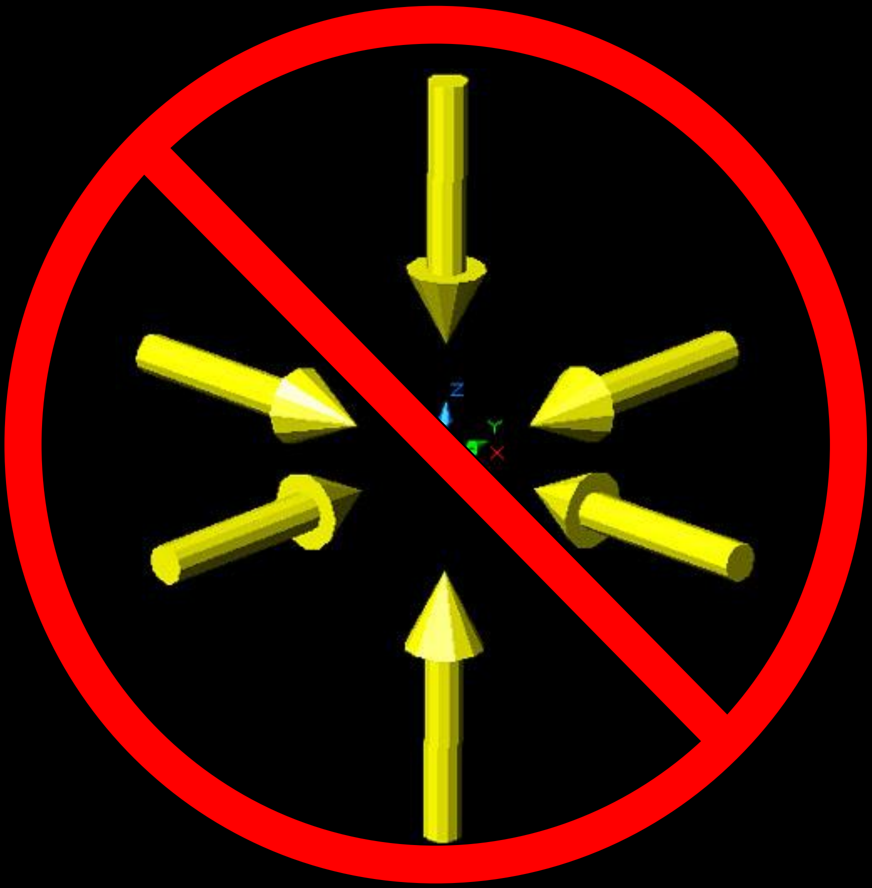
Ion Trap

Electric Field Vectors

NO! $\nabla \cdot \mathbf{E} = 0$



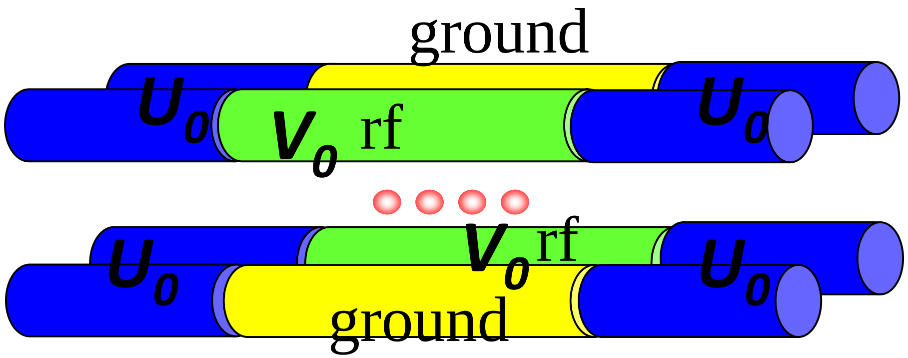
H.G. Dehmelt, Adv. At. Mol. Phys. 3, 5 (1967).
W. Paul, Rev. Mod. Phys. 62, 531 (1990).

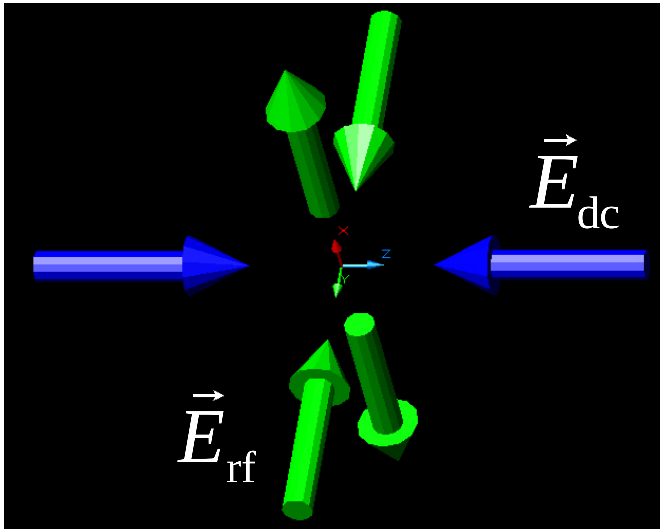
Wolfang Paul vs. Hans Dehmelt

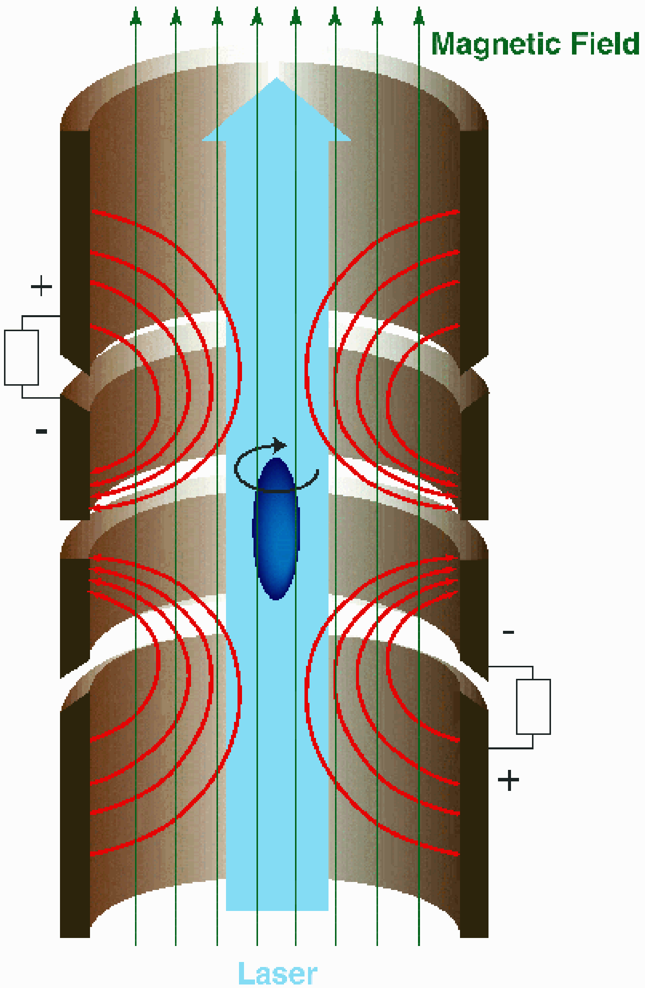
University of Bonn, Germany
Nobel Prize in physics (1989)

University of Washington, USA
Nobel Prize in physics (1989)

Paul Trap vs. Penning Trap



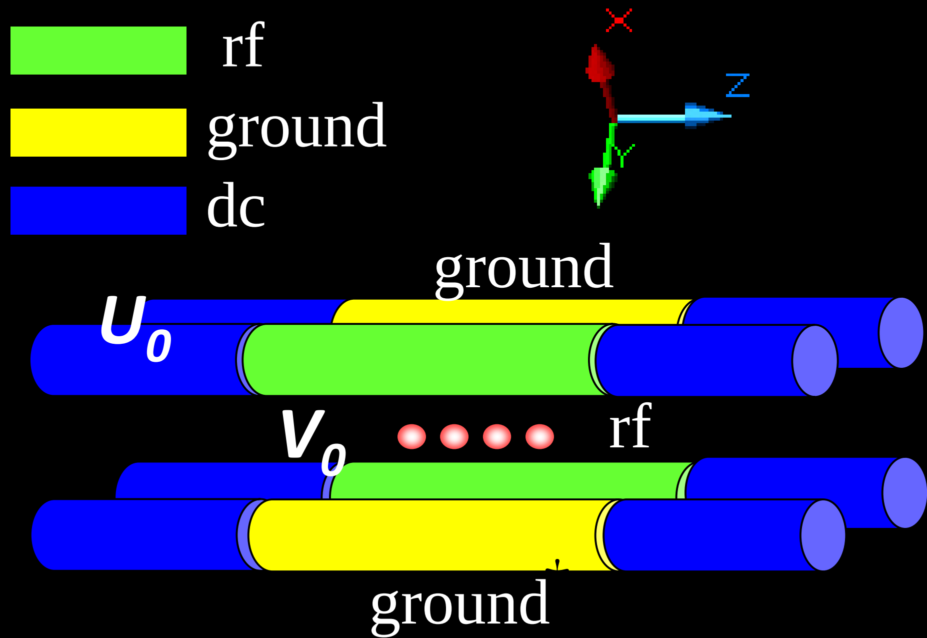


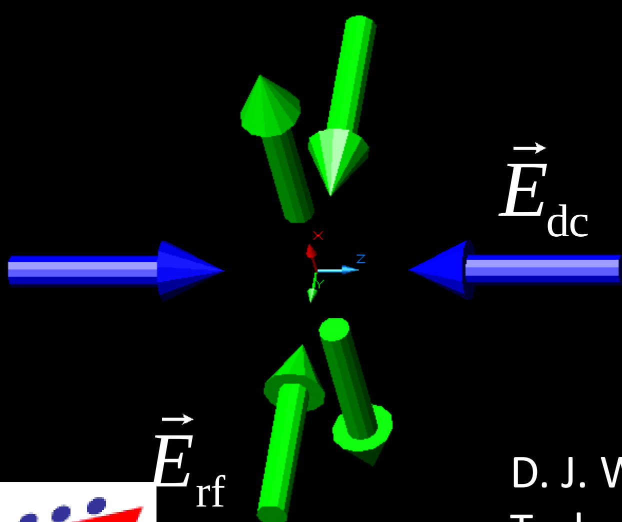


Paul Trap
– rotating electric field confinement

Penning Trap
– magnetic confinement

Linear Ion Trap





axial confinement - static!

$$\Phi(z) = (m\omega_z^2/2q)\,(z^2/2)$$

$$\omega_z^2 = 2\alpha q U_0/m, \quad \alpha \sim 1 \text{ (geom.)}$$

radial confinement -dynamic!

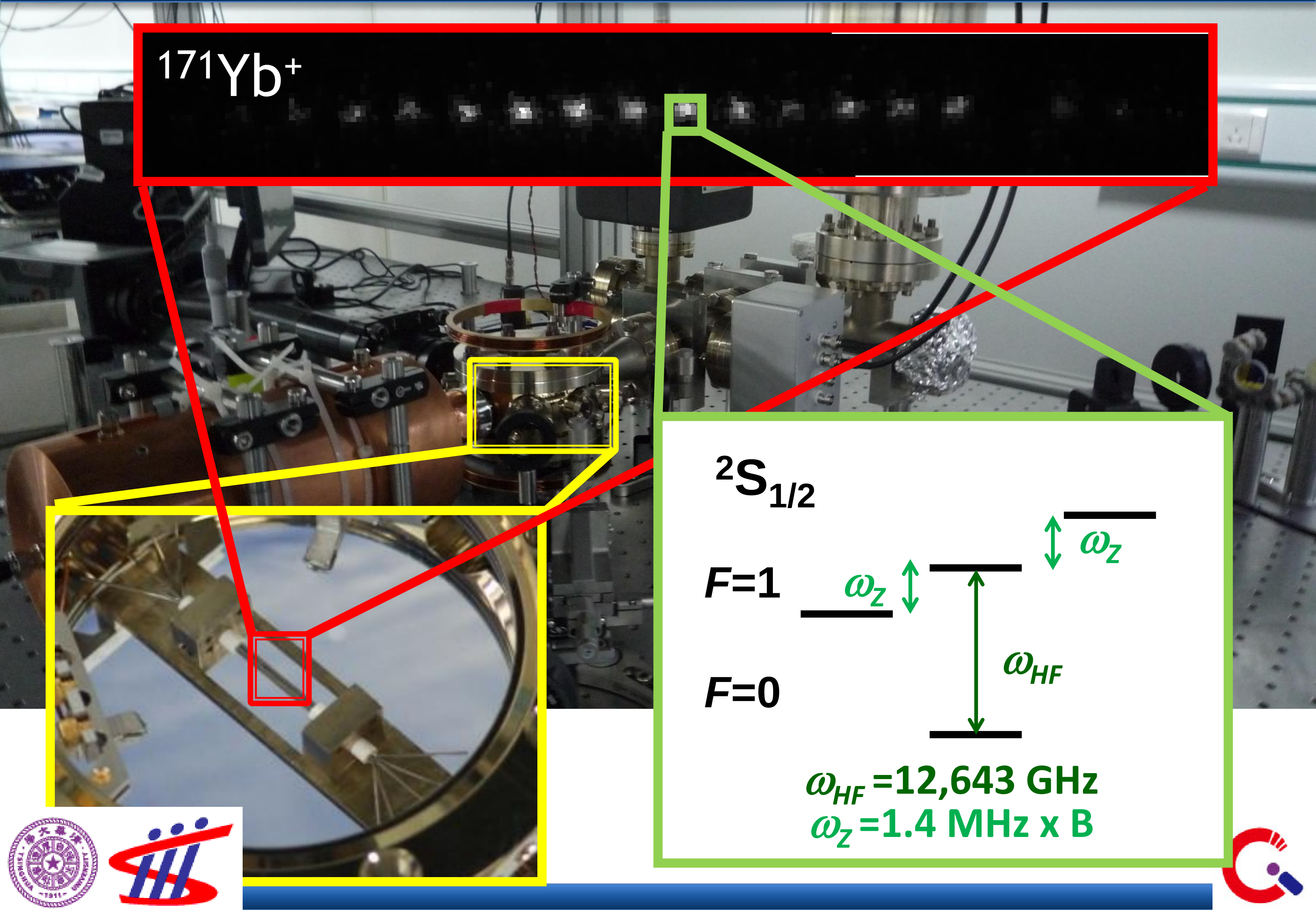
$$\Phi(r) = (m/2q)\,(\omega_r^2 - \omega_z^2/2)\,(r^2)$$

$$\omega_r^2 = q^2 V_0^2/(2m\Omega_{RF}\beta^4 r^4)$$

$$\omega_r < \Omega_{RF}, \quad \beta \sim 1 \text{ (geom.)}$$

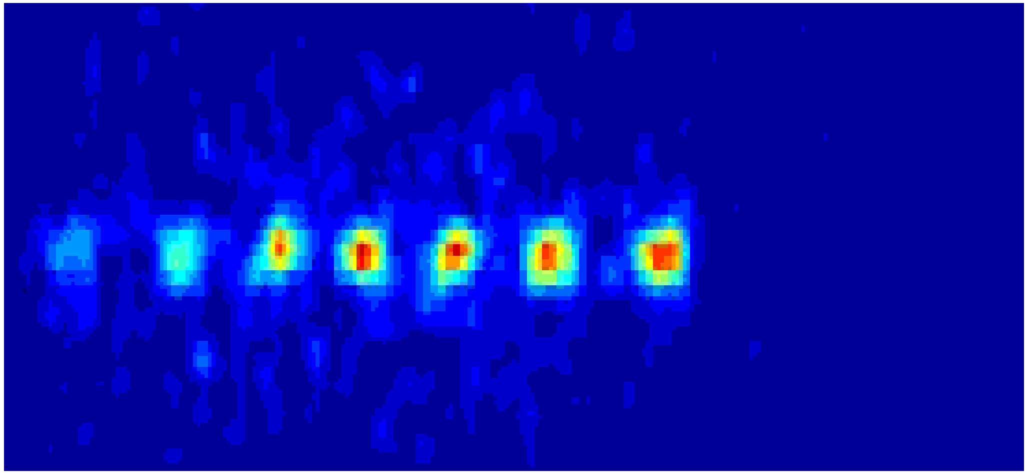
D. J. Wineland, et al., J. Res. Nat. Inst. Stand. Tech. 103, 259 (1998).

Linear Ion Trap @ Tsinghua University

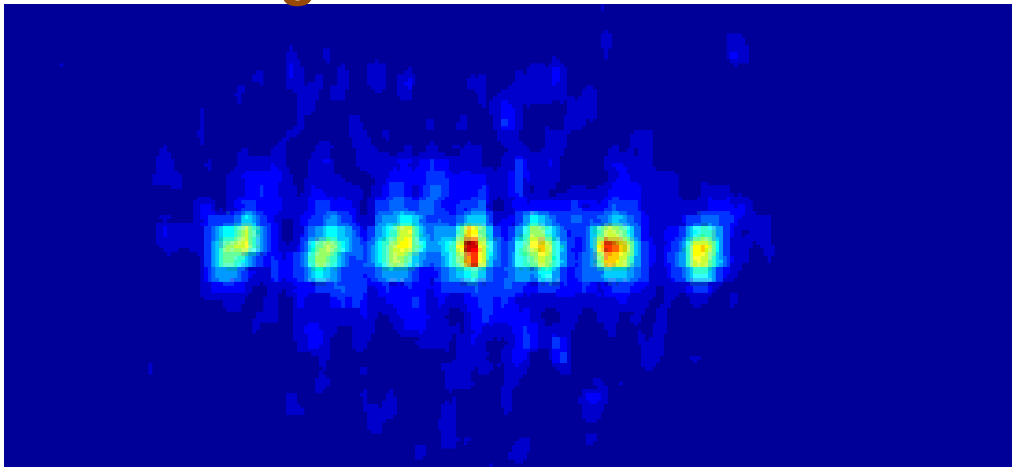


Collective Normal Modes

Center of mass mode



Breathing mode



$^{40}Ca^{+}$

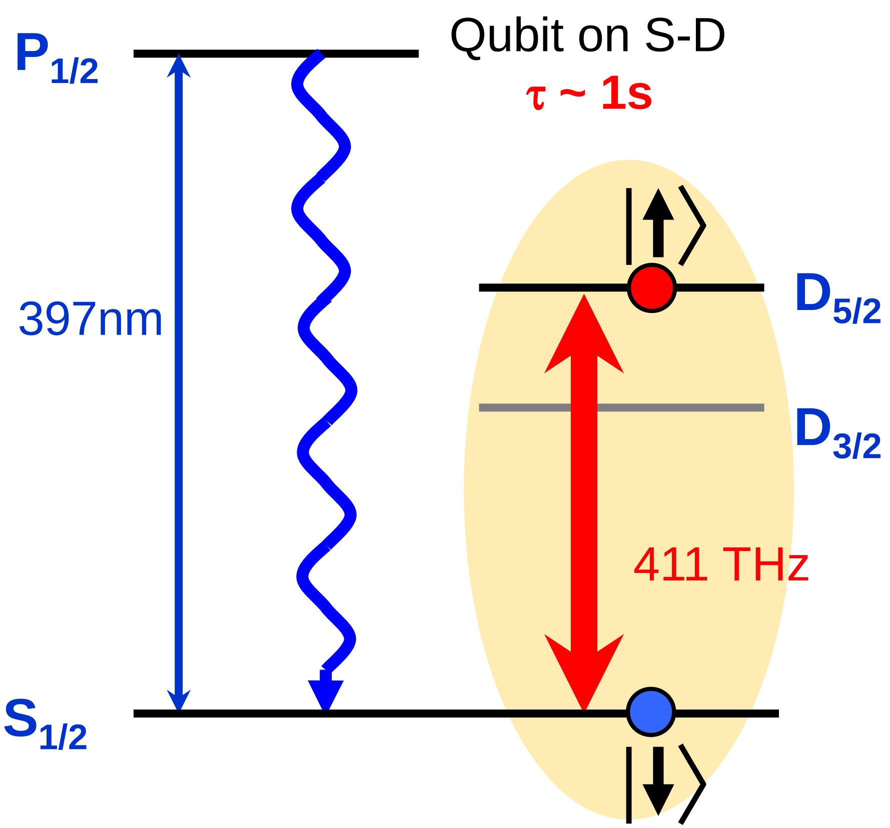
(Univ. Innsbruck)

D. F. V. James, Appl. Phys. B **66**, 181 (1998).

Qubits – long coherence time

Optical Qubit

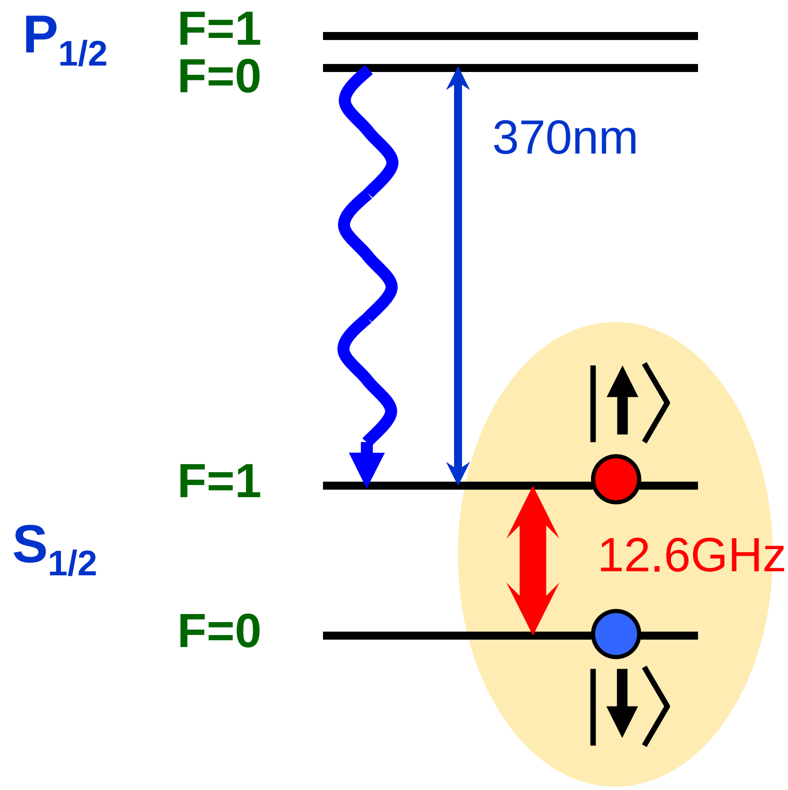
ex) $^{40}Ca^{+}$



$^{88}Sr^{+}$, $^{138}Ba^{+}$ …

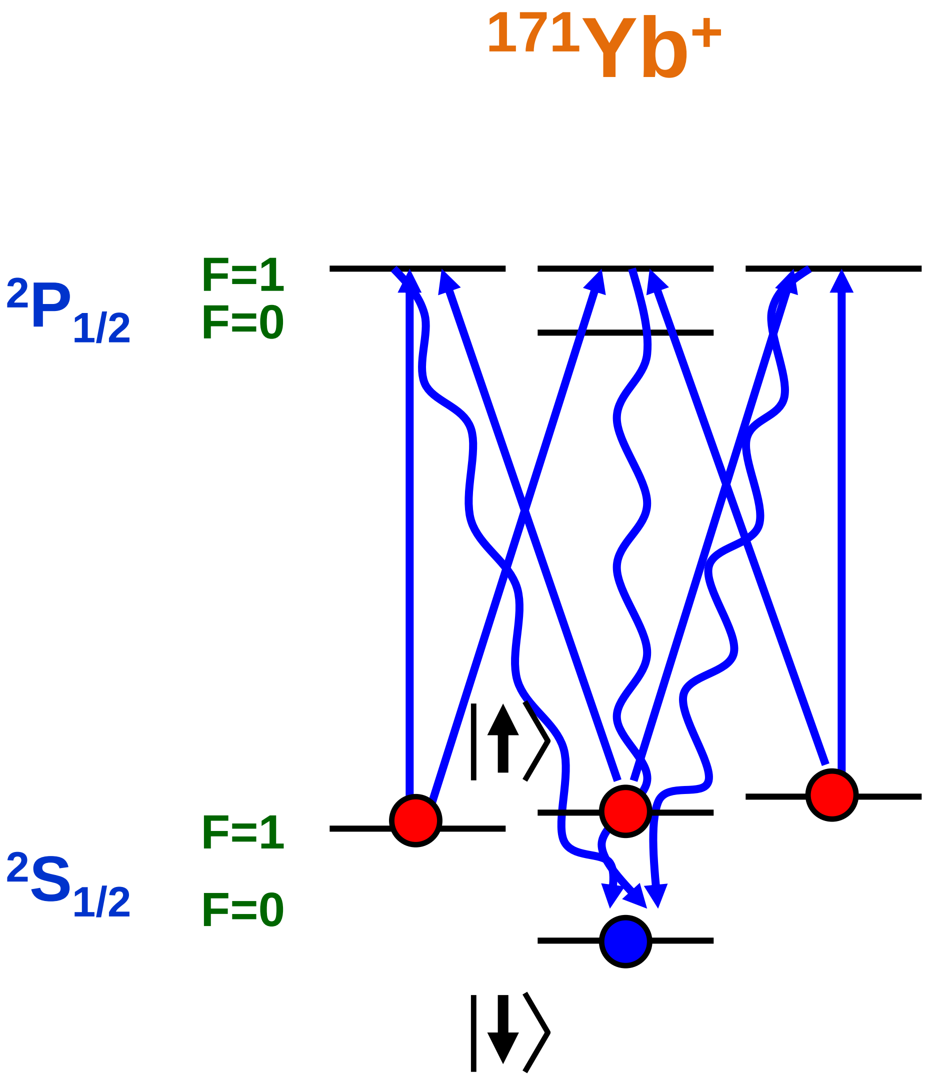
Hyperfine Qubit

ex) $^{171}Yb^{+}$



$^{9}Be^{+}$, $^{25}Mg^{+}$, $^{43}Ca^{+}$, $^{87}Sr^{+}$, $^{137}Ba^{+}$, $^{111}Cd^{+}$, $^{113}Cd^{+}$, $^{199}Hg^{+}$ …

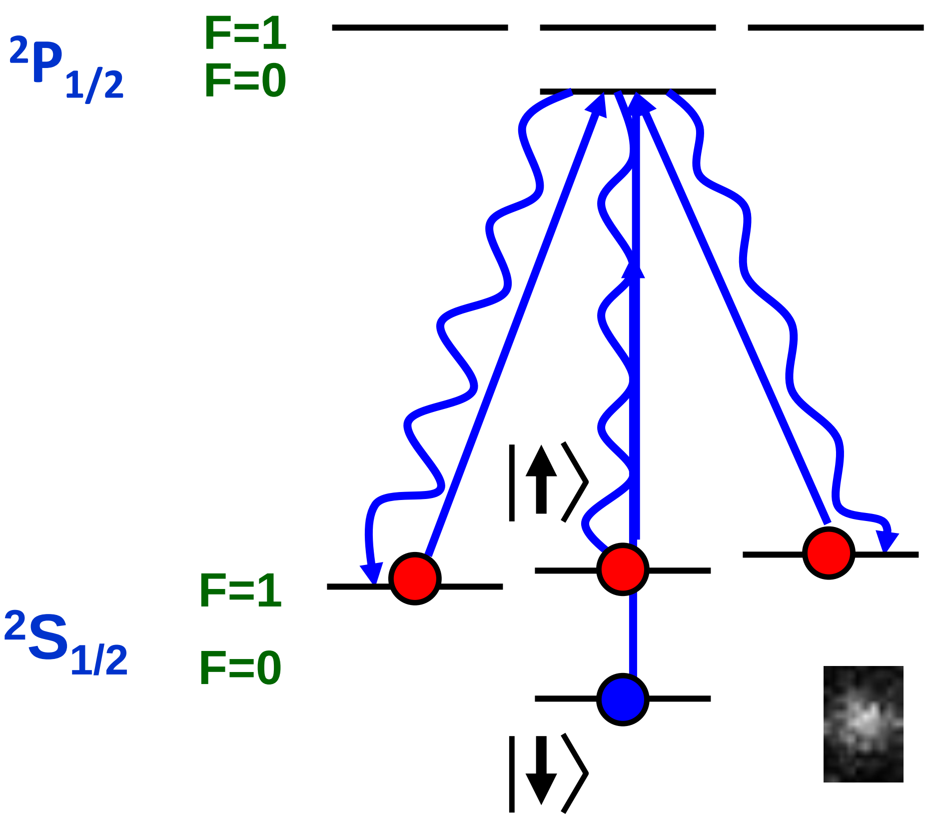
Initialization of Qubits



Initialization – optical pumping

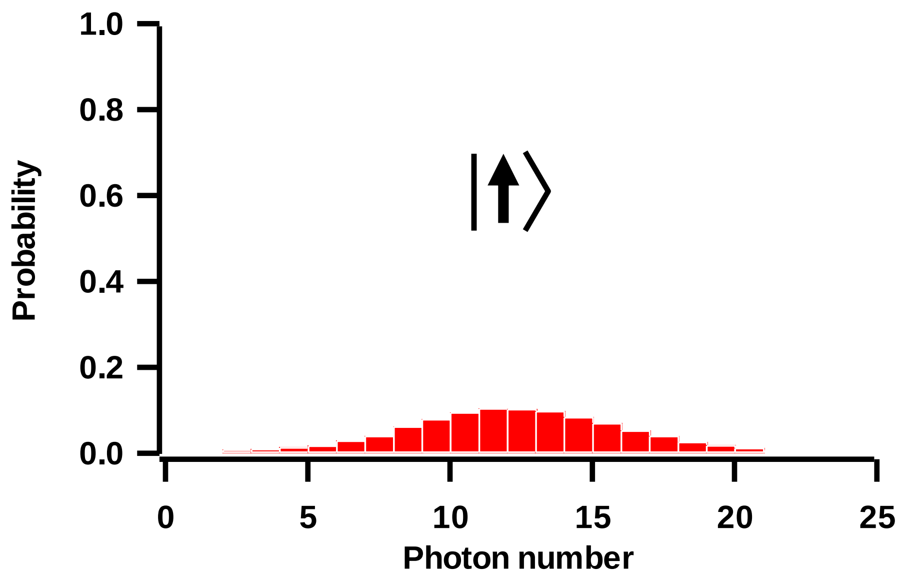
Detection of Qubits

$^{171}Yb^{+}$



Initialization – optical pumping

Detection – state dependent fluorescence

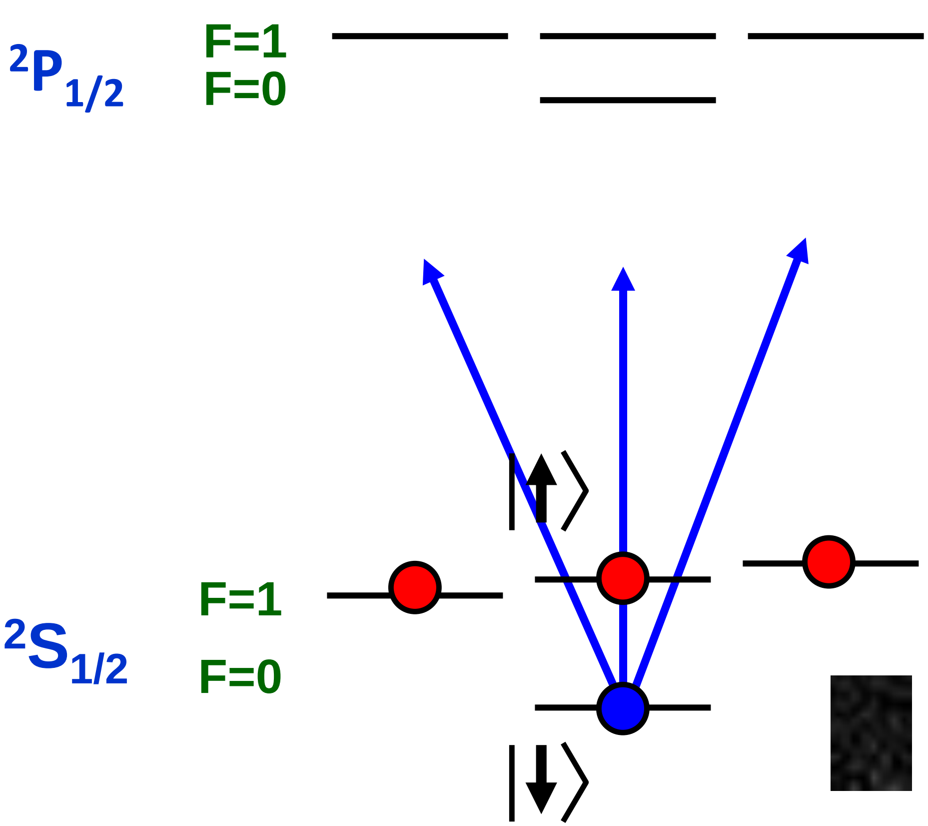


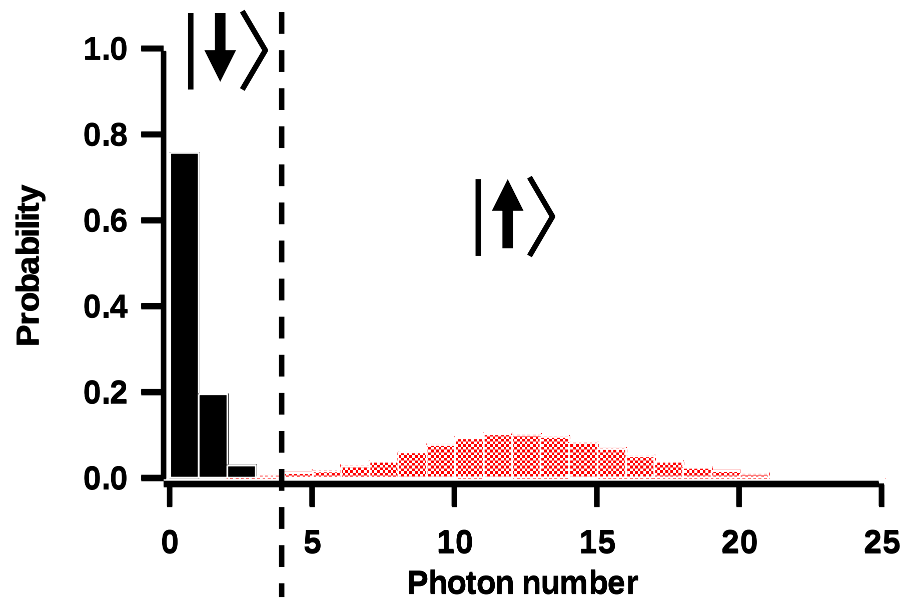
Detection of Qubits

$^{171}Yb^+$

Initialization – optical pumping

Detection – state dependent fluorescence





A. H. Myerson, et al., Phys. Rev. Lett. **100**, 200502 (2008).
A. H. Burrell, et al,, Phys. Rev. A **81**, 040302 (2010).

Outline

Introduction

- Why Quantum Computation?
- Current Status of Quantum Computation

Basic Requirements for the Quantum Computation

- Qubit, Initialization, Operation, Detection

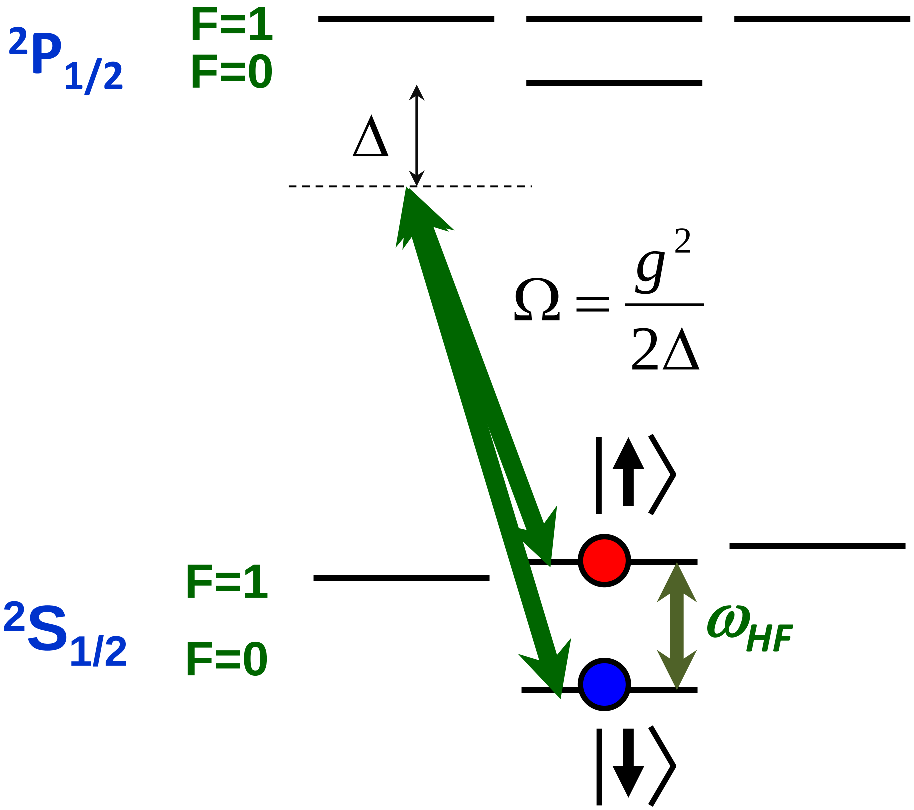
Qubit, Initialization, Detection in Ion Trap System

Operation in Trapped Ion System Part I

- Ion Laser Interaction
- Carrier, Red Sideband, Blue Sideband Transition

Single Qubit Operations

$^{171}Yb^{+}$

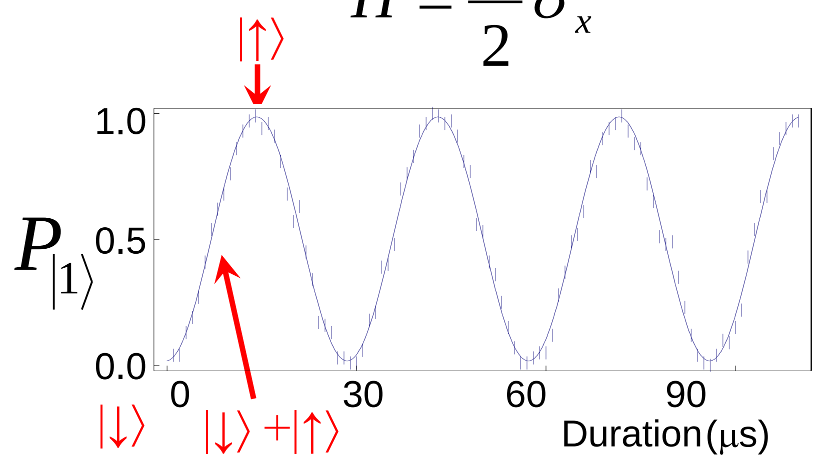


Initialization – optical pumping

Detection – state dependent fluorescence

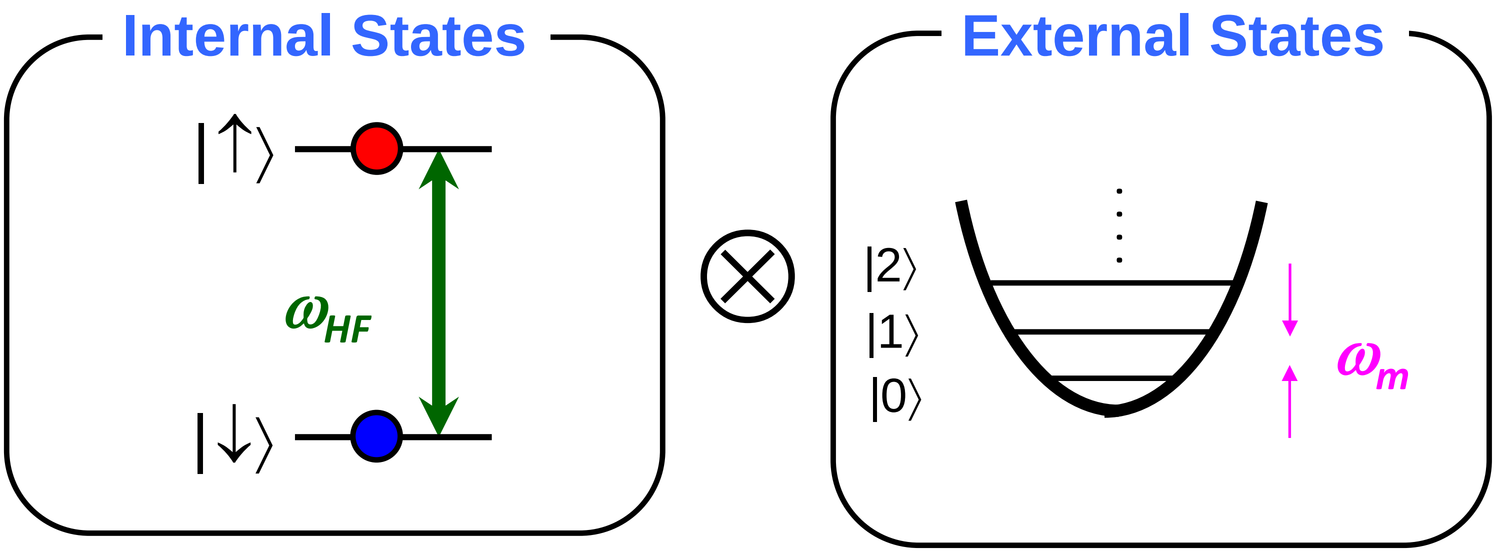
Coherent Operation
– Rabi oscillation by Raman Laser, Microwaves

$$H = \frac{\Omega}{2}\sigma_x$$

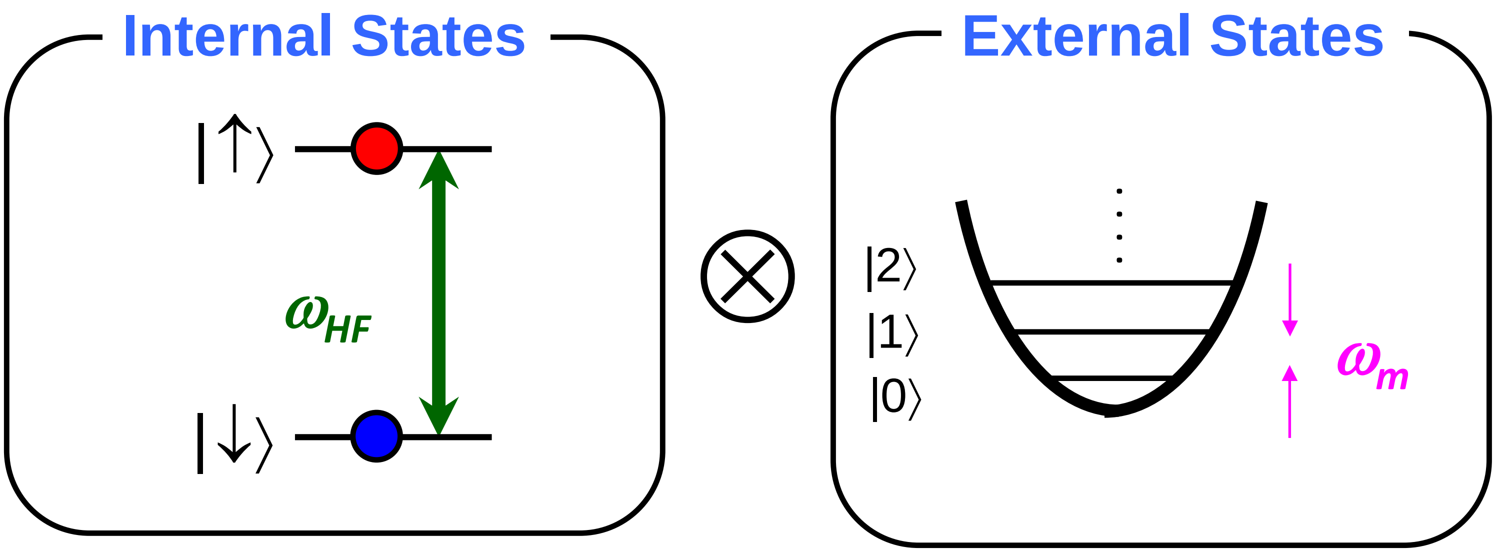


E. Knill, et al., Phys. Rev. A **77**, 012307 (2008).

A single ion system



A single ion system



$$H^{(e)} = \frac{\hbar\omega_{HF}}{2}\left(|\uparrow\rangle\langle\uparrow| - |\downarrow\rangle\langle\downarrow|\right)$$

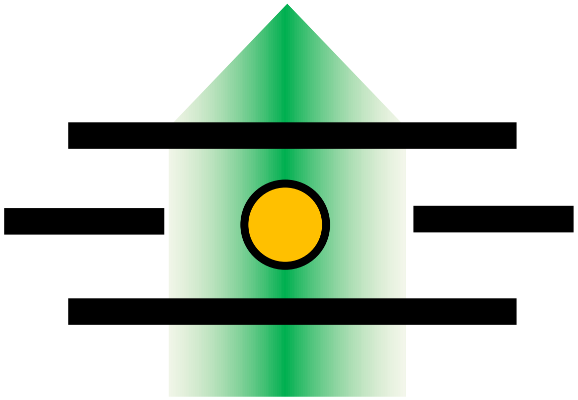
$$= \frac{\hbar\omega_{HF}}{2}\sigma_z$$

$$H^{(m)} = \frac{\hat{p}^2}{2m} + \frac{1}{2}m\omega_m^2\hat{x}^2$$

$$= \hbar\omega_m\left(a^+a + \frac{1}{2}\right)$$

Interaction between Laser and an Ion

$$E(\hat{x}) = E_0 \cos(k\hat{x} - \omega_L t + \varphi)$$



$$H^{(i)} = -\hat{\mu} \cdot E(\hat{x})$$

$$= \mu_0 (\hat{\sigma}_+ + \hat{\sigma}_-) \frac{E_0}{2} \left(e^{i(k\hat{x} - \omega_L t + \varphi)} + e^{-i(k\hat{x} - \omega_L t + \varphi)} \right)$$

$$= \frac{\Omega}{2} (\hat{\sigma}_+ + \hat{\sigma}_-) \left(e^{i(k\hat{x} - \omega_L t + \varphi)} + e^{-i(k\hat{x} - \omega_L t + \varphi)} \right)$$

where, $\Omega = \mu_0 E_0 / 2$

Interaction between Laser and an ion

Transformation to the interaction picture with

$$H_0 = H^{(e)} + H^{(m)} = \frac{\hbar\omega_{HF}}{2}\sigma_z + \hbar\omega_m\left(a^+a + \frac{1}{2}\right)$$

$$H_{\text{int}} = \hat{U}_0^+ H^{(i)} \hat{U}_0 = e^{(i/\hbar)H_0 t} H^{(i)} e^{-(i/\hbar)H_0 t}$$

$$= (\hbar/2)\Omega e^{(i/\hbar)H^{(e)}t}(\hat{\sigma}_+ + \hat{\sigma}_-)e^{-(i/\hbar)H^{(e)}t} e^{(i/\hbar)H^{(m)}t}\left[e^{i(k\hat{x}-\omega t+\varphi)} + e^{-i(k\hat{x}-\omega t+\varphi)}\right]e^{-(i/\hbar)H^{(m)}t}$$

$$= (\hbar/2)\Omega\left\{e^{i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}(\hat{\sigma}_+ + \hat{\sigma}_-)e^{-i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}\right\}\left\{e^{i\omega_m a^+ a t}\left[e^{i(k\hat{x}-\omega t+\varphi)} + e^{-i(k\hat{x}-\omega t+\varphi)}\right]e^{-i\omega_m a^+ a t}\right\}$$

$$e^{\alpha\hat{A}}\hat{B}e^{-\alpha\hat{A}} = \hat{B} + \alpha\left[\hat{A},\hat{B}\right] + \frac{\alpha^2}{2!}\left[\hat{A},\left[\hat{A},\hat{B}\right]\right] + \frac{\alpha^3}{3!}\left[\hat{A},\left[\hat{A},\left[\hat{A},\hat{B}\right]\right]\right]\ldots$$

Interaction between Laser and an ion

$$\left\{e^{i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)e^{-i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}\right\}$$

$$[\hat{\sigma}_z, \hat{\sigma}_+] = 2\hat{\sigma}_+ \quad [\hat{\sigma}_z, \hat{\sigma}_-] = 2\hat{\sigma}_-$$

$$\begin{aligned} e^{i\alpha\hat{\sigma_z}}\hat{\sigma}_+ e^{-i\alpha\hat{\sigma_z}} &= \hat{\sigma}_+ + 2\alpha\hat{\sigma}_+ + \frac{(2\alpha)^2}{2!}\hat{\sigma}_+ + \frac{(2\alpha)^3}{3!}\hat{\sigma}_+ + \ldots \\ &= e^{i2\alpha}\hat{\sigma}_+ \end{aligned}$$

$$\begin{aligned} e^{i\alpha\hat{\sigma_z}}\hat{\sigma}_- e^{-i\alpha\hat{\sigma_z}} &= \hat{\sigma_-} - 2\alpha\hat{\sigma}_- + \frac{(-2\alpha)^2}{2!}\hat{\sigma}_- + \frac{(-2\alpha)^3}{3!}\hat{\sigma}_- + \ldots \\ &= e^{-i2\alpha}\hat{\sigma}_-. \end{aligned}$$

$$\therefore \left\{e^{i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}\left(\hat{\sigma}_+ + \hat{\sigma}_-\right)e^{-i\frac{\omega_{HF}t}{2}\hat{\sigma}_z}\right\} = e^{i\omega_{HF}t}\hat{\sigma}_+ + e^{-i\omega_{HF}t}\hat{\sigma}_-$$

Interaction between Laser and an ion

$$H_{\text{int}} = \widehat{U}_0^+ H^{(i)} \widehat{U}_0 = e^{(i/\hbar)H_0 t} H^{(i)} e^{-(i/\hbar)H_0 t}$$

$$= (\hbar/2)\Omega e^{(i/\hbar)H^{(e)}t}\left(\widehat{\sigma}_+ + \widehat{\sigma}_-\right)e^{-(i/\hbar)H^{(e)}t} e^{(i/\hbar)H^{(m)}t}\left[e^{i(k\widehat{x}-\omega t+\varphi)} + e^{-i(k\widehat{x}-\omega t+\varphi)}\right]e^{-(i/\hbar)H^{(m)}t}$$

$\ldots\ldots$

$$= (\hbar/2)\Omega\widehat{\sigma}_+ e^{i\eta\left(ae^{-i\omega_m t} + a^+ e^{i\omega_m t}\right)} e^{-i(\delta t+\varphi)} + h.c.$$

$$\delta = \omega_L - \omega_{HF}$$

$$\hat{x} = x_0\left(ae^{-i\omega_m t} + a^+ e^{i\omega_m t}\right)$$

$$x_0 = \sqrt{\hbar/2m\omega_m}$$

$$\eta = kx_0 : \text{Lamb - Dicke parameter}$$

Carrier, Red sideband, Blue sideband

$$H_{\text{int}} = (\hbar/2)\Omega\left[\hat{\sigma}_+ e^{i\eta\left(ae^{-i\omega_m t}+a^+e^{i\omega_m t}\right)}e^{-i(\delta t-\varphi)} + \hat{\sigma}_- e^{-i\eta\left(ae^{-i\omega_m t}+a^+e^{i\omega_m t}\right)}e^{i(\delta t-\varphi)}\right]$$

$$\eta\sqrt{n+1} = kx_0\sqrt{n+1} << 1 : \text{Lamb-Dicke limit}$$

Stationary terms of H_{int} at particular values of δ

"CARRIER" $\delta = 0$

$$H_{carr} = (\hbar/2)\Omega\left[\hat{\sigma}_+ e^{i\varphi} + \hat{\sigma}_- e^{-i\varphi}\right]$$

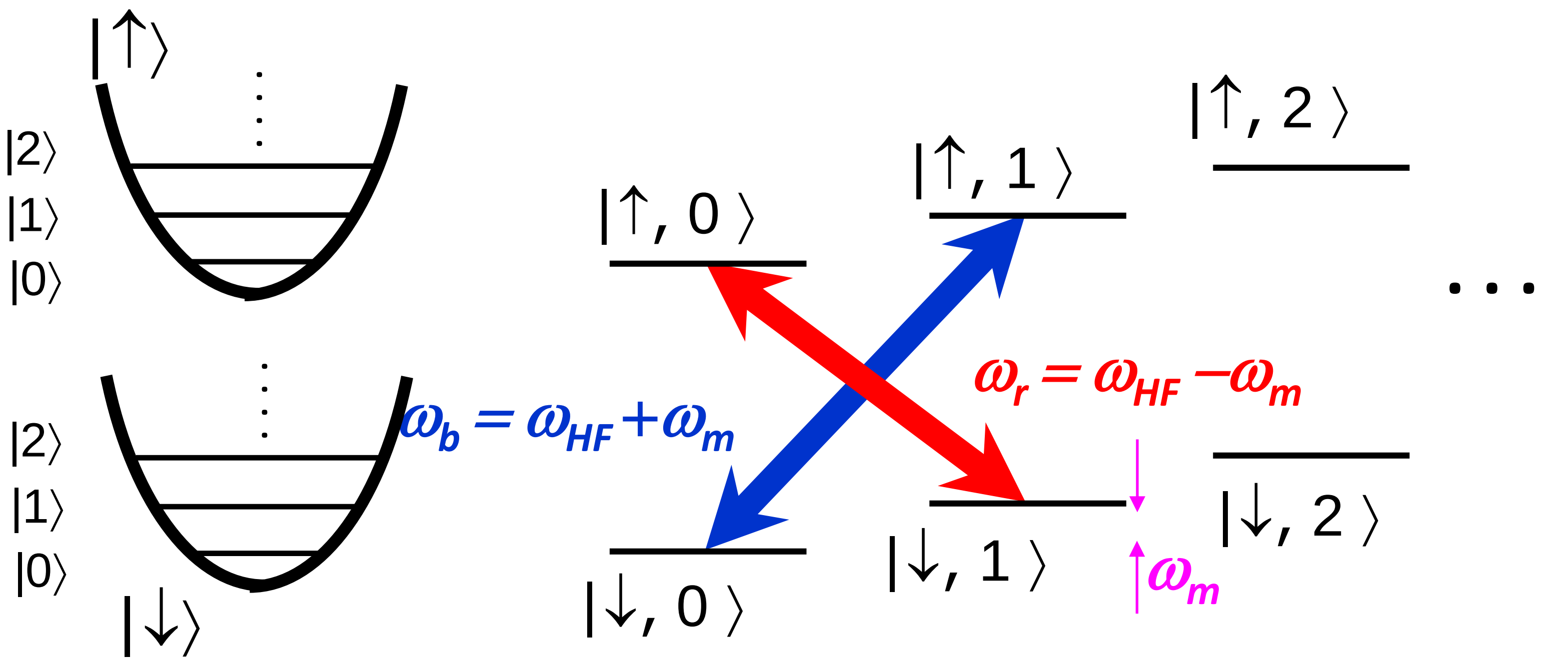
"Red Sideband" $\delta = -\omega_m$

$$H_{rsb} = (\hbar/2)\eta\Omega\left[\hat{\sigma}_+ a e^{i\varphi} + \hat{\sigma}_- a^+ e^{-i\varphi}\right]$$

"Blue Sideband" $\delta = +\omega_m$

$$H_{rsb} = (\hbar/2)\eta\Omega\left[\hat{\sigma}_+ a^+ e^{i\varphi} + \hat{\sigma}_- a e^{-i\varphi}\right]$$

Coupling Between Internal State and Motional Mode



$$H^{(e)} \otimes H^{(m)}$$

$$H_{bsb} = -i\hbar\eta\Omega\sigma^{+}a^{\dagger} + h.c.$$

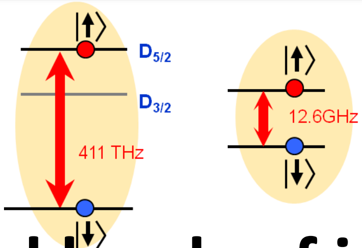
$$H_{rsb} = -i\hbar\eta\Omega\sigma^{-}a^{\dagger} + h.c.$$

Conclusion

Quantum Computation

- **Qubits**

$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, 2^N Hilbert Space



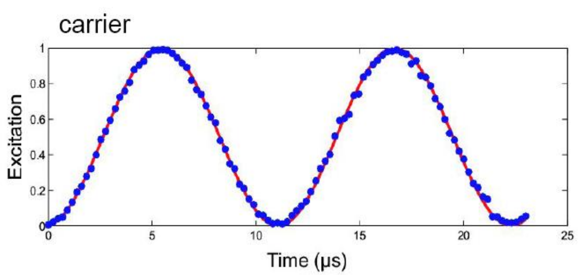
Internal levels of ions

- **Initialization**

Optical Pumping

- **Operations**
  - **1-qubit operations (rotations)**

    ⟶ **Superposition**



  - **2-qubit operations (CNOT gates)**

    ⟶ **Entanglements**

- **Detections of Qubits**

⟶ **Gain of Classical Information**

Fluorescent Discrimination