Operational quasi-probability to refute hidden variable models of macroscopic local realism

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Contents

- Wigner function (conventional appraoch)
- Operational quasi-probability function
- Inseparability for two qudits
- A “quantometer” to test if a process is quantum

Q. features

complementarity

non-locality

non-classicality

"… phenomena under different experimental conditions, must be termed complementary in the sense that each is well defined and that together they exhaust all definable knowledge about the object concerned."

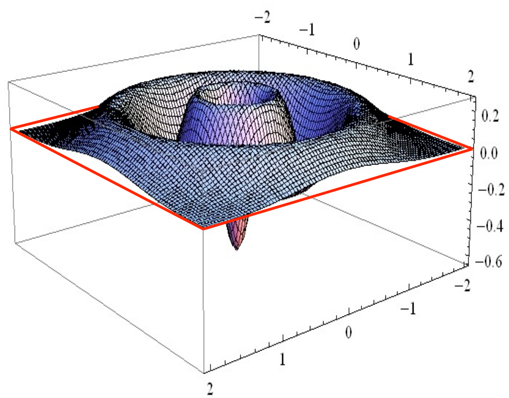
N. Bohr

"Quantum physics conflicts with local realism."

J. S. Bell

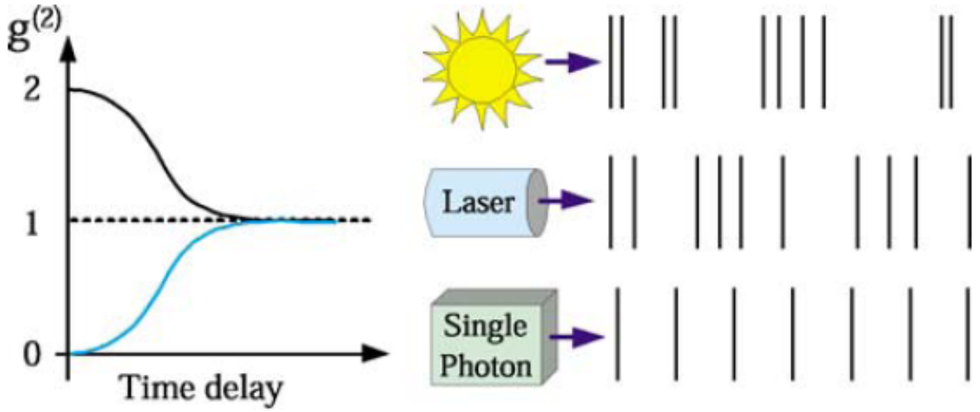
Negative distribution function

E. Wigner



Statistics and non-classicality

R. J. Glauber, Phys. Rev. 130, 2529 (1963); 131, 2766 (1963)



bunching

random

anti-bunching

Fig. thermal light(black), laser(dot), single photon source(blue).

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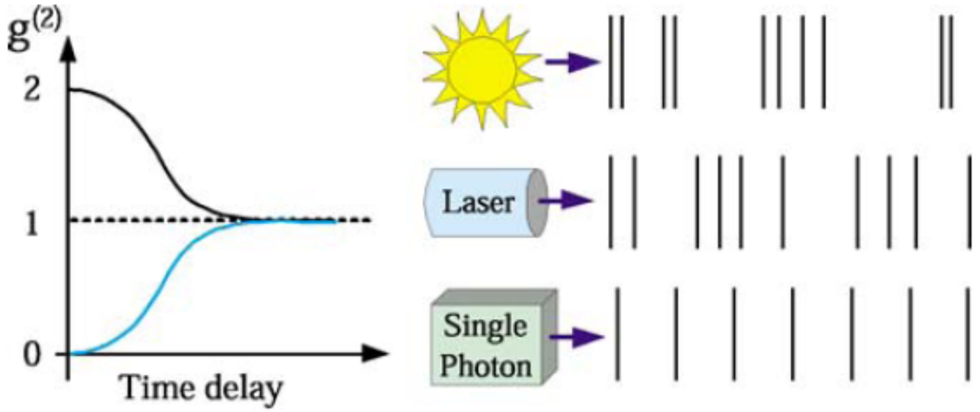

bunching

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classical statistics also predict.

quantum statistics only predict.

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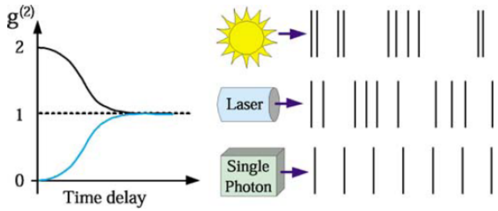


bunching
random
anti-bunching

classical statistics also predict.

quantum statistics only predict.

Fig. thermal light(black), laser(dot), single photon source(blue).

$$g^{(2)}(\vec{r},\tau) = \frac{\left\langle E^{(-)}(\vec{r},t)E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t)\right\rangle}{\langle E^{(-)}(\vec{r},t)E^{(+)}(\vec{r},t)\rangle\,\langle E^{(-)}(\vec{r},t+\tau)E^{(+)}(\vec{r},t+\tau)\rangle}$$; 2^{nd} order coherence

Statistical comparison provides

the insight in understanding quantum physics and

its non-classical features.

Wigner function

E. Wigner, Phys. Rev. 40, 749 (1932)

Quasi-probability distribution function

$$W(x,p)$$

corresponding to classical probability

distribution over phase space (x,p)

Wigner function

E. Wigner, Phys. Rev. 40, 749 (1932)

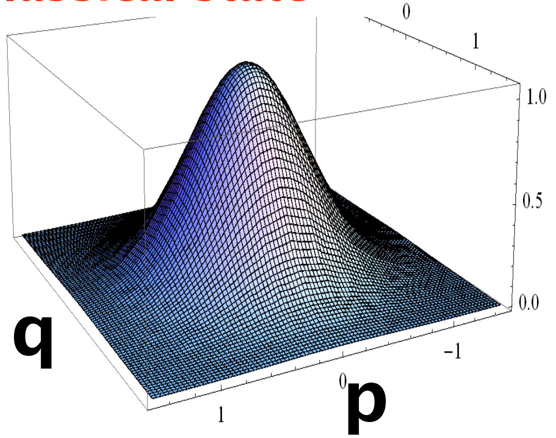
- provides every q. expectation

–real-valued

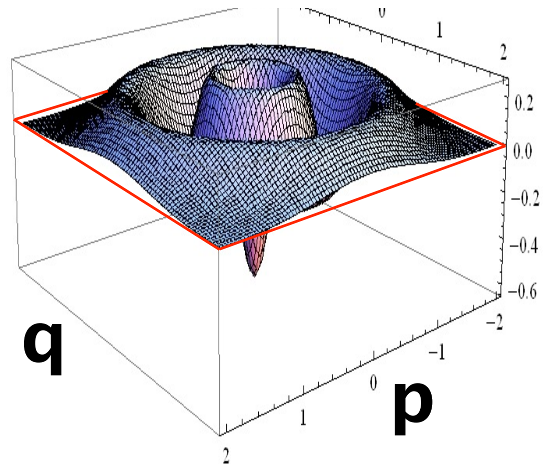
–negative due to uncertain principle
 => quasi-probability

–negativity is regarded as a signature
 of non-classical feature

classical state



nonclassical state



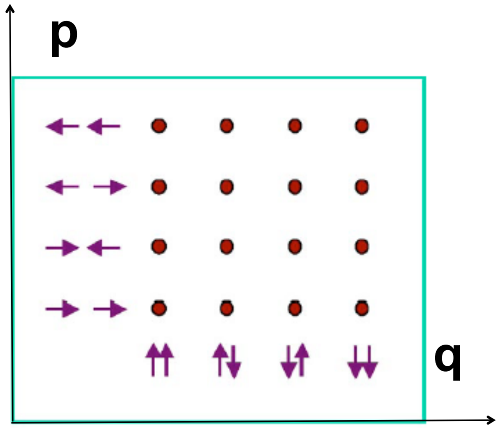
Discrete Wigner function

W. K. Wootters, Ann. Phys. (N.Y.) 176, 1 (1987)

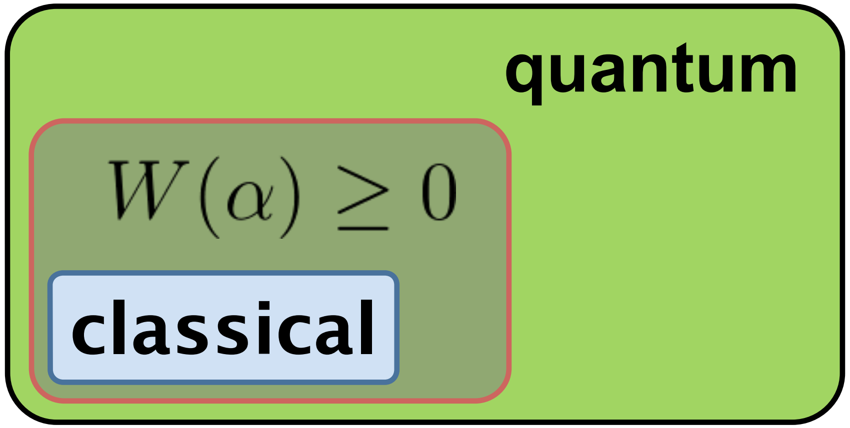
- $\hat{A}(\alpha)$; phase space point operator
- Wigner function is defined as

$$W(\alpha) = \mathrm{Tr}[\hat{A}(\alpha)\hat{\rho}]$$

- Classicality has nothing to do with hidden variables



4 x 4 phase space of two spins



Wigner function

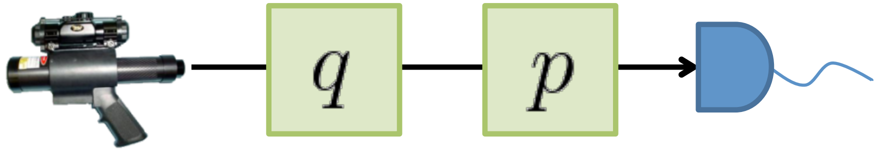
$\leftarrow\leftarrow$	$\frac{1}{4}$	0	0	0
$\leftarrow\rightarrow$	$\frac{1}{4}$	0	0	0
$\rightarrow\leftarrow$	$\frac{1}{4}$	0	0	0
$\rightarrow\rightarrow$	$\frac{1}{4}$	0	0	0
	$\uparrow\uparrow$	$\uparrow\downarrow$	$\downarrow\uparrow$	$\downarrow\downarrow$

Wigner function for $|\uparrow\uparrow\rangle$ state

Limits of Wigner function

Incommensurate with classical theory

Consecutive measurement of x and p in classical statistics



$$\Longrightarrow \langle qp \rangle = \int dq dp \, \mathrm{P}(q,p) \, q \, p$$

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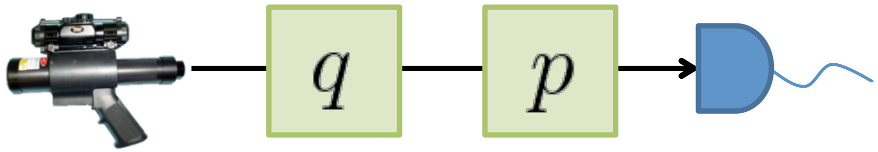

$$\Longrightarrow \langle qp \rangle = \int dq dp \, \mathrm{P}(q,p) \, q \, p$$

The same form in Wigner function

$$\int dq dp \, W(q,p) \, q \, p = \frac{1}{2} \langle \hat{q}\hat{p} + \hat{p}\hat{q} \rangle$$

implies completely other measurement.

Limits of Wigner function

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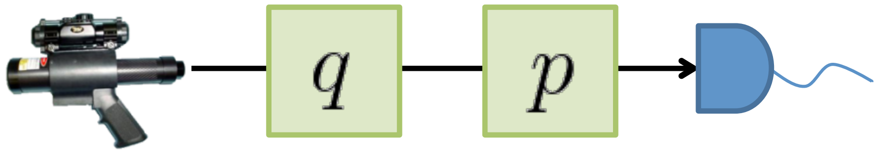
Consecutive measurement of x and p in classical statistics



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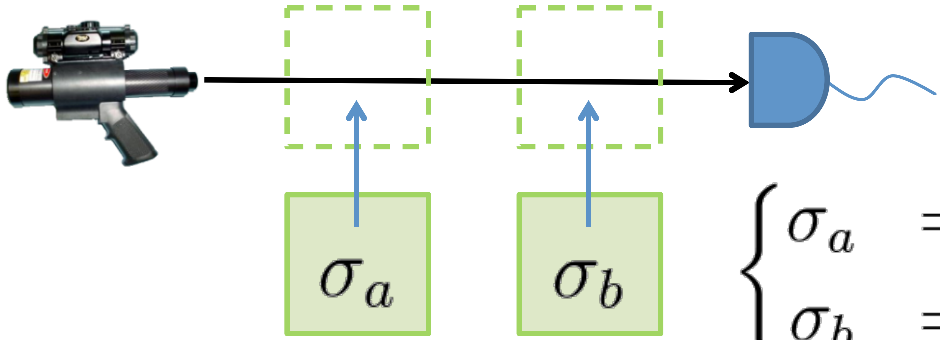
implies completely other measurement.

The functional form is incommensurate with what it stands for. Interpreting the functional forms as in classical theory leads to false conclusions.

The direct comparison of $W(q,p)$ & $\mathrm{P}(q,p)$ **is pointless.**

Commensurate approach

Spin ensemble to be observed in σ_a, σ_b, **both, or nothing.**



$$\begin{cases} \sigma_a &= \vec{a}\cdot\vec{\sigma} \\ \sigma_b &= \vec{b}\cdot\vec{\sigma} \end{cases}$$

Each σ **results in** $s = \pm 1$

avg / **obs**	$\langle e \rangle$	$\langle \sigma_a \rangle$	$\langle \sigma_b \rangle$	$\langle \sigma_a \sigma_b \rangle$
σ_a	X	O	X	O
σ_b	X	X	O	O

Classical model

Hidden variables with

- **Reality: The values of observables are predetermined before their measurements**
- **Locality: The outcomes do not depend on those spatially separated**
- **Noninvasive measurement: They are not altered by past nor future measurements**

Positive semi-definiteness

- **C. expectations for a given probability**

$$\chi_c^{nm} = \sum_{s_a, s_b = \pm 1} s_a^n s_b^m p(s_a, s_b)$$

eg) $\chi_c^{10} = \sum_{s_a} s_a p(s_a, s_b) = \langle s_a \rangle \quad \chi_c^{11} = \sum_{s_a, s_b} s_a s_b p(s_a, s_b) = \langle s_a s_b \rangle$

- **The probability function is restored by**

$$\frac{1}{2^2} \sum_{n,m} s_a^n \, s_b^m \, \chi_c^{nm} = p(s_a, s_b)$$

Positive semi-definiteness

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- **The probability function is restored by**

$$\boxed{\frac{1}{2^2} \sum_{n,m} s_a^n \, s_b^m \, \chi_c^{nm} = p(s_a, s_b)}$$

Quantum theory

Define quasi-probability function

$$w(s_a, s_b) \equiv \frac{1}{2^2} \sum_{n,m} s_a^n s_b^m \chi_q^{nm}$$

Discrte FT

$$\chi_q^{jk} = \sum_{s_1, s_2} s_a^j s_b^k w(s_a, s_b)$$

Quantum theory

Define quasi-probability function

$$w(s_a, s_b) \equiv \frac{1}{2^2} \sum_{n,m} s_a^n s_b^m \chi_q^{nm}$$

Operationally obtain q. expectations

$$\chi_q^{jk} = \sum_{s_1, s_2} s_a^j s_b^k p(s_b | s_a) p(s_a)$$

Quantum theory

All q. expectations

now have the same forms as the classical,

$$\chi_q^{nm} = \sum_{s_a, s_b} s_a^n s_b^m \, w(s_a, s_b) \quad \text{with} \quad w(s_a, s_b) \ngeq 0$$

$$\chi_c^{nm} = \sum_{s_a, s_b} s_a^n s_b^m \, p(s_a, s_b) \quad \text{with} \quad p(s_a, s_b) \geq 0$$

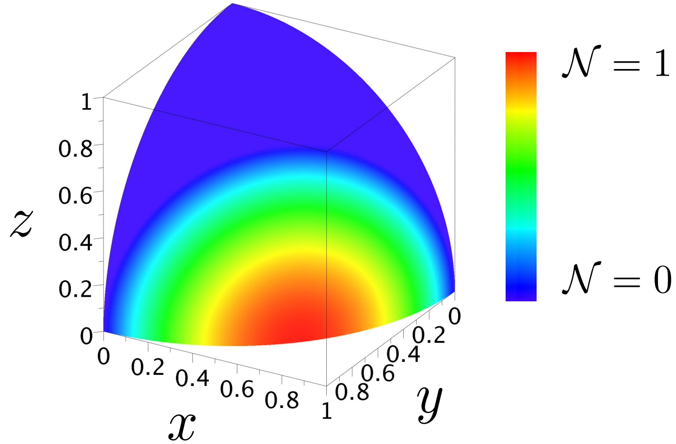
commensurate

enabling the direct comparison of both theories.

Single qubit with two obs.

- **Q. state**

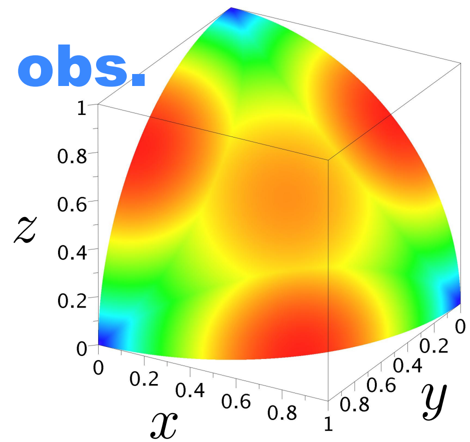
$$\hat{\rho} = \frac{1}{2}(\mathbf{1} + x\hat{\sigma}_x + y\hat{\sigma}_y + z\hat{\sigma}_z)$$



- **Commensurate QP**

$$w(s_x, s_y) = \frac{1}{4}(1 + s_x x + s_y y), \;\; s_{x,y} = \pm 1.$$

three obs.



- **Degree of nonclassicality**

$$N \equiv \sum_{\vec{s} \in \Xi} |w(\vec{s})| \;\; \text{s.t. } w(\vec{s}) < 0 \text{ for } \vec{s} \in \Xi$$

Generalization to a qudit

Define quasi-probability function

$$w(\vec{a}) = \frac{1}{2^n} \sum_{\vec{n}=0}^{d-1} \omega^{\vec{a}\cdot\vec{n}} \chi_q^{\vec{n}}$$

where $\omega = e^{i2\pi/d}$, $\vec{a} = (a_1, ..., a_k)$, and $\vec{n} = (n_1, ..., n_k)$

Obtain q. expectations operationally as

$$\chi_q^{\vec{n}} = \sum_{\vec{a}=0}^{d-1} \omega^{\vec{n}\cdot\vec{a}} p(a_k|a_{k-1}) \cdots p(a_2|a_1) p(a_1)$$

Correlation QP for two qudits

Suppose that two-qudit QP is given as

$$w(\vec{a}, \vec{b})$$

Define a marginal QP of spatial correlations

$$w_c(\vec{c}) = \sum_{\vec{a}, \vec{b}} \delta^n(\vec{c} = \vec{a} + \vec{b}) w(\vec{a}, \vec{b})$$

Inseparability in terms of MQP

For every separable state $\hat{\rho} = \sum_j p_j \hat{\rho}_j^a \otimes \hat{\rho}_j^b$

its marginal QP of spatial correlations is positive semidefinite:

$$w_c(\vec{c}) \geq 0, \quad \forall \vec{c}$$

Inseparability in terms of MQP

For a Werner state

$$\hat{\rho} = f\hat{\rho}_{\mathrm{MES}} + (1-f)\frac{I}{d}$$

the minimal value is given as

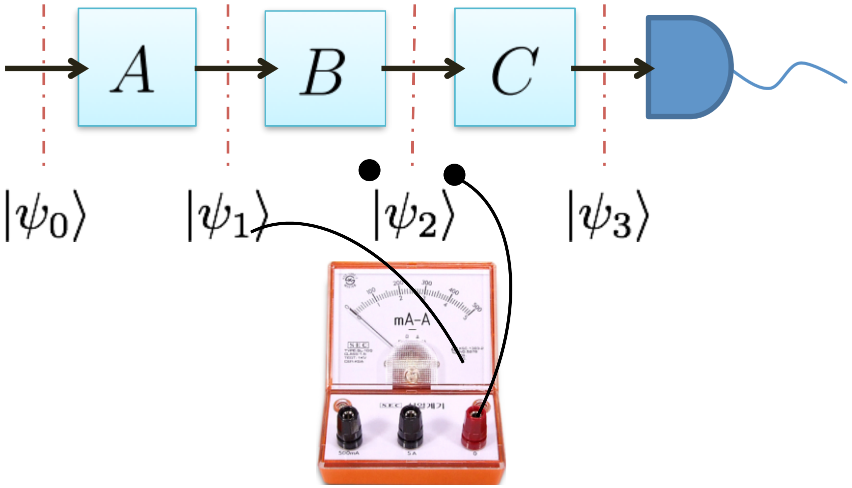
$$\min_{\mathrm{obs.}} w_c(\vec{c}) = \frac{1-(d+1)f}{d^{d+1}} < 0$$

when

$$f > 1/(d+1)$$

Quantometer

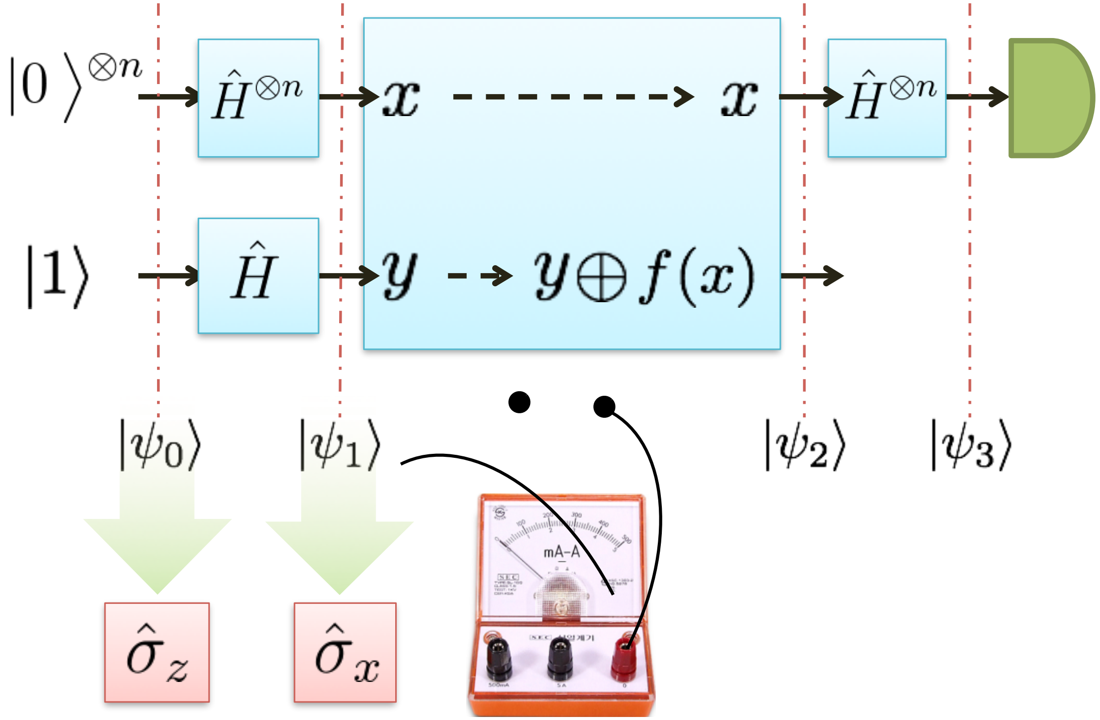
Assuming that, in information processing, c. variables satisfy macroscopic local realism, our QP tests if a given process is nonclassical.



C. Miquel, J. P. Paz, and M. Saraceno, PRA, 65, 062309 (2002).

P. Bianucci, C. Miquel, J. P. Paz, M. Saraceno, PLA, 297, 353 (2002).

Deutsch-Jozsa algorithm



$$|\psi_0\rangle = |0\rangle\,|1\rangle$$

$$|\psi_1\rangle = \left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]$$

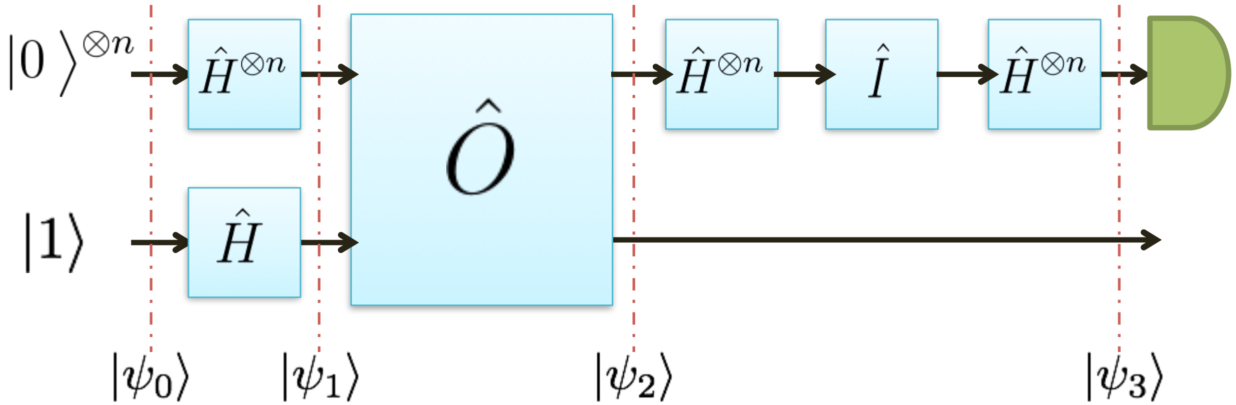
$$|\psi_2\rangle = \begin{cases} \pm\left[\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) = f(1) \\ \pm\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right]\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) \neq f(1) \end{cases}$$

$$|\psi_3\rangle = \begin{cases} \pm|0\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) = f(1) \\ \pm|1\rangle\left[\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right] & \text{if } f(0) \neq f(1) \end{cases}$$

Nonclassicality (N)

n	$\|\psi_0\rangle$	$\|\psi_1\rangle$	$\|\psi_2\rangle$	$\|\psi_3\rangle$
1 qubit	0	0	0	0
2 qubits	0	0	0	0
3 qubits	0	0	0.5	0.5

Grover's algorithm



Nonclassicality (N)

	$\lvert\psi_0\rangle$	$\lvert\psi_1\rangle$	$\lvert\psi_2\rangle$	$\lvert\psi_3\rangle$
3 qubits	0	0	0	NA
4 qubits	0	0.39	0.36	0.2

→ **nonclassical for 4 qubits**

Summary

Defined operational quasi-probability

- classical model: macroscopic local realism
- the same functional form as the classical
- any negative values refute the classical model

Generalized to qudits

- correlation QP is positive semidefinite for separable s.
- any negative values witness entanglement

Discussed possibility of quantometer