Dimensional deconstruction and skyrmions

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String-inspired modeling of QCD

Plan of the talk

- Vector meson dominance
- Hidden local symmetry: ρ, a_1
- Deconstruction and the fifth dimension
- AdS/CFT analogy
- A model: "cosh" metric
- Skyrmions

Hadrons

QCD correlations are dominated by resonances (large N_c)

- Pseudoscalar Goldstone bosons

  $\pi^a \sim i\bar{q}\gamma^5\tau^a q \sim \partial_\mu \mathscr{A}^\mu, \qquad \mathscr{A}^\mu \equiv \bar{q}\gamma^\mu\gamma^5\frac{\tau^a}{2}q,$

  $m_\pi = 0.14$ GeV, massless if $m_q = 0$.

  $\langle 0|\mathscr{A}^\mu|\pi\rangle = f_\pi p^\mu$, $f_\pi = 93$ MeV

- Vector mesons $\rho^a \sim \mathscr{V}^a_\mu \equiv \bar{q}\gamma^\mu\frac{\tau^a}{2}q,$

  $m_\rho = 0.77$ GeV $(m_\rho^2 \approx \frac{1}{2\alpha'})$.

  $\langle 0|\mathscr{V}^a_\mu|\rho^b\rangle = g_{\rho V}\epsilon_\mu\delta^{ab}$

- Axial vector mesons $a_1^a \sim \mathscr{A}^a_\mu \equiv \bar{q}\gamma^\mu\gamma^5\frac{\tau^a}{2}q,$

  $m_{a_1} = 1.23$ GeV $\approx 1.6 m_\rho$.

  $\langle 0|\mathscr{A}^a_\mu|a_1^b\rangle = g_{a_1 A}\epsilon_\mu\delta^{ab}$

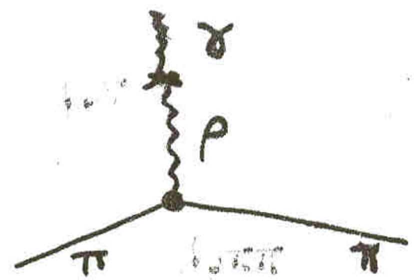
- Other resonances

Vector meson dominance (Sakurai 60s)

- ρ meson couples to other hadrons as a massive vector boson
- photon couples to hadrons through ρ

Immediate consequence:

$$\frac{g_{\rho V} g_{\rho\pi\pi}}{m_\rho^2} = 1,$$



$g_{\rho V}$ and $g_{\rho\pi\pi}$ can be determined from

$$\Gamma_{\rho^0 \to e^+ e^-} = \frac{4\pi\alpha^2}{3} \frac{g_{\rho V}^2}{m_\rho^3} = 6.85 \pm 0.11 \text{ keV}$$

$$\Gamma_{\rho \to \pi\pi} = \frac{g_{\rho\pi\pi}^2}{48\pi^2} m_\rho v_\pi^3 = 150 \text{ MeV}$$

Experimentally,

$$\frac{g_{\rho V} g_{\rho\pi\pi}}{m_\rho^2} \approx 1.2$$

Hidden local symmetry

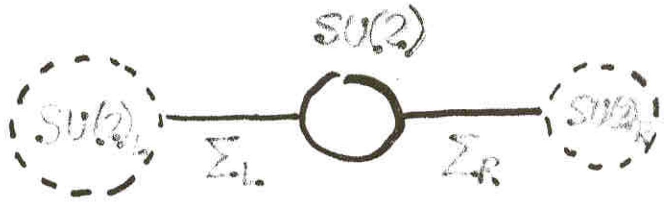
M. Harada's talk

Bando, Kugo, Yamawaki suggested that ρ is a gauge boson of a hidden local symmetry, broken by the vacuum:

$$\mathcal{L} = \frac{1}{2g^2}\mathrm{Tr}F_{\mu\nu}^2 + f^2\mathrm{Tr}(|D_\mu\Sigma_L|^2 + |D_\mu\Sigma_R|^2)$$

where

$$\begin{aligned} D_\mu\Sigma_L &= \partial_\mu\Sigma_L + \Sigma_L\rho_\mu \\ D_\mu\Sigma_R &= \partial_\mu\Sigma_R - \rho_\mu\Sigma_R \end{aligned}$$



Lagrangian invariant under SU$(2)_L \times$ SU$(2)_R \times$ SU$(2)_c$ symmetry

$$\begin{aligned} \Sigma_L &\to L\Sigma_L U^\dagger(x) \\ \Sigma_R &\to U(x)\Sigma_R R^\dagger, \qquad \Sigma = \Sigma_L\Sigma_R \to L\Sigma R^\dagger \\ \rho_\mu &\to U\rho_\mu U^\dagger + U\partial_\mu U^\dagger \end{aligned}$$

broken to SU$(2)_{L+R+c}$ by the vacuum (cf. CFL)

Hidden local symmetry (II)

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{SU}(2)_c \to \mathrm{SU}(2)_{L+R+c}$$

- 6 Goldstone bosons, 3 eaten by $\rho_\mu \Rightarrow$ 3 Goldstones (pions, $\Sigma = \Sigma_L \Sigma_R$), 3 massive vector bosons (ρ mesons).
- Georgi: if there was a limit of QCD where $m_\rho \ll \Lambda_{\mathrm{QCD}}$, then Lagrangian can be derived from chiral symmetry
- VMD: need to add term $\sim \mathrm{Tr}\partial_\mu \Sigma \partial_\mu \Sigma^\dagger$, with appropriate coefficient. Not Georgi's limit.
- Kawarabayashi-Suzuki-Riazuddin-Fayyazuddin (KSRF) ratio

$$\frac{m_\rho^2}{g_{\rho\pi\pi}^2 f_\pi^2} \sim \frac{m_W^2}{g^2 v^2}$$

is 4 in Georgi's limit. Experimentally close to the original KSRF value 2 [also =2 if $\mathrm{Tr}\partial_\mu \Sigma \partial_\mu \Sigma^\dagger$ is added]

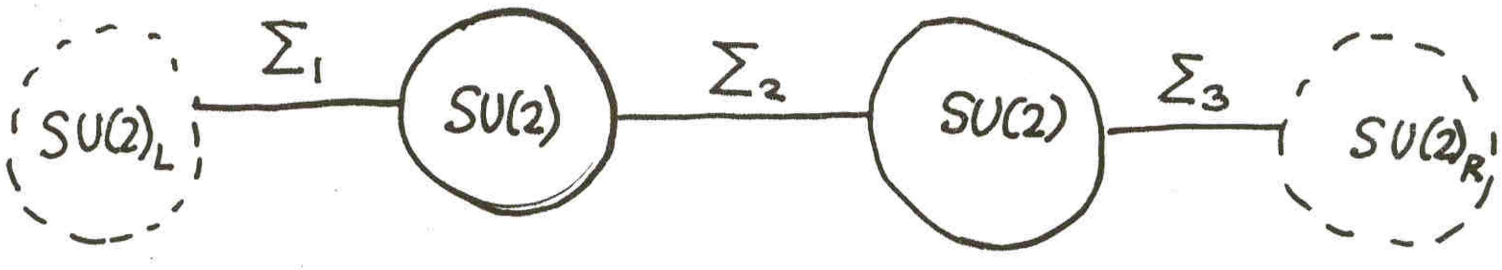
- Nice model, but not systematic if $m_\rho \sim \Lambda_{QCD}$

Hidden local symmetry with a_1

$m_{a_1} = 1.23\ \mathrm{GeV}$

Bando et al. extended the HLS construction to include the a_1 mesons

$$\mathcal{L} = f_1^2 \mathrm{Tr}(|D_\mu \Sigma_1|^2 + |D_\mu \Sigma_3|^2) + f_2^2 \mathrm{Tr}|D_\mu \Sigma_2|^2 + \frac{1}{2g^2} \mathrm{Tr}[(F^1_{\mu\nu})^2 + (F^2_{\mu\nu})^2]$$



Phenomenologically successful provided $f_1^2 = 2f_2^2$.

reproduces VMD

Questions :

- Why VMD ?
- Why hidden local symmetries model work well ?

$$\underbrace{SU(N_f)\ ,\quad SU(N_f)\times SU(N_f)}_{\underline{\text{GAUGE}}\text{ symmetries ??}}$$

Deconstruction

$$\mathcal{L} = \sum_{k=1}^{K+1} f_k^2 \mathrm{Tr}|D_\mu \Sigma^k|^2 - \sum_{k=1}^{K} \frac{1}{2} \mathrm{Tr}\left(F_{\mu\nu}^k\right)^2 .$$

where

$$\begin{aligned} D_\mu \Sigma^1 &= \partial_\mu \Sigma^1 + i\Sigma^1 (gA_\mu)^1 \\ D_\mu \Sigma^k &= \partial_\mu \Sigma^k - i(gA_\mu)^{k-1}\Sigma^k + i\Sigma^k (gA_\mu)^k, \qquad A_\mu^0 = A^{K+1} = 0 \\ D_\mu \Sigma^{K+1} &= \partial_\mu \Sigma^{K+1} - i(gA_\mu)^K \Sigma^{K+1} \end{aligned}$$

Symmetry

$$\mathrm{SU}(2)_L \times [\mathrm{SU}(2)_c]^K \times \mathrm{SU}(2)_R \to \mathrm{SU}(2)_{L+c_1+\cdots+c_K+R}$$

$\Rightarrow$ pions and a tower of K vector meson triplets.

Pions: $\Sigma = \Sigma_1 \Sigma_2 \cdots \Sigma_K$
Are they in the spectrum? $\rho(1450)$, $a_1(1640)$, $\rho(1700)$, etc.

Continuum limit: the fifth dimension

In the limit $K \to \infty$, k becomes a continuous variable u.
Σ can be thought of as a the fifth component of the gauge field,

$$\Sigma \approx 1 + iA_5$$

$\Sigma(x) = P\exp\left(i\int du\, A_5(u,x)\right)$

Theory becomes 5d YM

$$S = -\frac{1}{2g_0^2}\mathrm{Tr}\int_{-u_0}^{u_0} du \int d^4x\,\sqrt{g}\,e^{-2\phi}F_{\hat{\mu}\hat{\nu}}F^{\hat{\mu}\hat{\nu}}$$

propagating on the metric: $ds^2 = -du^2 + e^{2w(u)}\eta_{\mu\nu}dx^\mu dx^\nu$
and dilaton field: $\phi = \phi(u)$

$$f^2(u) = g_0^{-2}e^{2w-2\phi}, \qquad g^2(u) = g_0^2 e^{2\phi}.$$

Has to be understood as an effective theory. In practice, only tree graphs. OK if 5d gauge coupling is small (compared to [typical mass scale]$^{-1/2}$).

String theory and the fifth dimension of QCD?

- 60's: string theory suggested for strong interactions
- 1973: QCD discovered.
  String theory no longer a theory of strong interactions
- Mid 70's – mid 90's: QCD solidified its positions.
  Hopes for analytic understanding of confinement faded away.
  Some individuals believed string theory holds the key.
  Especially
- Polyakov: strings of QCD must propagate in 5 dimensions
- Maldacena conjecture (1997), concretized by
  Gubser-Klebanov-Polyakov and Witten: a string dual of $\mathcal{N} = 4$
  super YM.

AdS/CFT dictionary

4D gauge theory	5D gravity theory
operators	fields
dimension of operator	mass of field
global symmetries $SU(4)_R$	local 5D gauge symmetric $SU(4)$ 5D YM
triangle anomalies	5D Chern-Simon term
finite-temperature QFT	black hole
deconfinement	Hawking-Page phase trans
viscosity	graviton absorption
⋮	⋮

AdS/CFT connection

Suppose one wants to compute correlators of conserved currents. One turns on background gauge fields A_μ^L and A_μ^R,

$$\begin{aligned}\mathcal{L} = & f_1^2 \mathrm{Tr}|\partial_\mu \Sigma^1 - iA_\mu^L \Sigma^1 + i\Sigma^1 A_\mu^1|^2 + \cdots \\ & + f_{K+1}^2 \mathrm{Tr}|\partial_\mu \Sigma^{K+1} - iA_\mu^K \Sigma^{K+1} + i\Sigma^{K+1} A_\mu^R|^2\end{aligned}$$

One can think of the background fields as

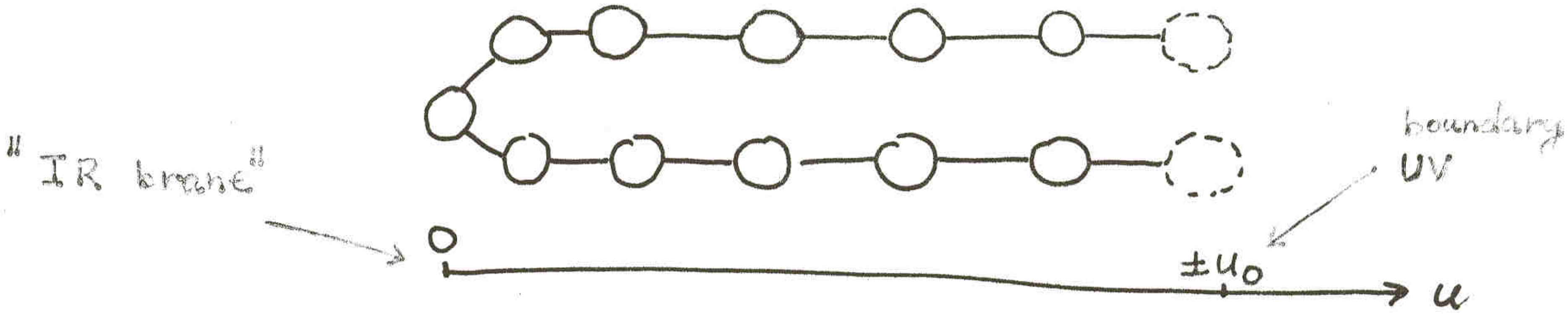
$$A_\mu^0 = A_\mu^L \qquad A_\mu^{K+1} = A_\mu^R$$

or in the continuum limit: $A_\mu(-u_0) = A_\mu^L, \quad A_\mu(u_0) = A_\mu^R$

$$\Rightarrow \boxed{Z[A_\mu^L, A_\mu^R] = e^{iS_{\mathrm{cl}}[A_\mu^{\mathrm{cl}}]}}$$

where A_μ^{cl} is the solution to the classical field equation that satisfies the boundary conditions. Similar to AdS/CFT.

AdS/CFT connection. II: Folding the moose



- Fifth dimesion $(0,\, u_0)$ instead of $(-u_0,\, u_0)$
- Two gauge fields: A^L_μ, A^R_μ, matching condition at $u = 0$. Natural from gauge/gravity duality point of view: conserved currents in 4d $\leftrightarrow$ gauge fields in 5d.
- Chiral symmetry breaking: by an infrared brane at $u = 0$, pion field as position of the brane in internal space

Physical observables in the continuum limit

Mass spectrum of vector mesons is determined by equation

$$g(f^2(gb_n)')' = -m_n^2 b_n$$

AdS/CFT:
hadrons= normalizable modes

these are ρ $(n=1)$, a_1 $(n=2)$, ρ' $(n=3)$, a_1' $(n=4)$, etc.

Pion decay constant: $$\frac{4}{f_\pi^2} = \int_{-u_0}^{+u_0} \frac{du}{f^2(u)}$$

$n \to \pi\pi$ amplitude: $$g_{n\pi\pi} = \frac{f_\pi^2}{4} \int_{-u_0}^{+u_0} \frac{du}{f^2(u)}\, g(u) b_n(u)$$

Vector meson decay constant: $$g_{nV,A} = -f^2(gb_n)'|_{u=u_0}$$

?

$\langle 0|V_\mu|n\rangle$

$\langle 0|A_\mu|n\rangle$

Vector meson dominance

Pion form factor

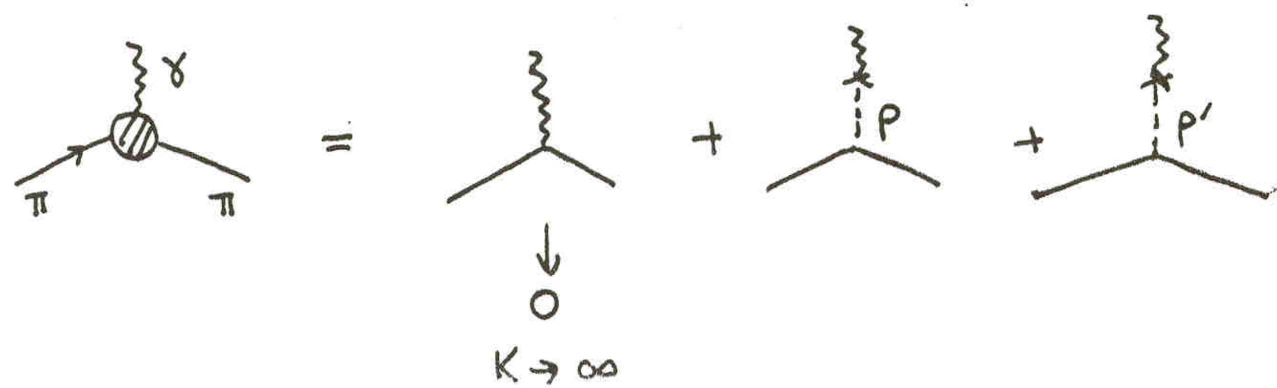
$$\langle \pi^a(p') | \mathscr{V}^c_\mu | \pi^b(p) \rangle = G_{V\pi\pi}(q) \epsilon^{abc} (p + p')_\mu \qquad q = p' - p$$

In the moose model:

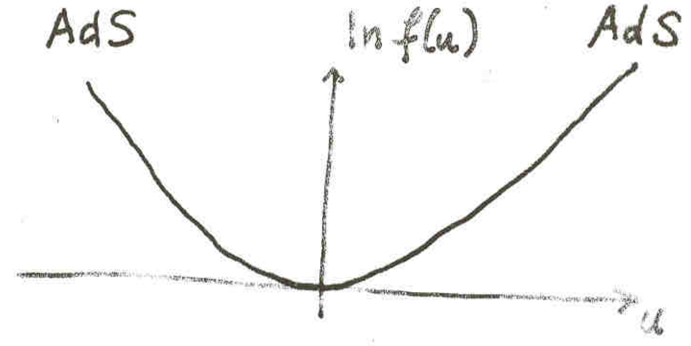
$$G_{V\pi\pi}(q) = \frac{f_\pi^2}{4 f_1^2} + \sum_n \frac{g_{nV} g_{n\pi\pi}}{Q^2 + m_n^2}$$

In the limit $K \to \infty$: $f_\pi \ll f_1$, pion form-factor given by sum over resonances.

VMD by a whole tower of vector mesons.



cosh metric



Choose $$f(u) = g_5^{-1} \Lambda \cosh u$$
$$g(u) = g_5$$

or equivalently $$ds^2 = -du^2 + \Lambda^2 \cosh^2 u \, dx^\mu dx_\mu$$

Two AdS$_5$ boundaries at $u \to \pm\infty$

$$ds^2 = -du^2 + e^{2u} dx^\mu dx_\mu = \frac{-dz^2 + dx^\mu dx_\mu}{z^2}, \qquad z = e^{-u}$$

match smoothly at $u = 0$.

- AdS boundaries important for matching to parton model
- No reason to prefer this metric over others, except for simplicity and exact solvability

Vector mesons spectroscopy

Hadronic spectrum: from Schrödinger's equation

$$(\cosh^2 u\, b_n')' = -\frac{m_n^2}{\Lambda^2} b_n$$

Solutions are

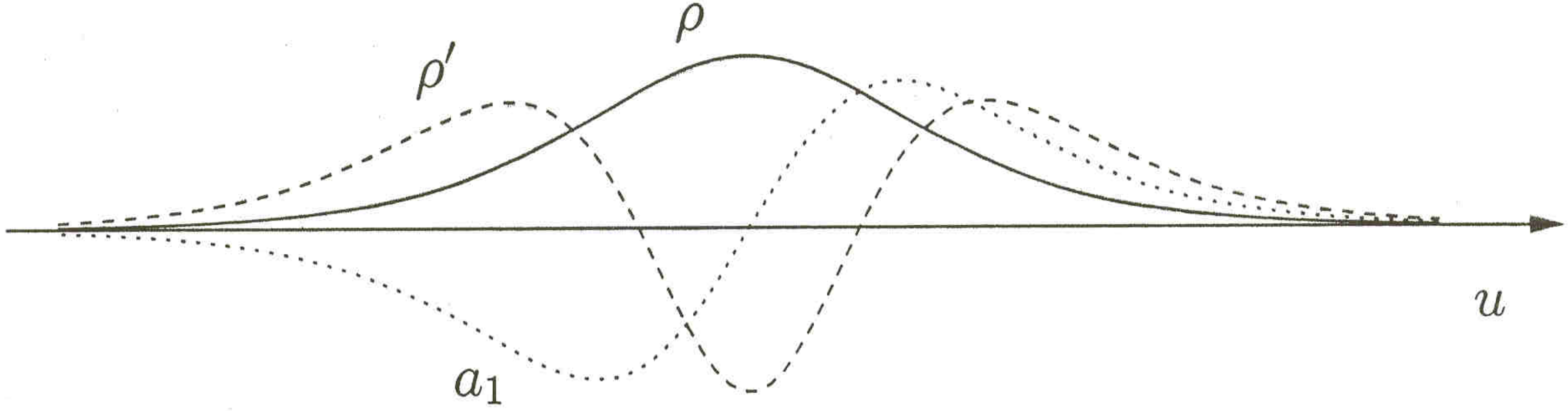
$$m_n^2 = n(n+1)\Lambda^2, \qquad b_n(u) = -c_n \frac{P_n^1(\tanh u)}{\cosh u}$$

In particular, $m_{a_1}/m_\rho = \sqrt{3}$. In the real world, $m_{a_1}/m_\rho \approx 1.6$

But: at large n, $m_n \sim n$ while theoretical prejudice is $m_n \sim n^{1/2}$.
For example: $m_{\rho'}/m_\rho = \sqrt{6}$ in the model, only 1.9 in real world.
Will close our eyes on this deficiency.

Wave functions in 5d

hadron = normalizable modes



- n-th meson: wave function has $n-1$ nodes.
- Couplings between mesons given by overlap integrals between wave functions,

$$g_{mnp} = \int du\, g(u)\, b_m(u) b_n(u) b_p(u)$$

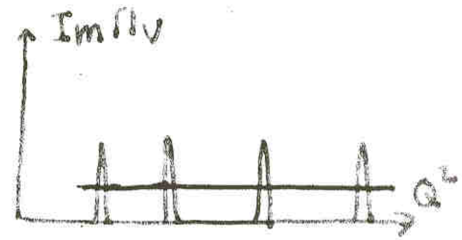
Matching to parton model

Decay constant of vector mesons:

$$g_{nV,A} = \sqrt{2n(n+1)(2n+1)}\,\frac{\Lambda^2}{g_5}$$

Define polarization operator:
$\langle \mathcal{V}^a_\mu(q)\mathcal{V}^b_\nu(-q)\rangle = \Pi_V(Q^2)(q_\mu q_\nu - q^2 g_{\mu\nu})$
then $\Pi_V(Q^2)$ is given by sum over resonances,

$$\Pi_V(Q^2) = \frac{2\Lambda^2}{g_5^2}\sum_{n\ \mathrm{odd}} \frac{2n+1}{Q^2+n(n+1)\Lambda^2}$$



At large Q^2, this becomes

Matching to QCD

$$\Pi_V(Q^2) = -\frac{1}{g_5^2}\ln(Q^2) = -\frac{N_c}{24\pi^2}\ln(Q^2) \longleftarrow \text{AdS boundaries}$$

$\Rightarrow g_5^2 = 24\pi^2 N_c^{-1}$. All quantities now can be expressed in term of m_ρ and N_c.

$$S = \int d^5x \sqrt{-g} \left(F_L^2 + F_R^2 + |\partial_\mu \Phi - A_L \Phi + \Phi A_R|^2 \right.$$
$$\left. + \Phi^\dagger \Phi (F_L^2 + F_R^2) + \cdots \right)$$

Match coefficients to OPEs of QCD

$$\Pi_A(Q^2) - \Pi_V(Q^2) \sim \frac{\langle \bar{q} q \rangle^2}{Q^6}$$

(was $\sim e^{-Q}$ in "open moose")

Phenomenology of pi and rho mesons

- Pion decay constant:

$$f_\pi^2 = \frac{2\Lambda^2}{g_5^2} = \frac{N_c}{24\pi^2} m_\rho^2$$

For $N_c = 3$, $f_\pi = 87$ MeV (exp. 93 MeV).

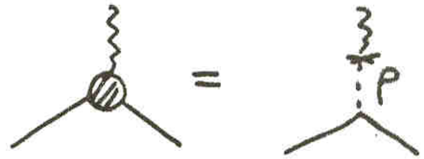
- Electromagetic width of ρ:

$$\Gamma_{\rho^0 \to e^+ e^-} = \frac{\alpha^2 N_c}{6\pi} m_\rho = 6.5 \text{ keV}$$

(exp. 6.85 ± 0.11 keV)

- Exact ρ-pole dominance of the pion form-factor:

$$g_{n\pi\pi} = 0, \qquad n = 1$$



due to a pecularity of the wave functions in fifth dimension.
$G_{V\pi\pi}(q) \sim (Q^2 + m_\rho^2)^{-1}$.

A pecularity of cosh model

$$g_{n\pi\pi} \sim \int du \frac{1}{f^2(u)} b_n(u)$$

By accident,

$$\frac{1}{f^2(u)} = b_1(u)$$

Orthogonality of wave functions means that $g_{n\pi\pi} = 0$ for $n > 1$

Comparison: Shifman-Vainshtein-Zakharov sum rules

SVZ sum rules: approximate matching between correlators calculated from QCD and resonance model

In simplest version gives

$$f_\pi^2 = \frac{N_c}{24\pi^2} m_\rho^2$$

same as our result.

$$\Gamma_{\rho^0 \to e^+e^-} = \frac{\alpha^2 e N_c}{18\pi} m_\rho$$

differs from our result by just $e/3$.

Lessons

- Single ρ-pole dominance in pion form-factors is accidental, specific for cosh metric
- The coupling of n-th meson to pions is given by an overlap integral

$$g_{n\pi\pi} = \frac{f_\pi^2}{4} \int_{-u_0}^{+u_0} \frac{du}{f^2(u)}\, g(u) b_n(u)$$

  For large n $b_n(u)$ is a rapidly oscillating function
  $\Rightarrow$ $g_{n\pi\pi}$ suppressed
  $\Rightarrow$ pion form-factor dominated by a few first poles.
- f_π/m_ρ and $g_{\rho V}$: closeness to exp. values somewhat a surprise.
- $g_{a_1 A}$ 50% too large compared to experiment/lattice. Predictions should worsen for higher resonances.

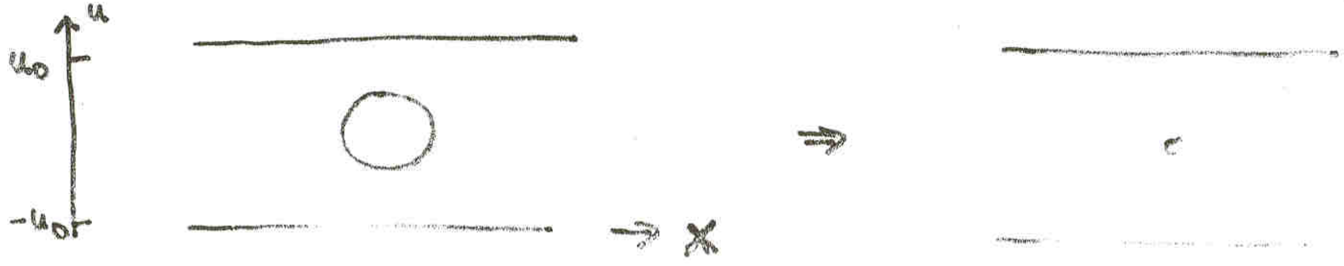
Skyrmion = instanton

- Instantons in 4d become to quasiparticles in 5d
- $A_5 = \dfrac{\tau \cdot x}{x^2(x^2+\rho^2)}$ looks like a hedgehog $\Sigma(x) = \exp\left(i \int du\, A_5(u,x)\right)$
- Instanton topological charge is equal to Skyrmion charge,

$$\frac{1}{32\pi^2}\int du\, d^3x\, \epsilon^{0\mu\nu\lambda\rho}\mathrm{Tr}F_{\mu\nu}F_{\lambda\rho} =$$
$$= \frac{i}{24\pi^2}\int d^3x\, \epsilon^{ijk}\mathrm{Tr}[(\Sigma^{-1}\partial_i\Sigma)(\Sigma^{-1}\partial_j\Sigma)(\Sigma^{-1}\partial_k\Sigma)]$$

$\Rightarrow$ instantons = Skyrmions = baryons

- Skyrmion mass $\sim$ instanton action: Skyrmion mass independent of size if small; needs stabilization.



Stabilization of skyrmions

- Trivial extension to include isoscalar: η, ω, f_1 mesons: replace SU(2) by U$(2) \sim$ SU$(2) \times$ U(1)
- Add 5d Chern-Simons terms, e.g.

$$\int d^5x \, \epsilon^{\mu\nu\lambda\rho\sigma} A_\mu F^a_{\nu\lambda} F^a_{\rho\sigma}$$

  - Coefficients fixed ($\sim N_c$) by matching to QCD anomalies
  - Couples baryon charge to ω meson and higher resonances: hard-core repulsion betwen baryons $\omega_\mu B_\mu$
- Stabilizes Skyrmions, preventing it from shrinking to a point.
- Skyrmions are prevented from swelling by curved spacetime
- Need to be numerically studied

An extension

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to appear

4D gauge theory	5d gravity
operators	fields
$\bar{q}_L \gamma^\mu q_L$	A^L_μ
$\bar{q}_R \gamma^\mu q_R$	A^R_μ
$(\bar{q}q,\ \bar{q} i\gamma^5 \tau^a q)$	Φ
dimension	mass
$[\bar{q}q] = 3$	$m^2_\Phi = -\frac{3}{R^2}$
m_q	$\frac{1}{z}\Phi \big\vert_{z\to 0}$
$\langle \bar{q}q \rangle$	$\frac{1}{z}\left(\frac{\Phi}{z}\right)' \big\vert_{z\to 0}$

Summary

- We have seen connection between
  - Models based on hidden local symmetry
  - Ideas of gauge/gravity duality, 5th dimension

  via deconstruction.
- Simplest open-moose model gives phenomenologically interesting results.
- To do:
  - Include explicit breaking of chiral symmetry
  - Include scalar mesons (σ), excited π's
  - Find models with Goldstones which have gravity dual.