Parity of Pentaquarks from QCD sum rules

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1. **Why is there a controversy**
2. **What is the “2 hadron reducible KN intermediate states”**
3. **Reanalysis of QCD sum rule**
4. **Summary**

Theoretical Controversy over Parity of $\Theta+$

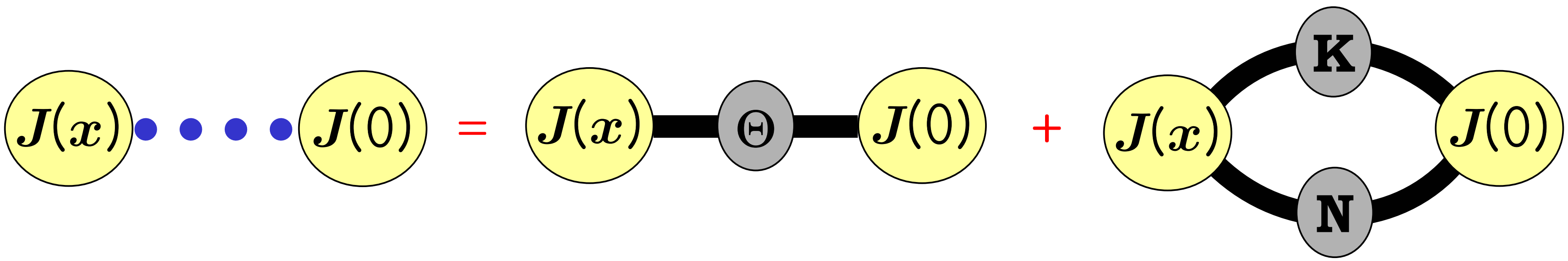
Non perturbative method to determine $\Theta+$ parity

Lattice QCD	*QCD sum rules*
Csikor, Fodor, Katz, Kovacs : Hep-lat/0309090, Sasaki, Hep-lat/03100014, Takahashi, Kunihiro, pentaquark04 → Predict "–" Parity	Sugiyama, Doi, Oka: Hep-ph/0309271 → Predict "–" Parity
Mathur, F.Lee, et.al., Hep-ph/0406196 Tokyo. Inst. Tech.,. pentaquark04 → No signal for Θ	Kondo, Morimatsu, Nishikawa Hep-ph/0404285 → Predict "+" Parity

Common Problem

How big is the “K-N 2 Hadron reducible contributions” ?

Lattice QCD	QCD sum rules
$< J_{\Theta}(0), J_{\Theta}(t) >_{lattice}$ Large t $= \lambda_1 \exp(-t m_{\Theta}) + \lambda_2 \exp(-tE_{KN}) + ..$	$\Pi(q^2) = \int dx \exp(iqx) < J_{\Theta}(x), J_{\Theta}(0) >_{OPE}$ Borel transformation $= \lambda_1 \exp(-m_{\Theta} / M^2) + \lambda_2 \exp(-E_{KN} / M^2) + ..$



QCD sum rule

$$\Pi_{OPE}(q^2) = \sum c_n \frac{< O_{2n} >}{q^{2n}} = \int ds \frac{\rho_{phen}(s)}{s - q^2}$$

Borel Transformation

$$\sum c_n \frac{< O_{2n} >}{n! M^{2n}} = \int ds \exp(-s / M^2) \rho_{phen}(s)$$

OPE is obtained from
$-q^2$ → large expansion

ρ_{phen} (s) should include
low energy states

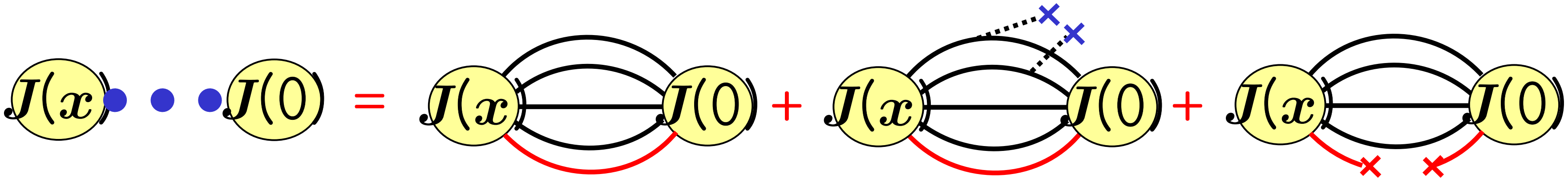
If Independent of M

Extract ground state properties such as m_Θ

Sugiyama, Doi, Oka Sum rule : OPE

■ **OPE calculation of** $\Pi(q^2) = \int dx \exp(iqx) < J_\Theta(x), J_\Theta(0) >$

valid at $-q^2 \rightarrow$ large



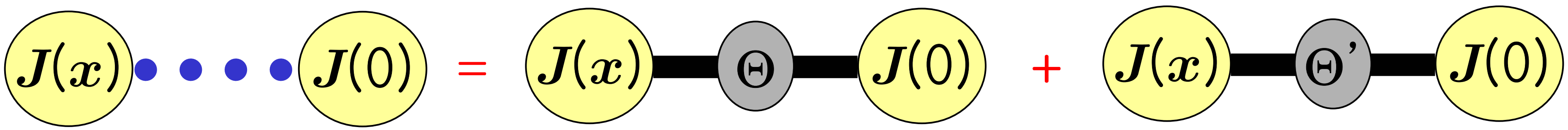
$$\Pi(q^2)_{OPE} \xrightarrow{-q^2\to\infty} c_0 \log(-q^2) + c_4 \frac{<\alpha_s G^2>}{q^2} + c_6 \frac{<m_s \bar{s} G s>}{q^6} + ..$$

Good OPE : *no contribution from* $\frac{<(\bar{u}u)(\bar{d}d)>}{q^6}$

Sugiyama, Doi, Oka Sum rule : Phen. side

■ **Phenomenological side**

$$\Pi(q^2) \rightarrow B.T.\int dx \exp(iqx) < J_\Theta(x), J_\Theta(0) >= \sum_n \lambda_n \exp(-m_n / M^2)$$



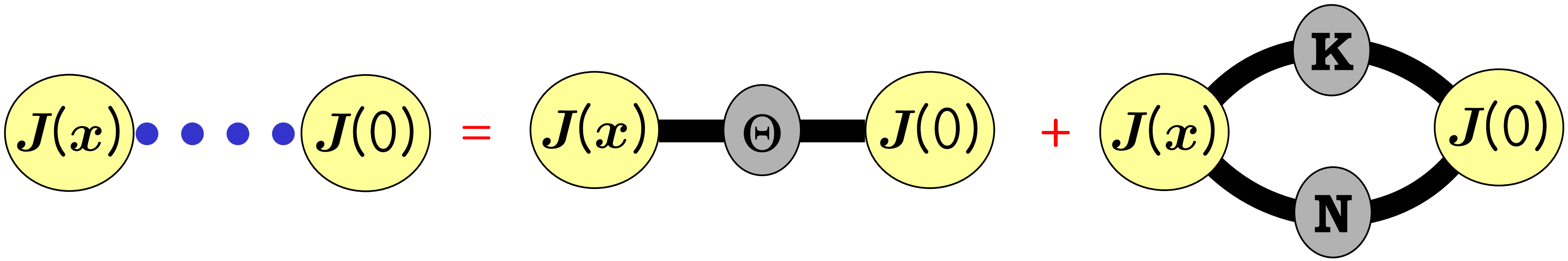
$$\sum_n \lambda_n \exp(-m_n / M^2) \xrightarrow{M^2 \rightarrow small} \lambda_1 \exp(-m_\Theta / M^2) + c \int_{S_0 > m_\Theta} ds \exp(-s / M^2)$$

All excited states are included in the $S_0 = (1.8\text{GeV})^2 > m_\Theta$

comparing with OPE → **P = -**

Kondo, Morimatsu, Nishikawa:

- **Correlation should include the "K-N 2 Hadron reducible contribution"**



$$\Pi(q^2) = \Pi^{2Hadron-Irreducible}(q) + \int \frac{d^4p}{(2\pi)^4} \Pi_N(p)\Pi_K(p-q)$$

It is important to include the K−N state, because $E_{KN} < m_{\Theta}$

→ **Question is how and where?**

Kondo, Morimatsu, Nishikawa: in the OPE side

$$\Pi_{OPE}(q^2) - \lambda \int \frac{d^4 p}{(2\pi)^4} \Pi^N_{OPE}(p) \Pi^N_{OPE}(p-q) = \Pi_{phen}^{2Hadron-Irreducible}$$

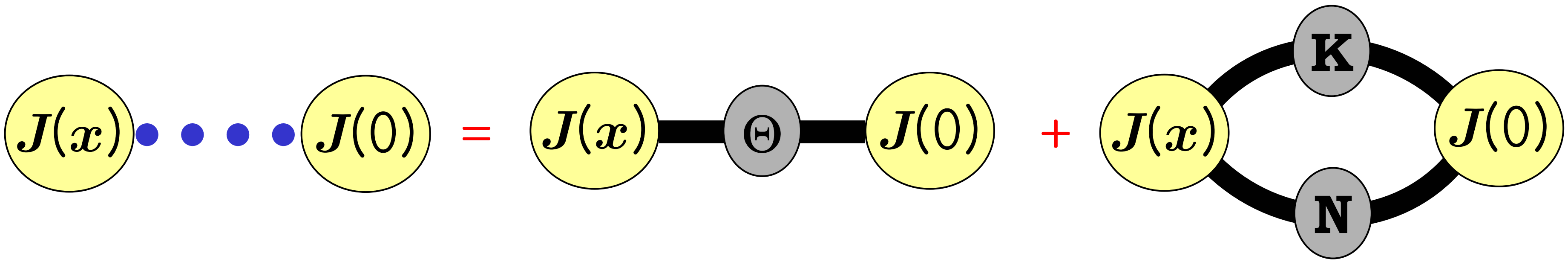
$\Pi_{OPE}(q^2)$	$\Pi^N_{OPE}(p)$	$\Pi^N_{OPE}(p-q)$	
Large $-q^2$	Large $-p^2$	Large $-(p-q)^2$	← OPE by KMN with $\lambda=1$
Large $-q^2$	Large $-p^2$	Large $-(p-q)^2$	← OPE in Sum rule should be obtained for Large $-q^2$
	Small $-p^2$	Large $-(p-q)^2$	

Moreover, it is not clear how big λ should be?

$$< 0 | J_K(x) J_N(x) | KN > \neq < 0 | J_K(x) | K > \times < 0 | J_N(x) | N >$$

Alternatively way of including K-N 2HR,

- **One could put K-N 2 Hadron reducible contribution in the phenomenological side,**



$$\Pi_{OPE}(q^2) = \Pi^{2Hadron-Irreducible}(q) + \int \frac{d^4p}{(2\pi)^4} \Pi_N(p)\Pi_K(p-q)$$

$$\lambda_N \lambda_M i \int \frac{d^4p}{(2\pi)^2} \frac{i}{\not{p} - m_N} \frac{i}{(p-q)^2 - m_K^2} \lambda_N^* \lambda_M^*$$

Where, the only unknown is $\lambda_N \lambda_M = <0|J_\Theta|KN>$

Reanalysis of QCD sum rules

- **Use same J_Θ as in Sugiyama, Doi, Oka sum**

$$J_\Theta = \varepsilon^{abc}\varepsilon^{def}\varepsilon^{cfg}(u_a^T C d_b)(u_d^T C\gamma^5 d_e)C\overline{s}_g^T$$

- **Our estimate of Matrix element**

$$<0|J_\Theta|KN> = -\frac{i}{f_\pi}<0|[Q_5^K, J_\Theta]|N> = -\frac{i}{f_\pi}<0|J_N|N>$$

($<0|J_N|N>$ → λ_N)

Where,

$$J_N = \varepsilon^{abc}\varepsilon^{def}\varepsilon^{cfg}\{(u_a^T C d_b)(u_d^T C\gamma^5 d_e)C\gamma^5\overline{d}_g^T + (u_a^T C\gamma^5 s_b)(u_d^T C\gamma^5 d_e)C\overline{s}_g^T + (u_a^T C d_b)(u_d^T C d_e)C\overline{s}_g^T\}$$

λ_N Can be obtained from

$$\Pi_N(p) = i\int d^4x \exp(ipx) <0|J_N(x), \overline{J}_N(0)|0>$$

Does it really couple to the ground state nucleon with positive parity

Estimating λ_N

- $$\Pi_N(p) = i\int d^4x \exp(ipx) < 0 | J_N(x), \bar{J}_N(0) | 0 >= -\gamma^5 \{\Pi_\gamma \not{p} + \Pi_1\} \gamma^5$$

$$\frac{1}{\pi}\mathrm{Im}\,\Pi_\gamma = A = 3\times\frac{q_0^{11}}{5!5!2^{10}7\pi^8} + 4\times\frac{q_0^7}{3!5!2^8\pi^6} m_s < \bar{s}s > + 3\times\frac{q_0^7}{3!5!2^{10}\pi^6} < \frac{\alpha_s}{\pi}G^2 > - 4\times\frac{q_0^5}{4!3!2^9\pi^6} m_s < \bar{s}Gs >$$

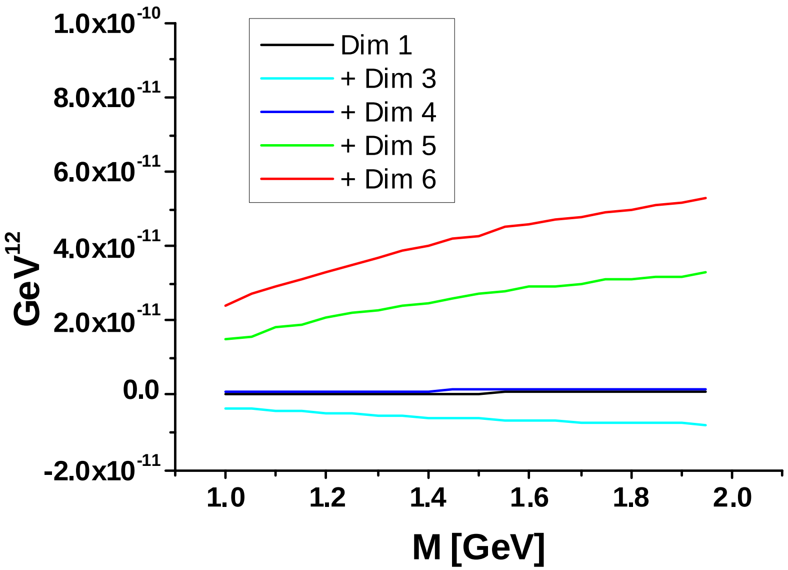
$$\frac{1}{\pi}\mathrm{Im}\,\Pi_1 = B = 2\times\frac{q_0^{10} m_s}{5!5!2^{10}\pi^8} - \frac{q_0^8}{4!5!2^7\pi^6}(2 < \bar{s}s > - < \bar{d}d >) + \frac{q_0^6}{4!3!2^9\pi^6}(2 < \bar{s}Gs > - < \bar{d}Gd >)$$

These terms gives positive parity as expected for nucleon

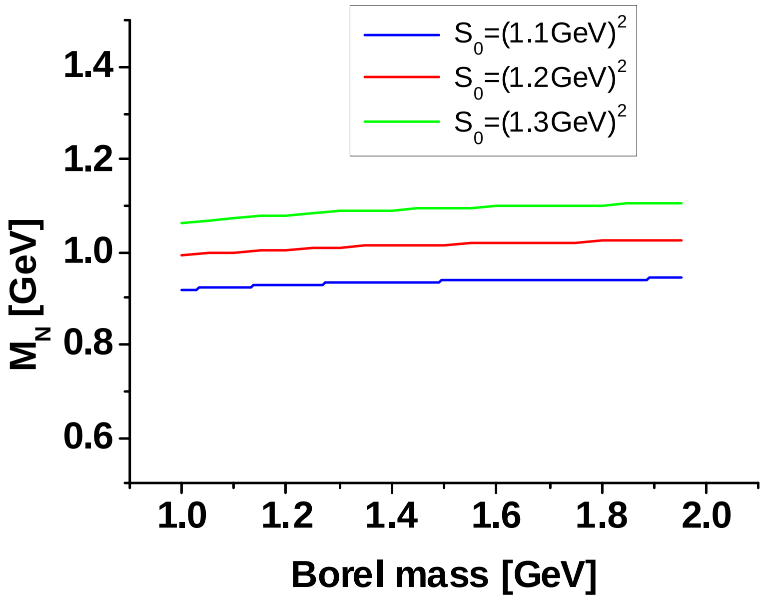
Just as the corresponding term gives negative parity for the Θ

Indeed positive parity

$$\left|\lambda_N^+\right|^2 \exp(-m_N^2 / M^2)$$



$$m_N^2$$

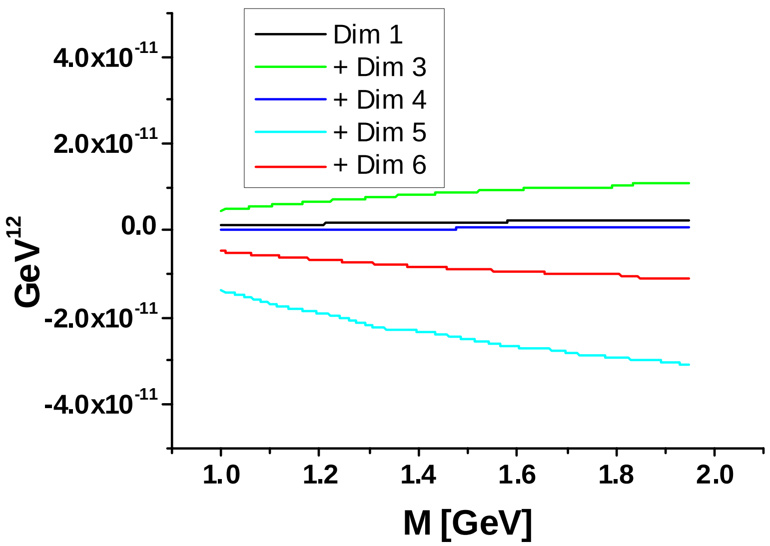


Consistent result for + parity and mass for nucleon with

$$\left|\lambda_N^+\right|^2 \approx 10^{-10} \ \ \mathrm{GeV}^{12}$$

If negative parity

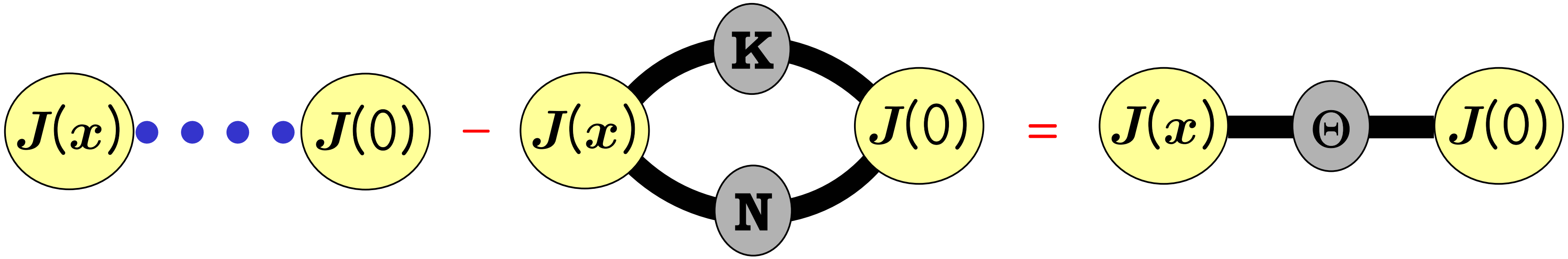
$$\left|\lambda_N^-\right|^2 \exp(-m_N^2 / M^2)$$



$\left|\lambda_N^-\right|^2 < 0$ → gives Inconsistent Result

Hence J_N couples to ground state nucleon, which has positive parity with $\left|\lambda_N^+\right|^2 \approx 10^{-10}\ \text{GeV}^{12}$

Reanalysis of Sugiyama, Doi, Oka sum rule after subtracting the K-N 2HR contribution

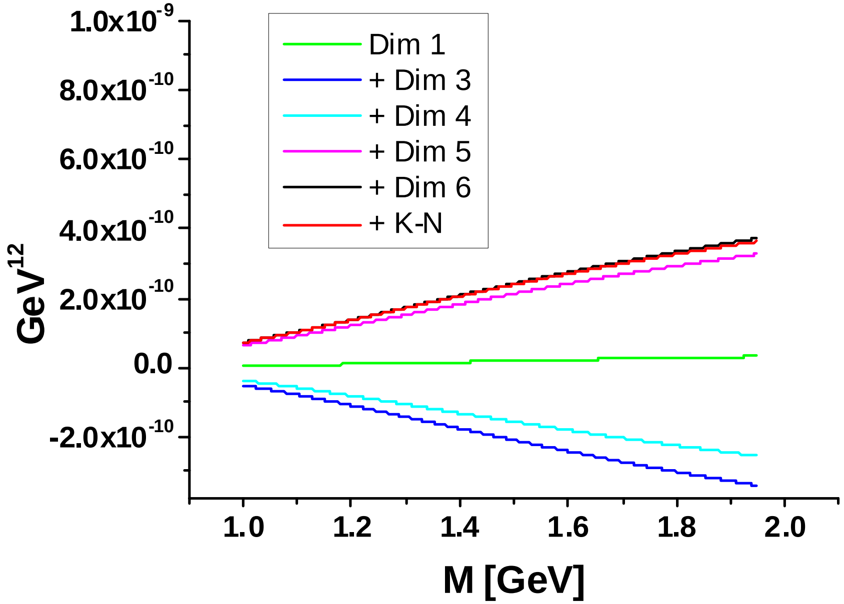


$$\Pi_{OPE}(q^2) - \int \frac{d^4p}{(2\pi)^4} \Pi_N(p)\Pi_K(p-q) = \Pi^{2Hadron-Irreducible}(q)$$

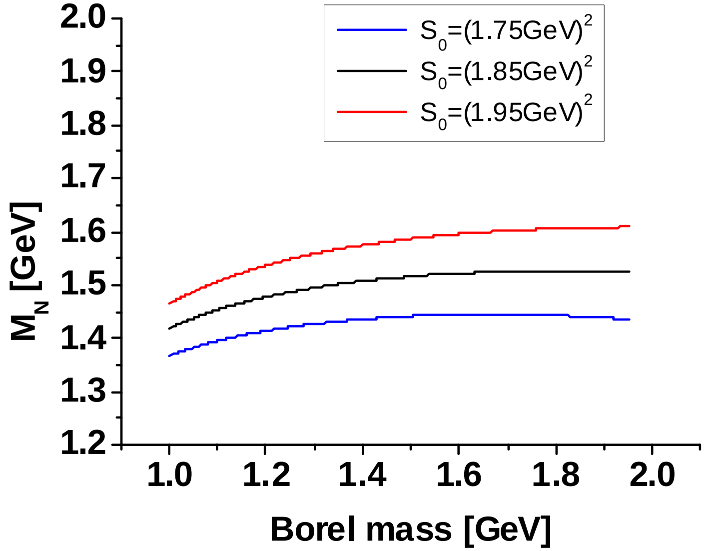
$$-\frac{\left|\lambda_N^+\right|^2}{f_\pi^2}\gamma^5 i\int \frac{d^4p}{(2\pi)^2}\frac{i}{\not{p}-m_N}\frac{i}{(p-q)^2-m_K^2}\gamma^5$$

QCD sum rules for Θ with K-N 2HR cont.

$$\left|\lambda_{\Theta}^{+}\right|^{2} \exp(-m_{\Theta}^{2} / M^{2})$$



$$m_{\Theta}^{2}$$

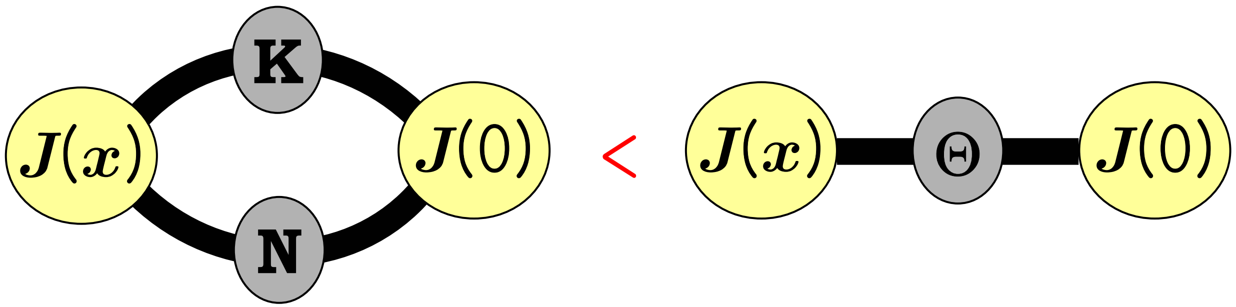


Reanalysis shows

1. K-N 2HR contributions constitutes less than 5% of the OPE. Hence QCD sum rule predicts negative parity for the Θ as in the original Sugiyama, Doi, Oka sum rule

2. Why is the K-N contribution so small?

$$\left| < 0 \,|\, J^5_\Theta \,|\, \Theta > \right|^2 = 10 \times \left| < 0 \,|\, J^5_N \,|\, N > \right|^2$$

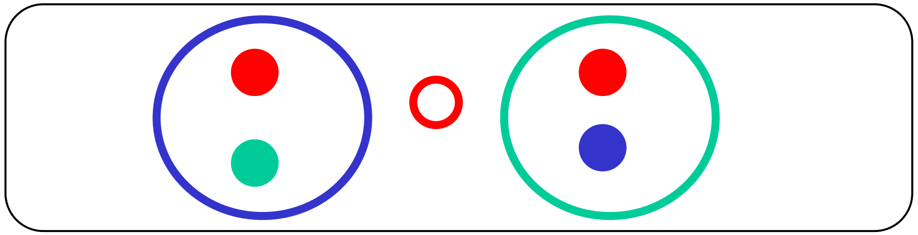


Could this be an artifact of the current

- Jaffe Wilczek

L=0, S=1/2 L=1 → P=+

Θ



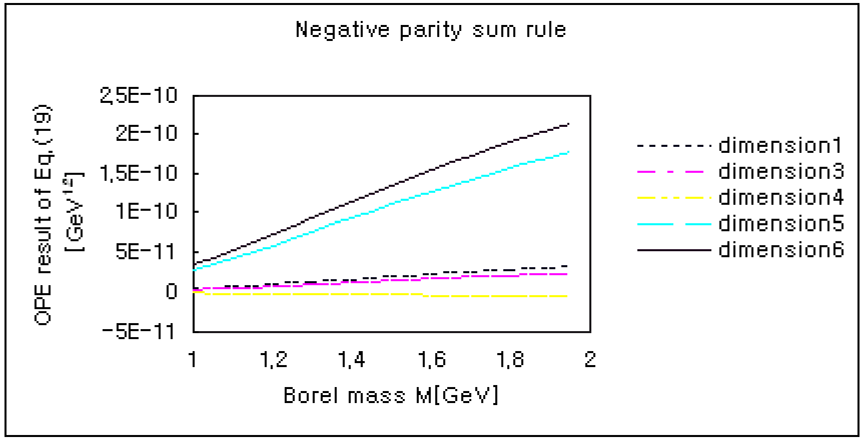
$$J_{\Theta} = \varepsilon^{abc}\varepsilon^{def}\varepsilon^{cfg}(u_a^T C d_b)(u_d^T C\gamma^5 d_e)C\overline{s}_g^T$$

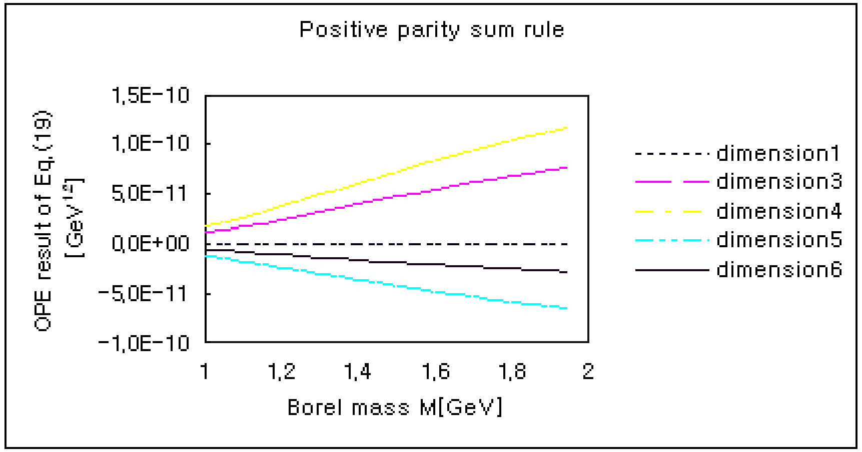
Jaffe Wilczek picture is better represented by (hep-ph/0401034)

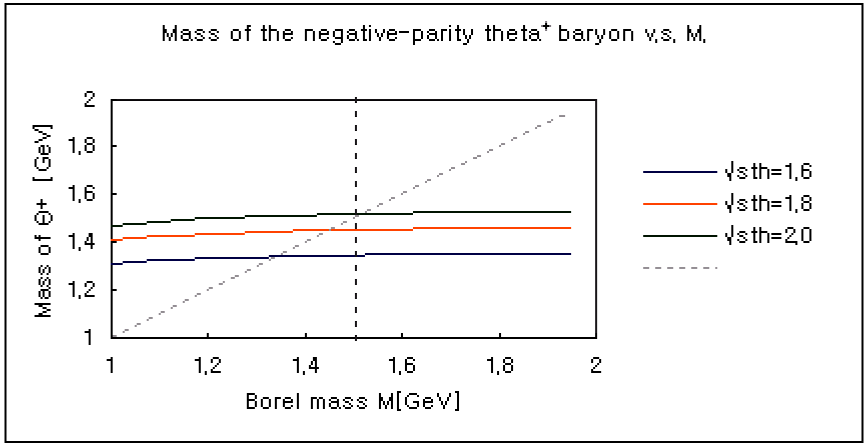
$$J_{\Theta} = \varepsilon^{abc}\varepsilon^{def}\varepsilon^{cfg}(u_a^T C\gamma^5 d_b)(D_\mu[u_d^T C\gamma^5 d_e])\gamma^\mu\gamma^5 C\overline{s}_g^T$$

Repeated the QCD sum rule with this current (Y. Kwon, B. Lee, SHL 04)

Again, predict negative parity







Summary

1. Reanalyzed the QCD sum rule for Θ with direct coupling to K-N states → parity is still negative

   as a by product

   Nucleon sum rule with 5 quark component →positive parity and $\left| < 0 | J_{\Theta}^{5} | \Theta > \right|^2 = 10 \times \left| < 0 | J_{N}^{5} | N > \right|^2$

2. Different current gives same result