

Nuclear EFT and

Nuclear Many-Body Problems

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# 1. Introduction

Examples of phenomena

we are interested in

## 2. Computational Methods (not an exhaustive list)

## 3. Applications

## 4. Summary

cf. Talks by  
T.-S. Park  
and  
Young-Ho Song

# 1. Introduction

- Many astrophysically interesting electroweak processes involving light nuclei and low energy-momentum
- In particular, weak processes related to the Sun

pp fusion



hep fusion



$\nu d$  reactions (SNO experiments)

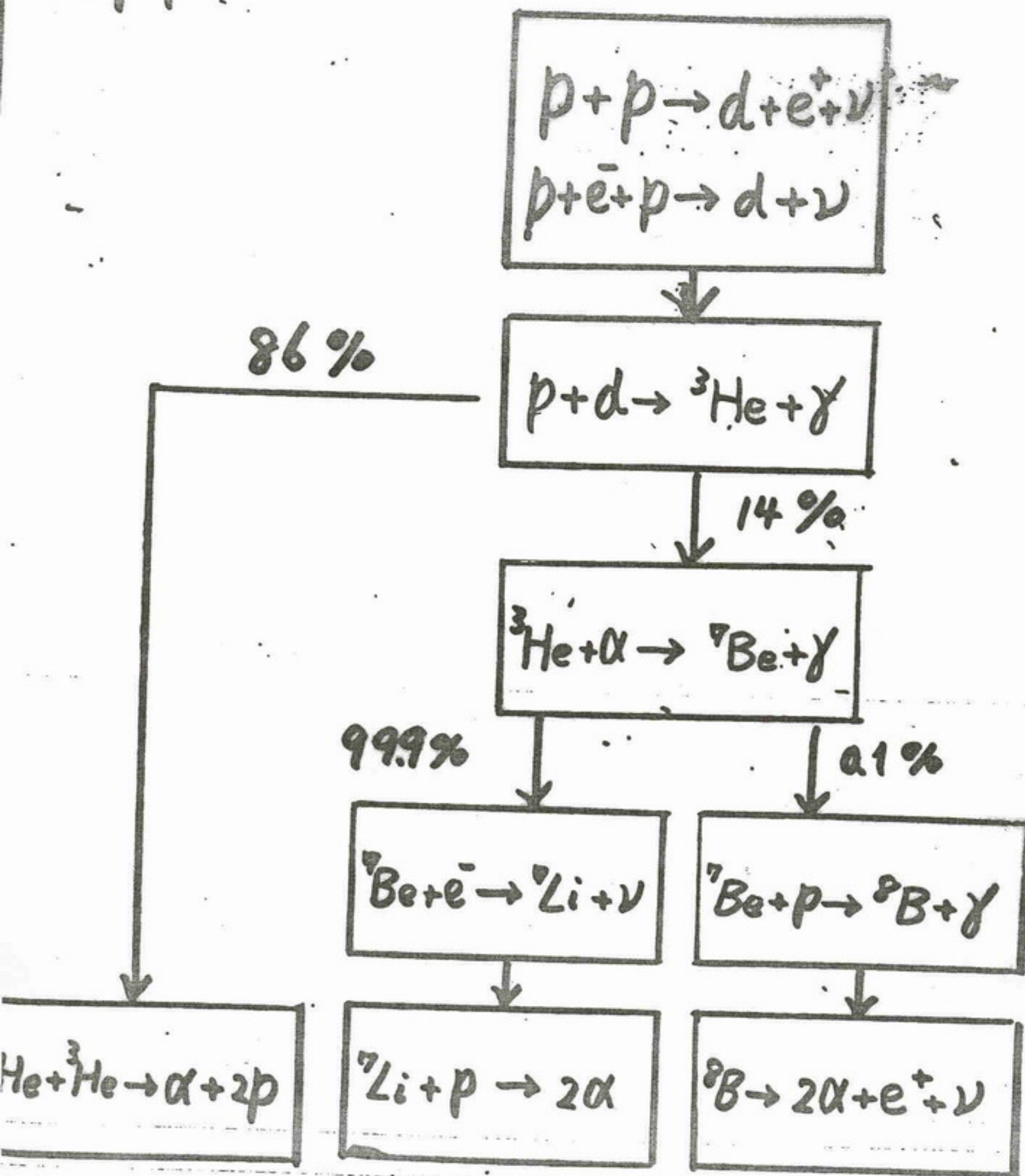


$$x = e, \mu, \tau$$

# pp-chain

$$p+p+p+p \rightarrow \alpha + 2e^+ + 2\nu_e$$

1-3



pp-I

pp-II

pp-III

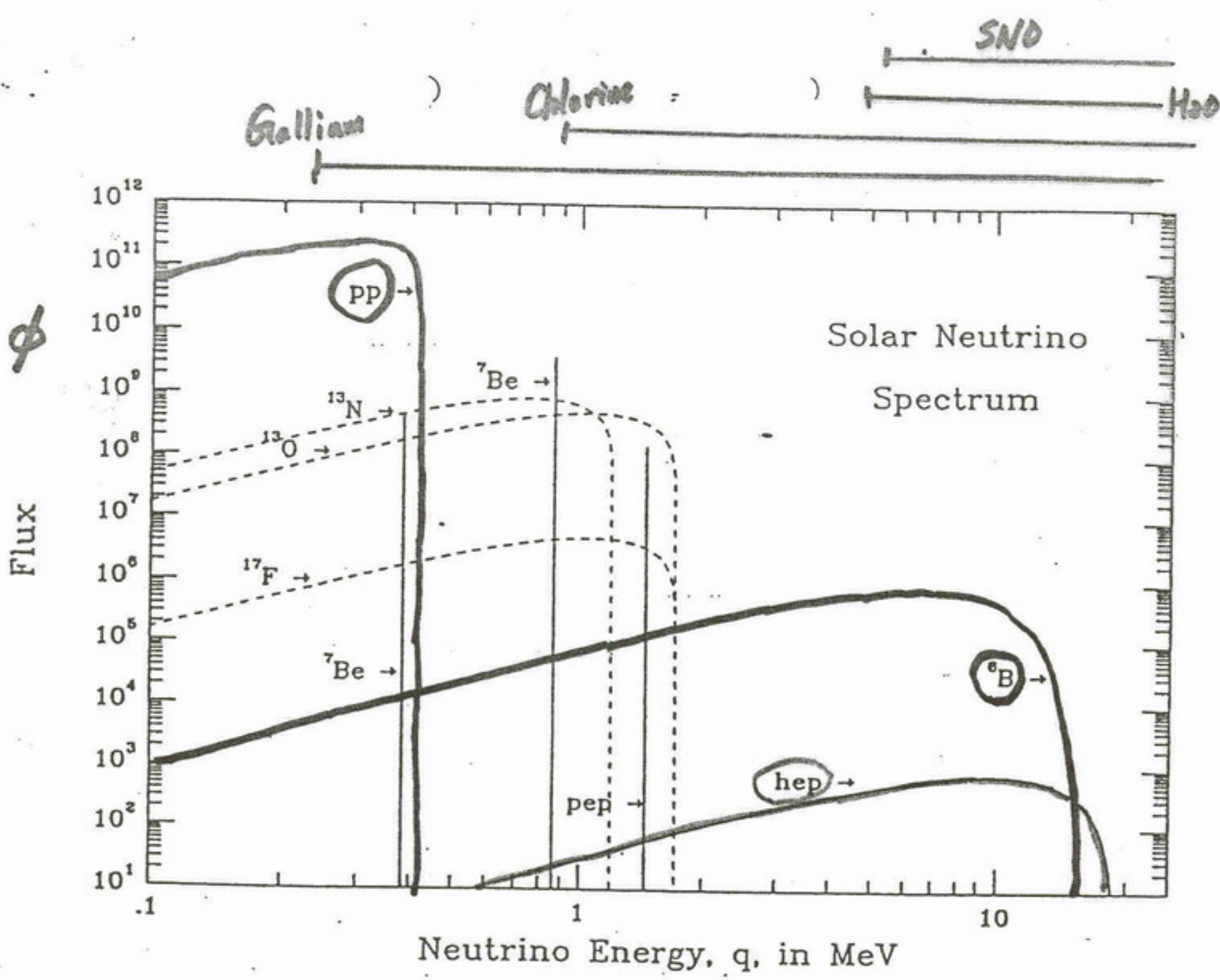
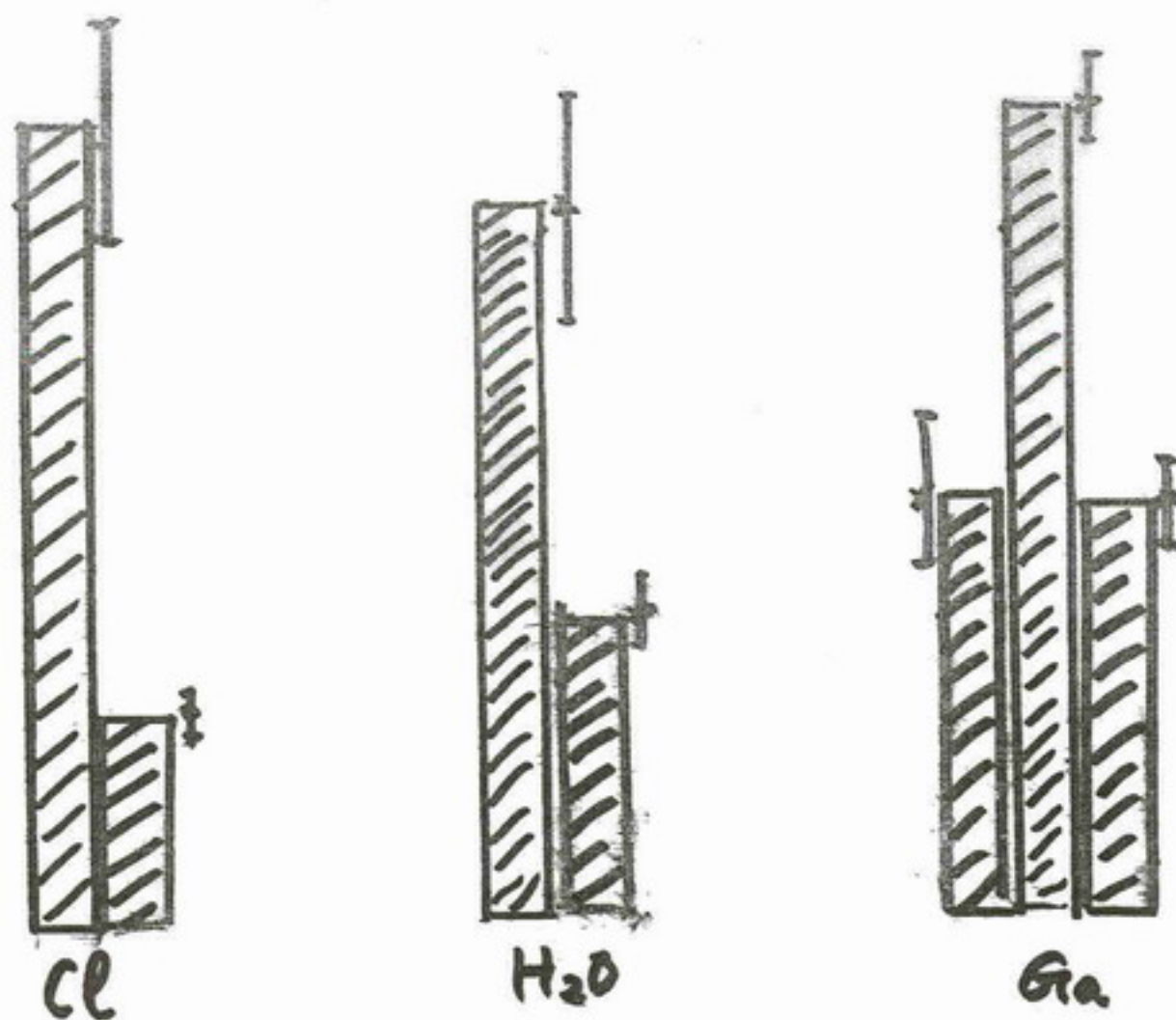


FIG. 2

Solar neutrino flux calculated in SSM  
(Standard solar model)  
J. N. Bahcall

$$\phi = \begin{cases} \# \text{ of } \nu\text{'s} / \text{cm}^2 \cdot \text{sec MeV} & \text{for continuous spectrum} \\ \# \text{ of } \nu\text{'s} / \text{cm}^2 \cdot \text{sec} & \text{for line spectrum} \end{cases}$$



*The solar neutrino problem*

 SSM prediction

 measured value

*The solar neutrino problem*

*prior to the first SNO result (2001)*

SNO can detect

$$\nu_e + d \rightarrow e^- + p + p \quad : \text{CC}$$

$$\nu_x + d \rightarrow \nu_x + p + n \quad : \text{NC}$$

( $x = e, \mu, \tau$ )

as well as

$$\nu_e + e^- \rightarrow \nu_e + e^- \quad \text{CC + NC}$$

NC is flavor-independent

→ Can determine the total flux of solar neutrinos regardless of their flavors

SNO results

Ahmad et al., P.R.L. 87(2001) 071301;  
89(2002) 011301; 011302

⊙  $\phi_\nu^{\text{tot}}$  agrees with  $\phi_\nu^{\text{tot}}(\text{SSM})$

⊙  $\frac{\phi_{\nu_e}}{\phi_\nu^{\text{tot}}} = 0.34$

→ Neutrino oscillations

$$\left\{ \begin{array}{l} \sigma_{\text{calc}}^{\text{cc}} \equiv \sigma(\nu d \rightarrow e^- + p + p)_{\text{calc.}} \\ \sigma_{\text{calc}}^{\text{nc}} \equiv \sigma(\nu x d \rightarrow \nu x + n + p)_{\text{calc.}} \end{array} \right.$$

important for these considerations

How reliable are they?

Similar questions for pp and hep.

## 2. Methodology in nuclear physics (a very limited list)

### ① SNPA

Standard nuclear physics approach  
(potential model)

### ② EFT

Effective field theory

### ③ MEEFT (More Effective EFT) or EFT\*

the latest development

# §1 SNPA (Standard nuclear physics approach) 2-2

SNPA is based on

$$H^{SNPA} = \underbrace{\sum_i t_i + \sum_{i < j} V_{ij}^{phenom}}_{\text{or}} + \sum_{i < j < k} V_{ijk}^{phenom} + \dots$$

$$\mathcal{L} = \mathcal{L}^{SNPA}(N, \pi, \Delta, \rho, \omega, \sigma \dots)$$

$H^{SNPA}$ ,  $\mathcal{L}^{SNPA}$  determined phenomenologically to reproduce 1-body and 2-body data (up to  $\pi$  threshold)

Once  $H^{SNPA}$  is fixed, find realistic wave functions by solving

$$H|\Psi_A\rangle = E|\Psi_A\rangle$$

Practically, no approximation up to  $A=11$  !!

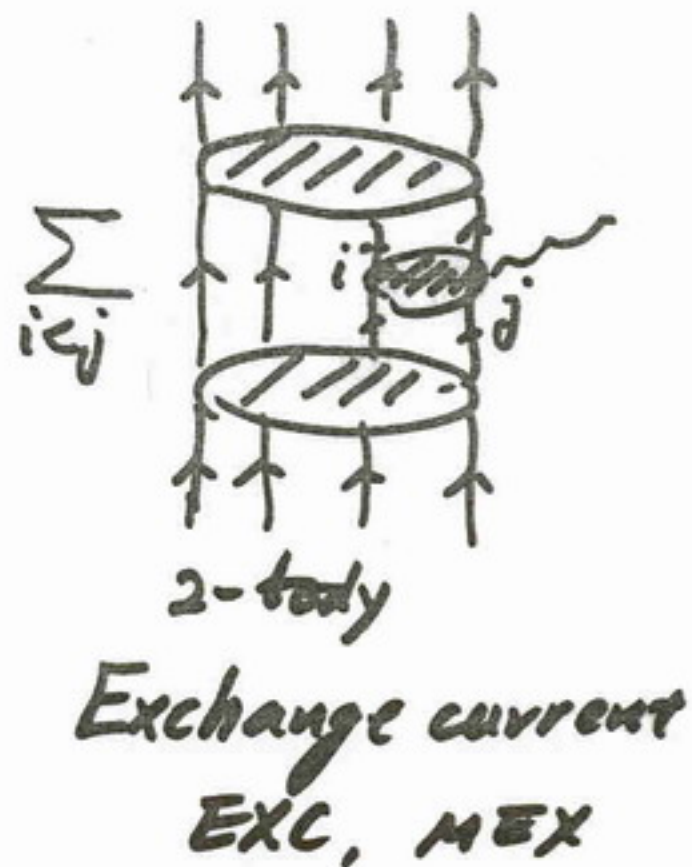
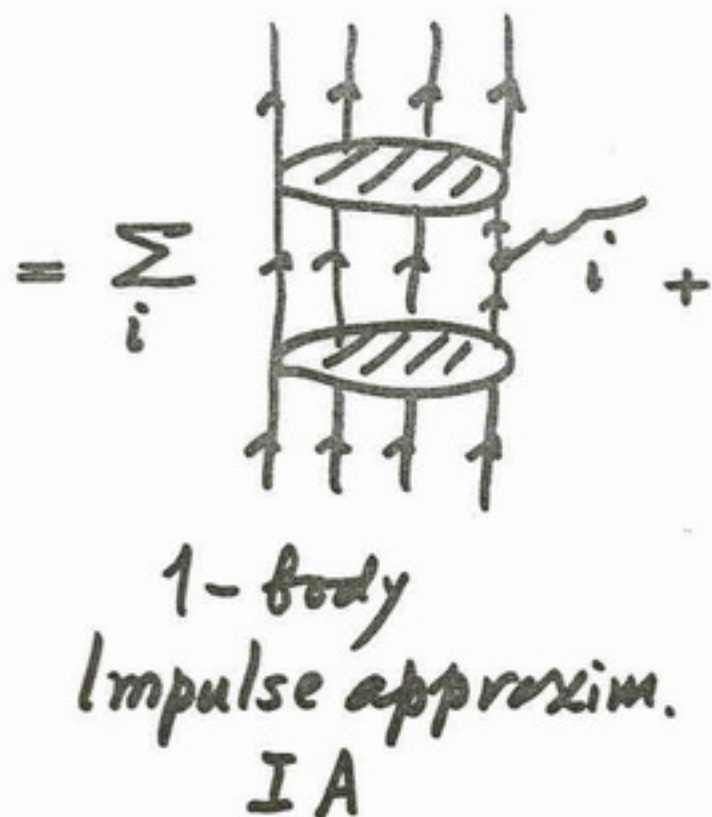
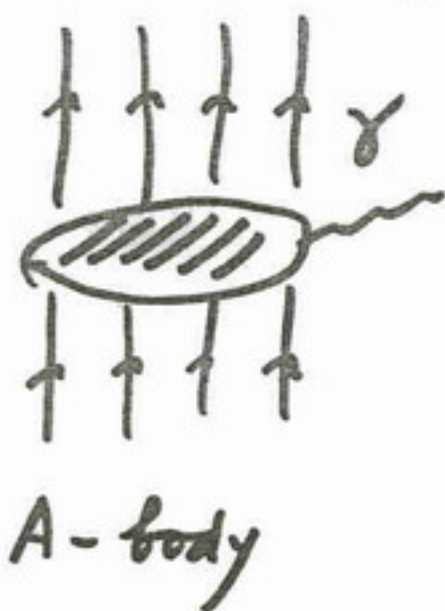
No truncation problems.

No need to employ approximate many-body technique such as RPA, ...

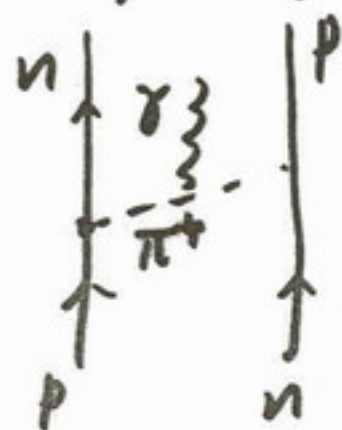
# Responses to external probes

2-3

(electromagnetic and weak currents)

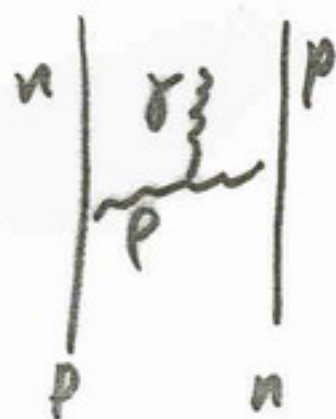


## Example of EXC



$\mathcal{L}_{NN\pi}, \mathcal{L}_{NN\rho}, \mathcal{L}_{N\Delta\pi}, \text{etc.}$

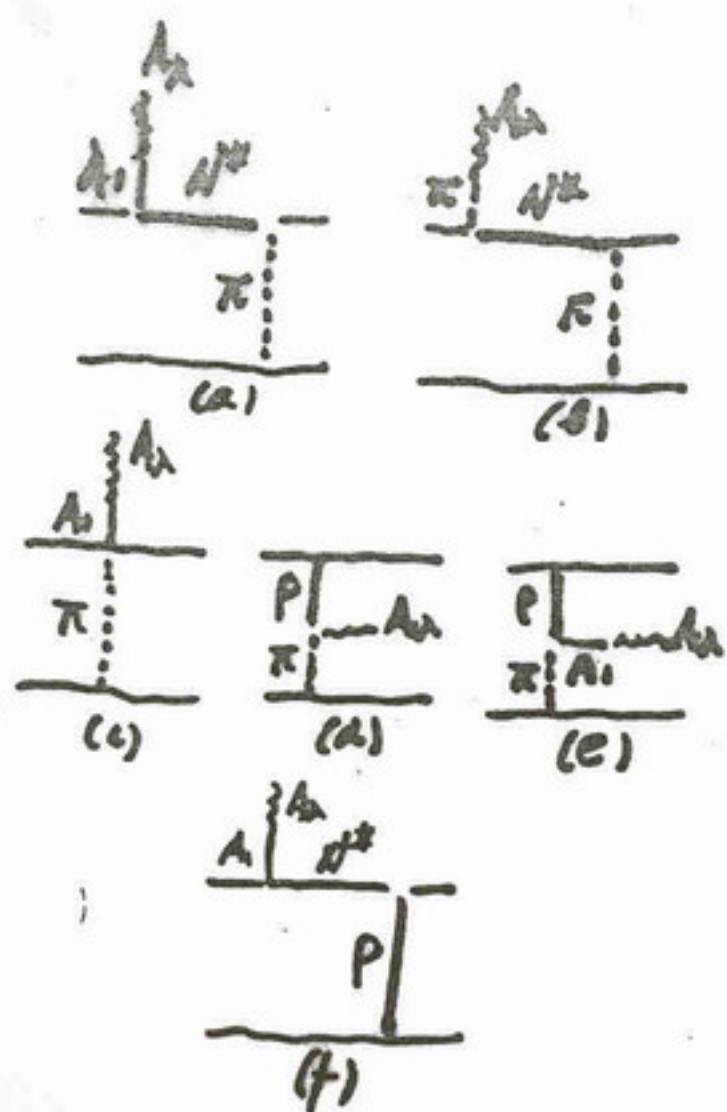
$\mathcal{L}_{\gamma\pi\pi}, \text{etc.}$



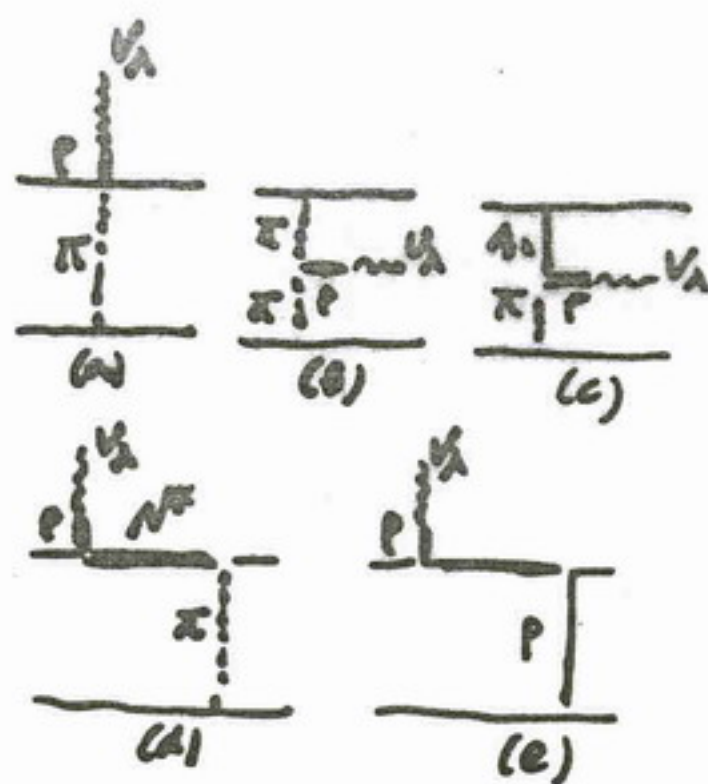
determined phenomenologically

to satisfy

CVC, PCAC, current algebra, etc.



Exchange currents for  $A_\lambda$



Exchange currents for  $V_\lambda$

$$\underline{\mu = \langle \Psi | J | \Psi \rangle}$$

IA + EXC (derived phenomenologically)

Use of "realistic" wave functions  
obtained by solving exactly  
 $H^{SNPA} |\Psi\rangle = E |\Psi\rangle$ .

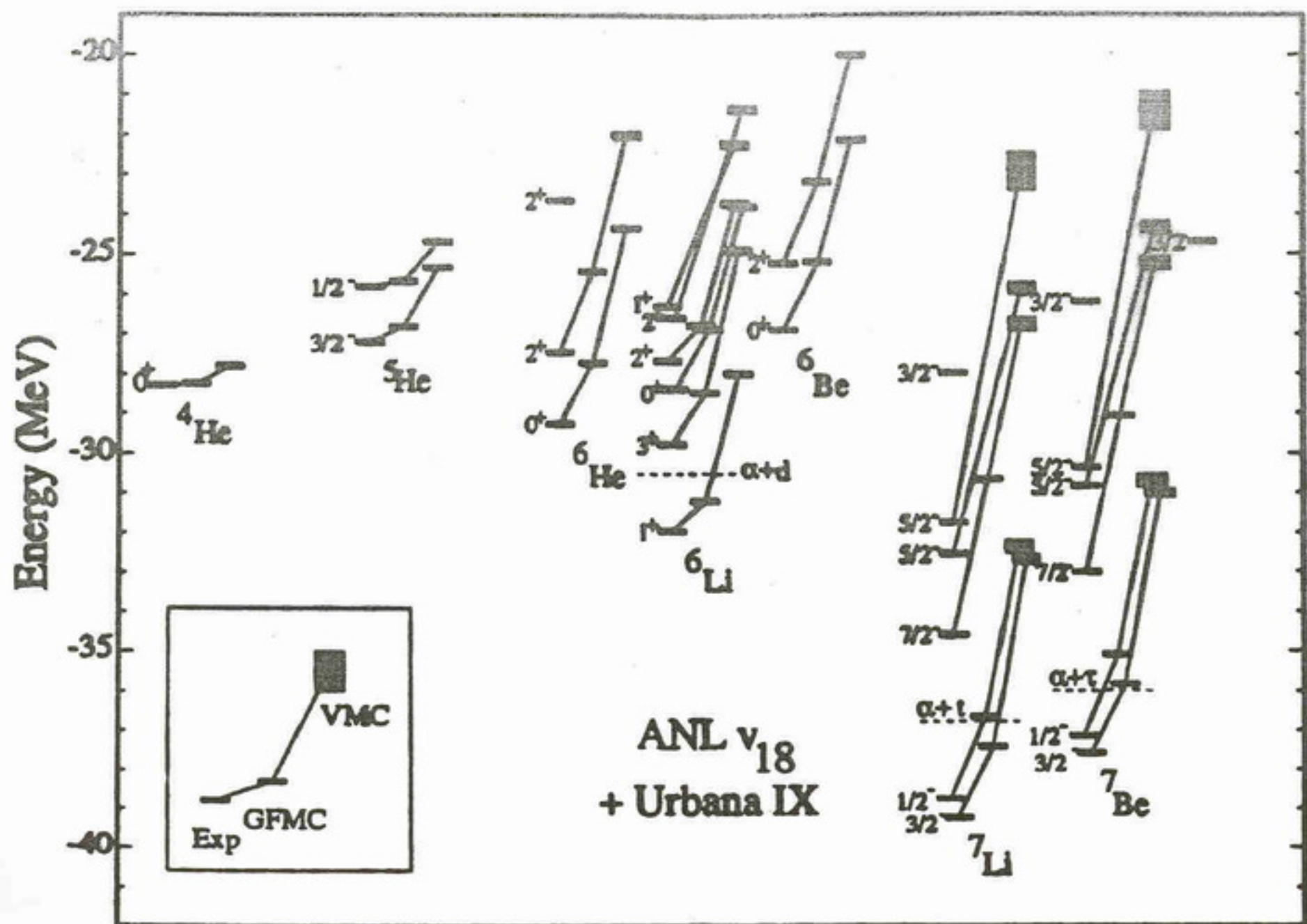
→

SNPA

Great success !!

see e.g.,

Carlson and Schiavilla,  
Rev. Mod. Phys. 70 ('98) 743.

10/12  
2-51

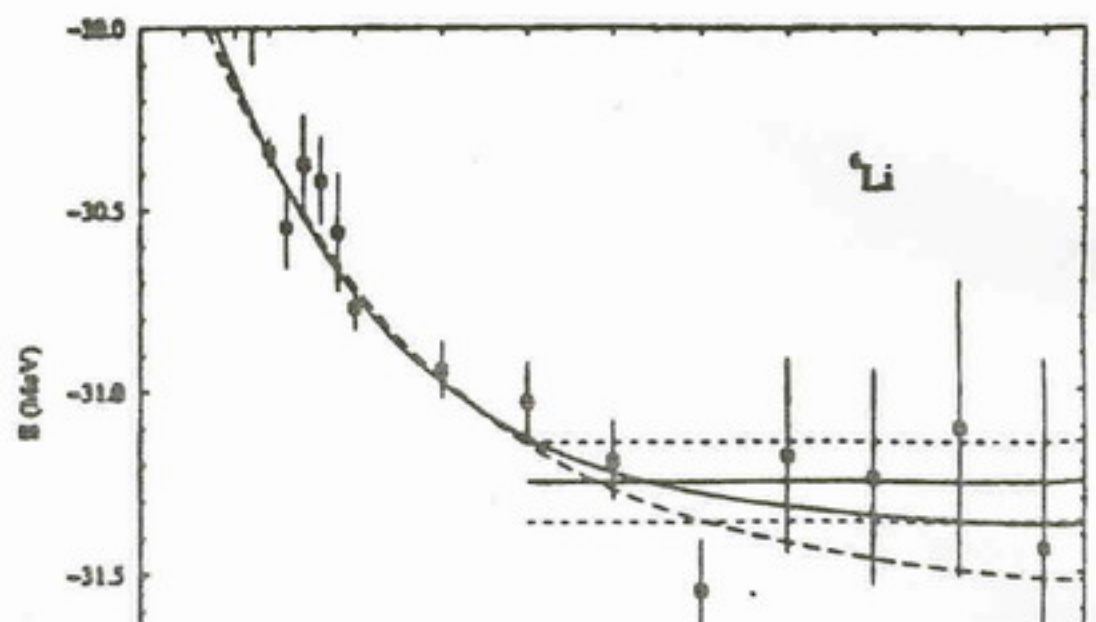
Color) Energy spectra of  $A = 4-7$  nuclei, obtained in variational Monte Carlo (VMC) and Green's-function Monte Carlo calculations with the Argonne  $v_{18}$  two-nucleon and Urbana model IX three-nucleon interactions. Both the central value and one-standard-deviation error estimate are shown. GFMC results are a variational bound obtained by averaging from  $\tau = 0.06 \text{ MeV}^{-1}$ .

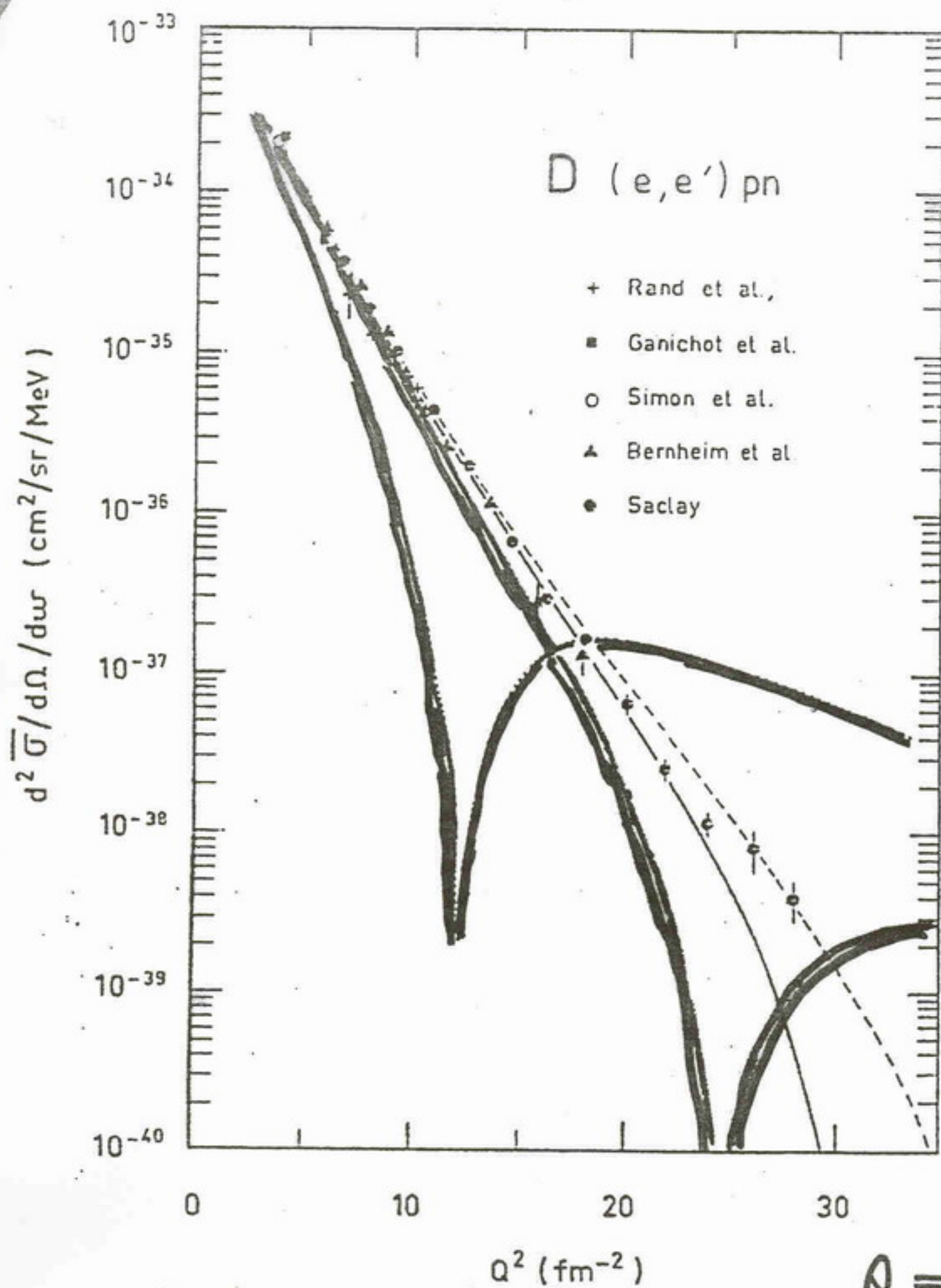
8. Thus this calculation yields approximately 1.5% of the experimental spin-orbit splitting in  $^6\text{Li}$ .

Green's-function Monte Carlo calculations produce a decreasing upper bounds to the true energy as simulation time  $\tau$  is increased. For  $^5\text{He}$ , the calculations appear to be well converged—indeed, little dependence on  $\tau$  is seen for any observable for  $\tau > 0.03 \text{ MeV}^{-1}$ . Hence the only uncertainty remaining is the difference between the full isospin-dependent Hamiltonian and a simpler static  $v_8$  model treated in perturbation theory. While this is apparently a good approximation in  $A = 3$  and 4, for  $A = 6-7$  systems it remains to be tested.

Recent sets of calculations for  $A = 6-7$  have recently been completed (Pudliner *et al.*, 1995, 1997), and the results are quite encouraging. Figure 7 compares the

Similar calculations in  $A = 4$  and 5 show that the results are well converged. Statistical fluctuations that occur in all path-integral calculations of fermion systems (Schmidt and Kalos, 1984) limit the calculations to  $\tau < 0.1 \text{ MeV}^{-1}$ .





$E_p = 1.5 \text{ MeV}$

$Q \equiv p_e - p_{e'}$

— impulse

— pion-exchange

---  $\pi + \rho$

— full ( $\pi + \rho + \Delta$ )

theory  
J. Mathiot,  
N.P. A412('84)  
201

# FIGURES

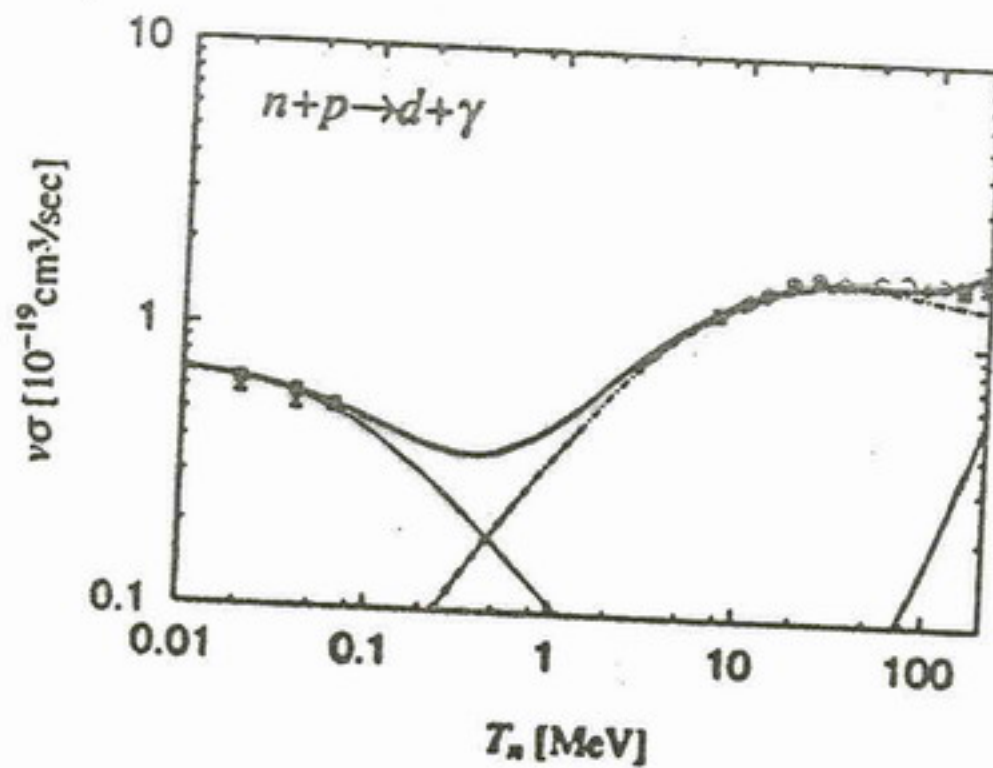


FIG. 1. Total cross section for radiative neutron capture. The solid curve corresponds to the results of our full calculation including the IA and exchange currents and all the multipole amplitudes. The dashed and dash-dotted curves show the individual contributions of the magnetic-dipole and electric-dipole amplitudes, respectively. The data are taken either from the neutron capture reaction itself [43], or from its inverse process [44,45], with the use of detailed balance for the latter.

ANL V18 used

2-5<sup>11</sup>  
1-5<sup>11</sup>  
2-5<sup>11</sup>

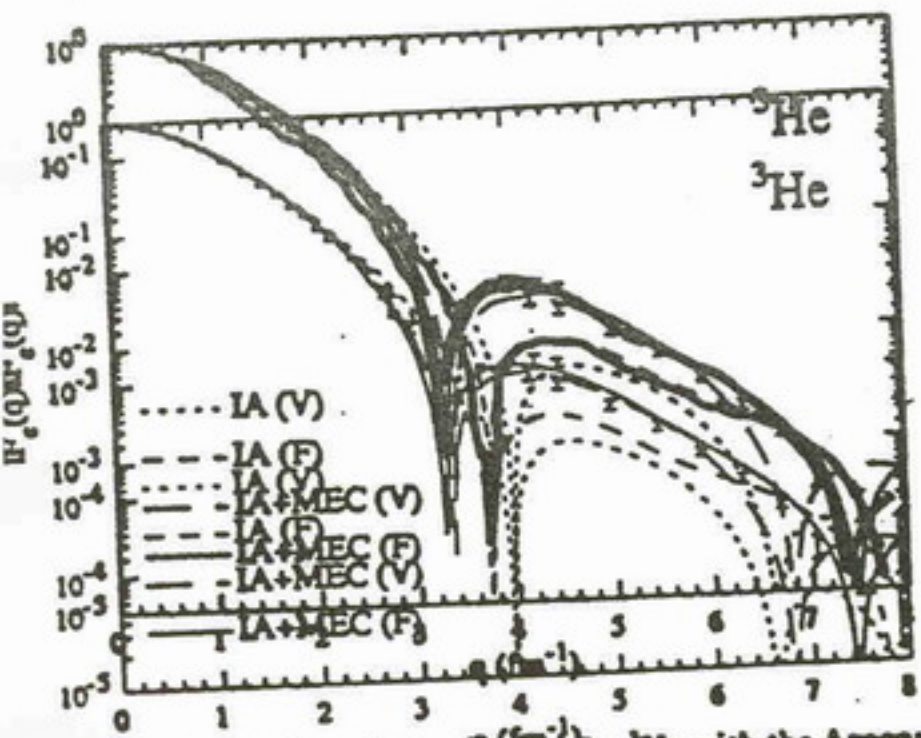


FIG. 8. Charge form factor  $|F_c(q)|$  for  ${}^3\text{He}$  with the Argonne  $v_{14}$  + Urbana VIII Hamiltonian calculated in impulse approximation. Legend: IA (V) (dotted), IA (F) (dashed), IA (V) + MEC (V) (dash-dot), IA (F) + MEC (F) (solid), IA (V) + MEC (V) (long-dash), IA (F) + MEC (F) (short-dash).

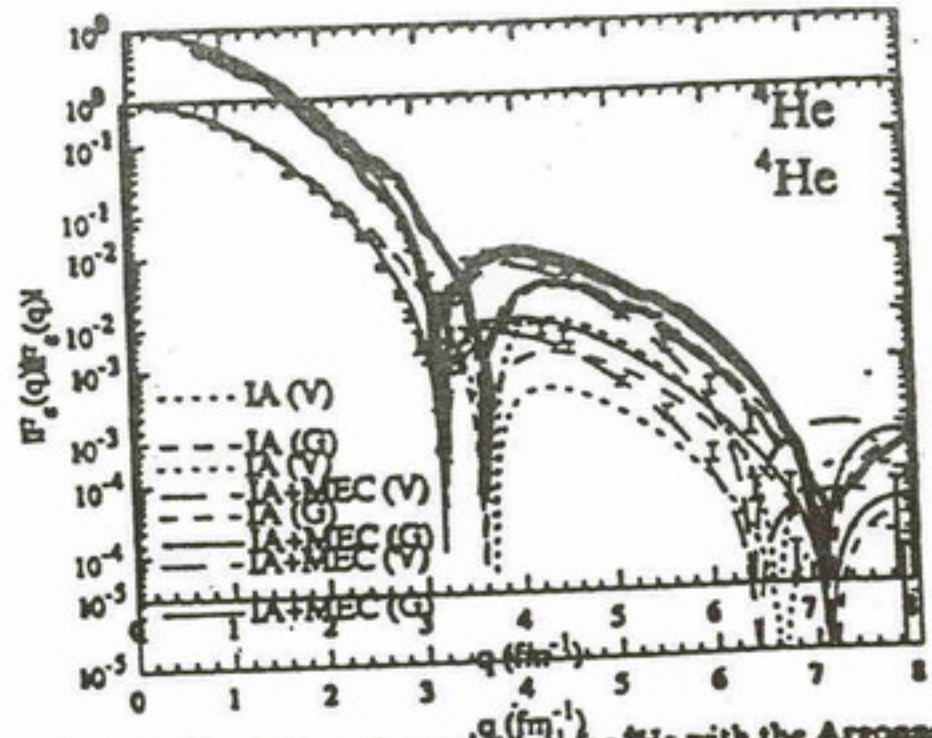


FIG. 9. Charge form factor  $|F_c(q)|$  for  ${}^4\text{He}$  with the Argonne  $v_{14}$  + Urbana VIII Hamiltonian calculated in impulse approximation. Legend: IA (V) (dotted), IA (G) (dashed), IA (V) + MEC (V) (dash-dot), IA (G) + MEC (G) (solid), IA (V) + MEC (G) (long-dash), IA (G) + MEC (V) (short-dash).

Despite its great successes,  
there is still room for a new approach  
based on "effective field theory (EFT)";  
at least for certain classes of nuclear  
phenomena.

## § 2 EFT (Effective field theory)

2-7

① How is SNPA related to QCD?

The fundamental theory of strong interactions for quarks and gluons

vs. nuclei composed of hadrons

Does SNPA respect the symmetries of QCD?  
in particular, chiral symmetry?

② Do we have a guiding principle in choosing  $V_{ij}$  (phenom)?

What is an expansion parameter?

What is a measure of errors in SNPA?

③ Why do we need "high-energy" information (up to  $E_{cm} \sim 150 \text{ MeV}$ ) to understand "low-energy" phenomena?

We <sup>Chen-Kuan</sup> wish to formulate a nuclear theory in which

- (i) the <sup>Brown</sup> symmetries of <sup>sol</sup> QCD are respected,
- (ii) there is a <sup>transl</sup> well-defined expansion scheme,
- (iii) the <sup>history, T</sup> separation of low- and high-energy scales is clear.

→ Nuclear Effective Field Theory (EFT)

S. Weinberg, Phys. Lett. B 251 ('90) 288



Explosion of research activities

For a recent review, see, e.g.,

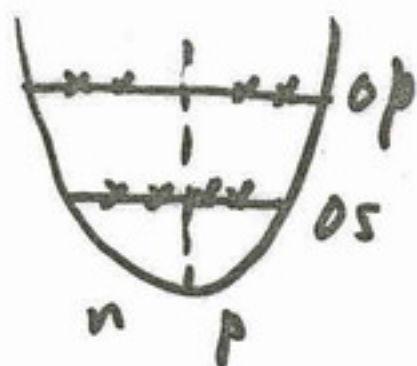
P. Bedaque & U. van Kolck,  
Ann. Rev. Nucl. Part. Sci., 52, 339 (2002)

The basic idea of EFT is not foreign to nuclear physics

Cohen-Kurath's treatment of p-shell nuclei

$$|\Psi_{\text{true}}^A\rangle \approx |(os)^u (op)^{A-u}\rangle$$

$$|\Psi_{\text{true}}^A: uTJ\rangle = \sum_k |(op)^{A-u} \alpha T J: k\rangle C_k$$



$$H_{\text{true}}^{\text{III}} \rightarrow H_{\text{p-shell}} = \sum_i t_i + \sum_{\alpha, \alpha'} |(op)^u \alpha' T J\rangle \langle (op)^u \alpha' T J | \tilde{H} | (op)^u \alpha T J \rangle \cdot \langle (op)^u \alpha T J |$$

$H_{\text{SNPA}}$

The model Hamiltonian

characterized by "low-energy constants" (LEC's)

$$h_{\alpha\alpha'}(TJ) \equiv \langle (op)^u \alpha T J | \tilde{H} | (op)^u \alpha' T J \rangle$$

$h_{\alpha\alpha'}(TJ)$  subsumes "high-energy" physics

core polarization

$\Delta$ -excitation, etc.

Use the observed energy spectra of p-shell nuclei to fix LEC's

→ prediction for the remaining levels

a nucleonic observable  $O_N$   
 (Transition operator)

In general

$$\langle f: \text{full} | \sum_{i=1}^A O_N(i) | i: \text{full} \rangle$$

$$\neq \langle f: P | \sum_{i=1}^A O_N(i) | i: \text{full}^P \rangle$$

This difference is called the  
Core-polarization effect

Furthermore

$$\begin{array}{ccc} \hat{O} & \xrightarrow{\text{Truncation to } P} & \sum_{i=1}^A O_N(i) \\ \uparrow & & \\ \text{in the world of } \mathcal{L}_{hadron} & & \end{array}$$

In general, we expect some deviation from  $\sum_{i=1}^N O_N(i)$

Impulse approx.

$\Rightarrow$  meson-exchange currents (MEC)

# effective transition operator method

T-5-7  
~~3-5-8~~  
2-9-83

① Choose for  $|A\rangle, |B\rangle$  reasonably realistic wave functions.

② 
$$\langle B || \sum_{i=1}^N \theta_i || A \rangle = \sum_{\substack{B', A' \\ b, a}} [B || B' \times \theta] \cdot [A || A' \times a]$$
  
$$= \langle B || \theta || a \rangle$$
  
single-particle label

③  cfp's many-body problem part assumed to be given by the model w.f.

④  $\langle B || \theta || a \rangle$  acquires modifications due to  
i) more complicated components in w.f.  
ii) explicit degrees of mesons

⑤  $\vec{\Sigma} \vec{\sigma} \rightarrow g_L^{\text{eff}} \vec{\Sigma} \vec{\ell} + g_A^{\text{eff}} \vec{\Sigma} \vec{\sigma} + g_P^{\text{eff}} [\gamma_2(\vec{\sigma}) \times \vec{\sigma}]^{(1)}$

⑥ Determine  $g_L^{\text{eff}}, g_A^{\text{eff}}, g_P^{\text{eff}}$  through  $\chi^2$  fitting to the available data

⑦ Predict  $\langle B || \vec{\Sigma} \vec{\sigma} || A \rangle$  for unknown transitions

2-9-84 F-5-E  
2-9  
~~2-9-84~~

# References for ETOM

B.A. Brown and B.H. Wildenthal

Atom. Data Nucl. Data Tables, 33 ('85) 347

W. Bentz

A. Arima, K. Shimizu and H. Hyuga,

Adv. Nucl. Phys. 18 ('87) 1

I.S. Townor, Phys. Rep. 155 ('87) 263

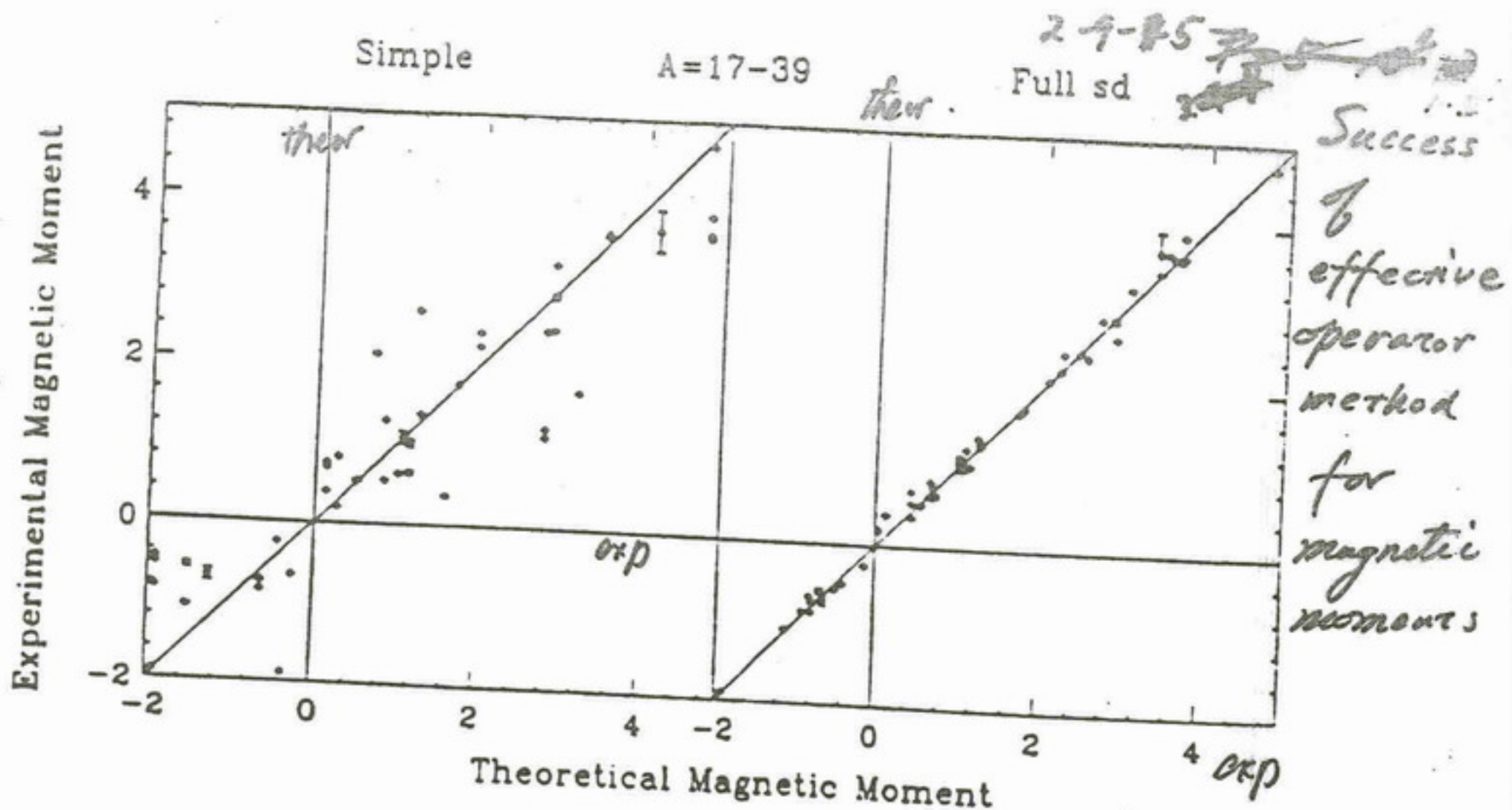


Figure 1: Comparison of experimental (y) and theoretical (x) magnetic moments for 1s0d-shell nuclei. The theoretical values for the right-hand side were obtained with the full 1s0d-shell basis, and those on the left-hand side were obtained with only the largest component in the wave function (corresponding to the extreme single-particle model). Free nucleon g-factors are used in both cases.

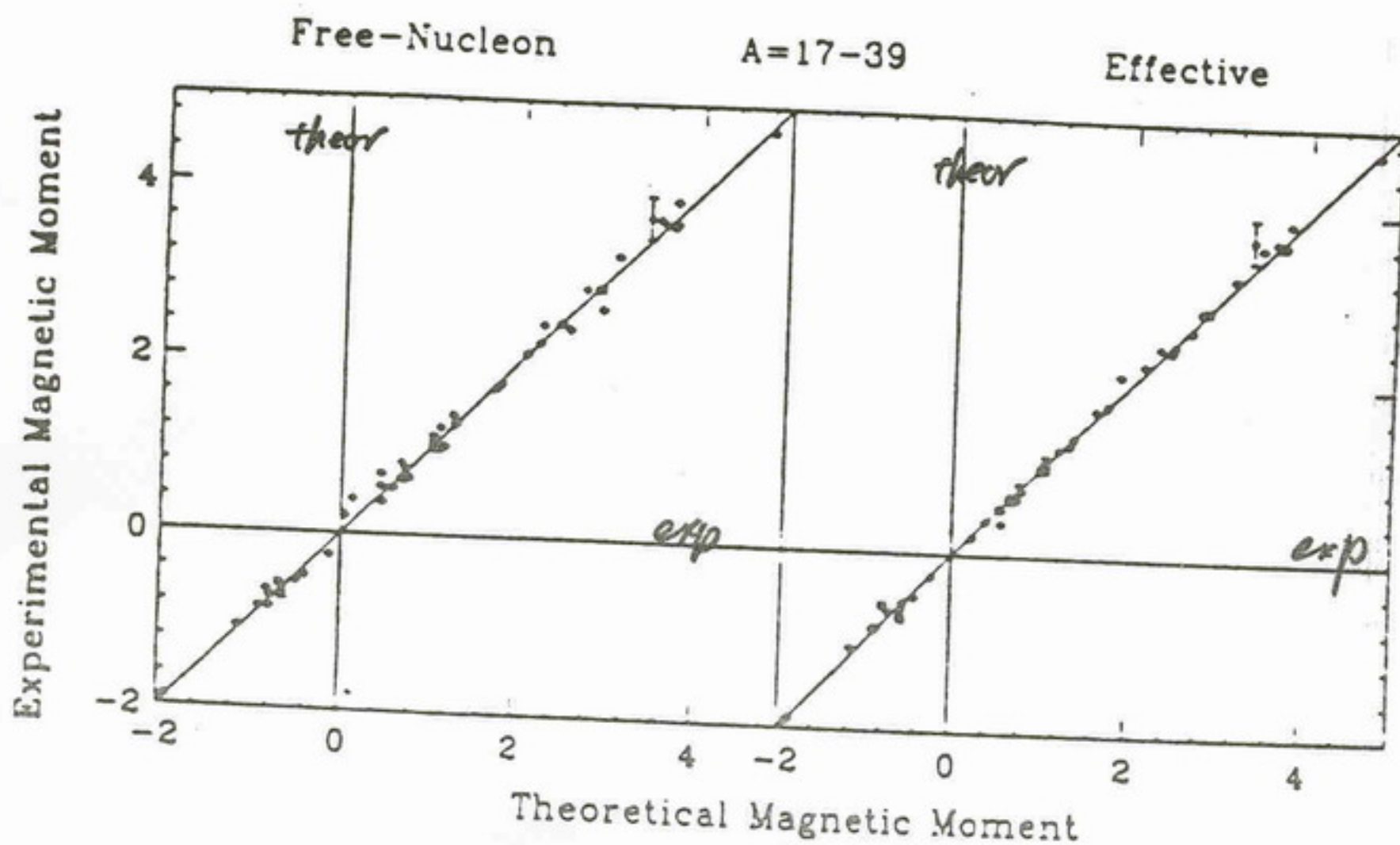


Figure 2: Comparison of experimental (y) and theoretical (x) magnetic moments for 1s0d-shell nuclei. The theoretical values for both sides were obtained with the

- The problems of SNPA  
persist in this type of  
"conventional" Effective operator  
methods

- What is the expansion parameter ?
- Are the one-body operators sufficient ?
- Non-hermiticity due to truncations ?

etc. ...

Back to our case !!

Suppose we are interested in nuclear phenomena characterized by an energy-momentum scale  $Q$ , say,  $Q \lesssim m_\pi$  (pion mass).

Relevant degrees of freedom

$\psi_N$ : nucleon (fields)

$\phi$ : pion (field)

⇒ We should be able to describe our low-energy phenomena in terms of  $\psi_N$  and  $\phi$ .

$$\mathcal{L}_{\text{QCD}}(g, \bar{g}, A_\mu) \overset{\text{related}}{\longleftrightarrow} \mathcal{L}_{\text{EFT}}(\psi_N, \bar{\psi}_N, \underbrace{\phi}_{\text{denoted collectively}})$$

$$\int [dg] [d\bar{g}] [dA_\mu] e^{i \int d^4x \mathcal{L}_{\text{QCD}}(g, \bar{g}, A_\mu)} \\ = \int [d\phi] e^{i \int d^4x \mathcal{L}_{\text{EFT}}(\phi)}$$

- Write down all possible monomials consisting of  $\psi$ ,  $\bar{\psi}$  and  $\phi$  and their derivatives that are consistent with the symmetries of QCD (chiral symmetry, etc.)  
Hermiticity preserved.

- $\vec{D}\phi \leftrightarrow Q$  : Typical momentum of the system  
 $\rightarrow$  Expansion in  $\frac{Q}{\Lambda}$  ( $\Lambda \approx 1 \text{ GeV}$ )

A small quark mass  $m_q$  makes the mass of a Goldstone boson  $M_\pi$  deviate from 0  
 $m_\pi/\Lambda$  also appears as an expansion parameter

- $\Rightarrow$   $\chi$ PT (chiral perturbation theory)  
 if the nucleon is treated as almost static  
 $\rightarrow$  HB $\chi$ PT (heavy baryon  $\chi$ PT)

# HBF (Heavy Baryon Formalism) of Heavy Quark Formalism 2-12

$$N \longrightarrow B(x) \equiv e^{i(v \cdot x) m_N} N(x)$$

$$v \equiv (1, 0, 0, 0)$$

$$\mathcal{L}(N, \bar{N}) \Rightarrow \mathcal{L}_{\text{eff}}(B)$$

$$B = (P_+ + P_-) B \equiv B_+ + B_-$$

upper comp.

lower comp.

$$P_{\pm} \equiv \frac{1}{2}(1 \pm \gamma_4)$$

$$B_- \propto \frac{1}{m} B_+$$

$$\mathcal{L}_{\text{eff}}(B) \Rightarrow \mathcal{L}_{\text{eff, eff}}(B_+) \equiv \mathcal{L}_{\text{HBF}}(B_+)$$

$$\frac{\partial}{\partial t} N \sim m_N N \quad \text{large!}$$

$$\frac{\partial}{\partial t} B \sim (m_N - m_N) \quad \text{small!}$$



The coefficient of certain terms may not be known.

↔ Low-energy constants (LEC's) need to be fixed empirically.

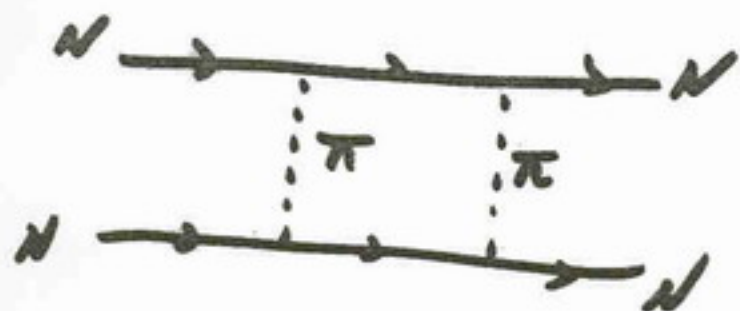
2-14

$\chi$ P.T. successfully used for the pion sector  
and 1-nucleon sector ( $\pi N$  scattering,  $\mu$ -p capture, etc.)

How about  $A$ -nucleon systems ( $A \geq 2$ )?

→ Nuclear  $\chi$ P.T. à la Weinberg - of van Kolck, thesis

"Dangerous" intermediate states



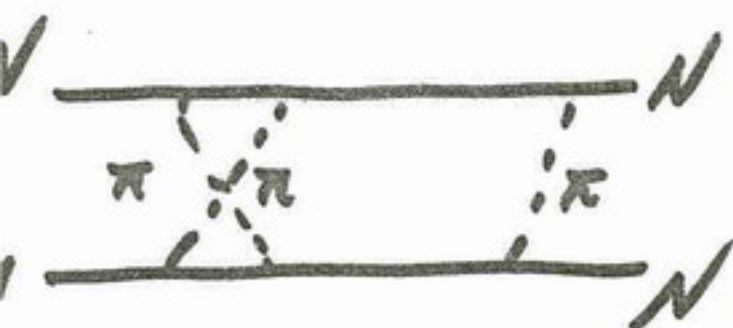
Very low excited states possible  
in nuclei !!

→ Perturbation breaks down

→ Reducible and irreducible diagrams.  
Treat them separately

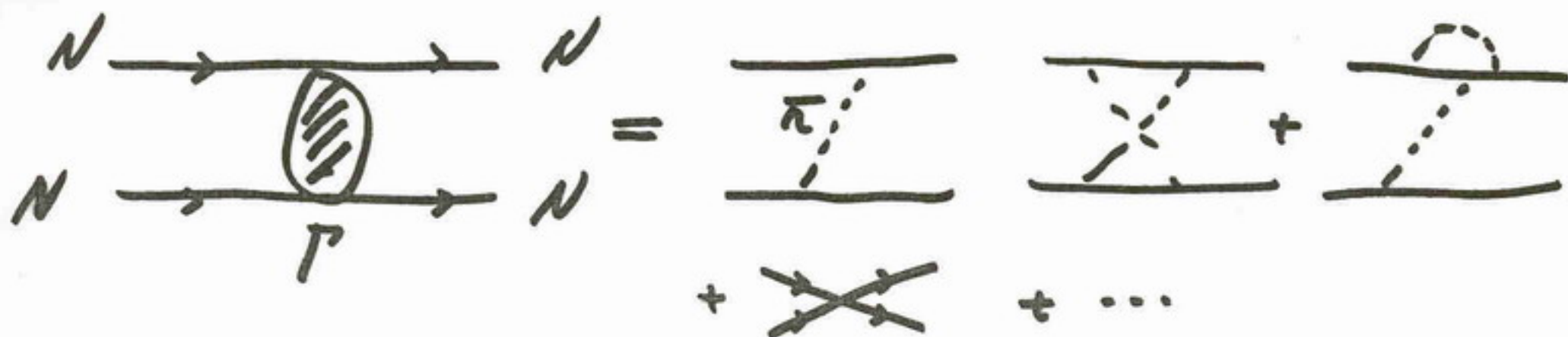


irreducible



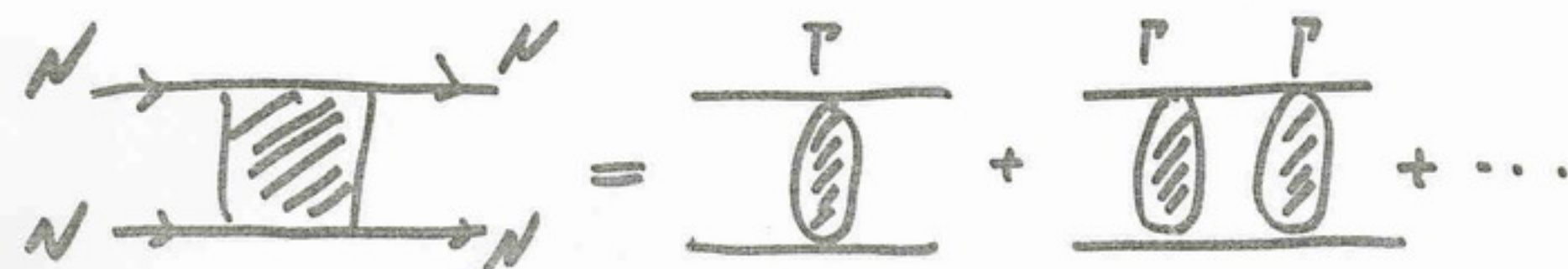
reducible

● Apply  $\chi$ PT to irreducible diagrams  
eg., N-N force



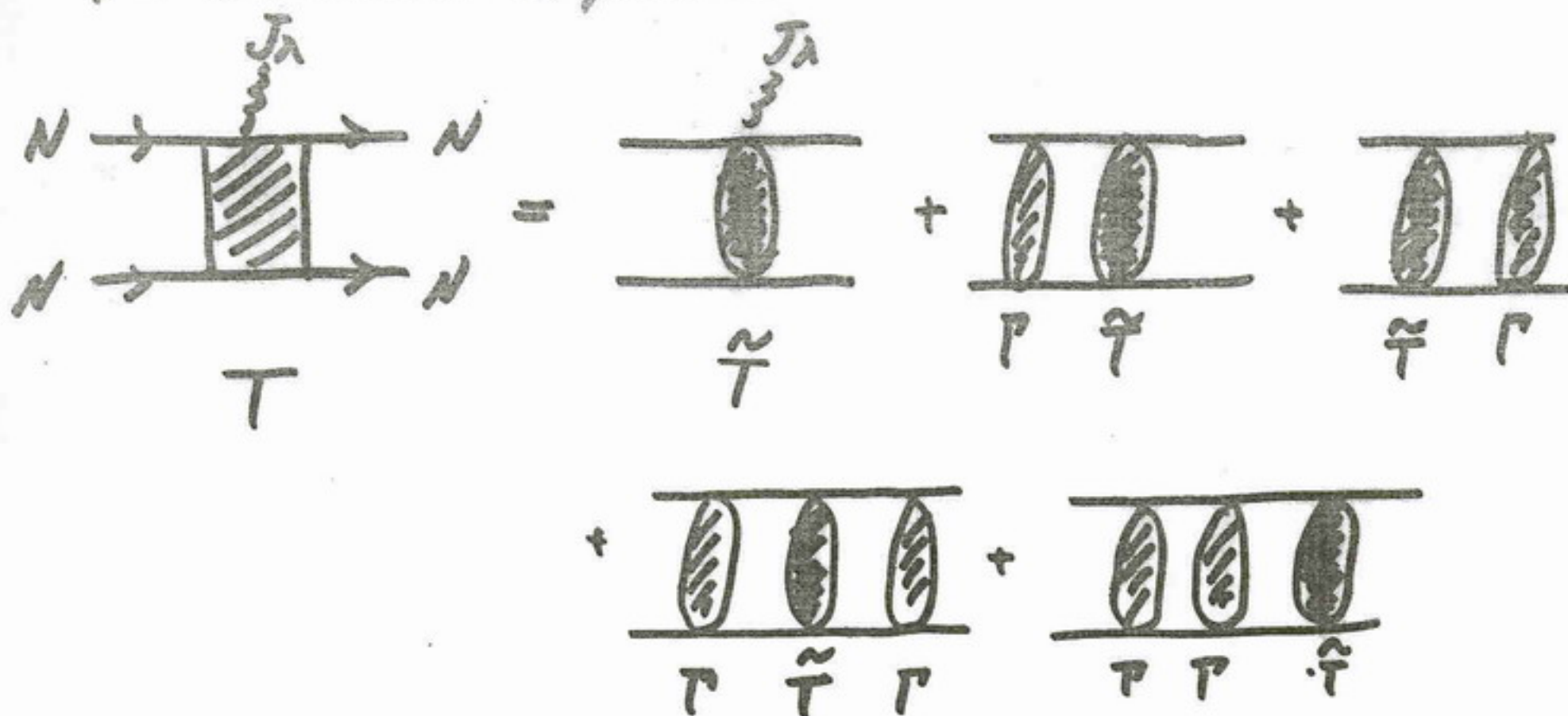
within a given order of  $\chi$  expansion

Use  $T$  as a potential in  
Lippman-Schwinger eq. or Schrödinger eq.



For electroweak probes

2-16



$\tilde{T}$  : irreducible diagrams

$$\underline{T = \langle \tilde{\Psi}_f | \tilde{T} | \tilde{\Psi}_i \rangle}$$

$|\tilde{\Psi}_i\rangle, |\tilde{\Psi}_f\rangle$  : distorted waves obtained by solving

$$(H_0 + V) |\tilde{\Psi}\rangle = E |\tilde{\Psi}\rangle.$$

If carried out faithfully,

→ an "ab initio" calculation

# Criticism of Weinberg counting

→ PDS (power divergence subtraction) KSW

M. Lutz, hep-ph/9606301;  
 U. van Kolck, Mainz 1997;  
 D. Kaplan, M. Savage and M. Wise  
 Nucl. Phys. B528 ('98) 329;  
 Phys. Lett. B 424 ('98) 390.

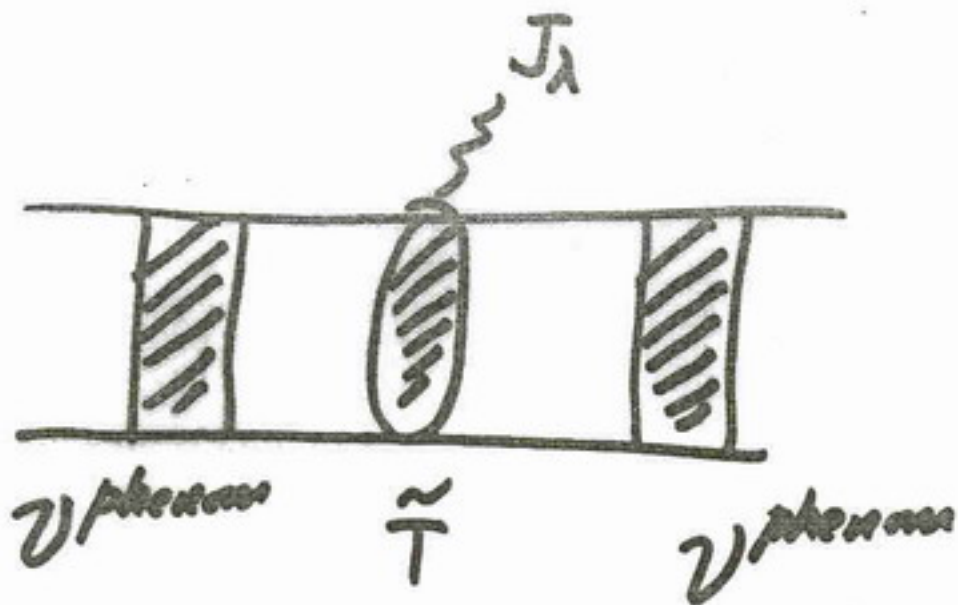
Careful study of the convergence properties  
of W and KSW counting.

S. Beane, P. Bedaque, M. Savage & U. van Kolck,  
 Nucl. Phys. A700, 397 (2002).

In practice,

2-18

## Hybrid nuclear $\chi$ PT



(i)  $|\tilde{\Psi}_i\rangle, |\tilde{\Psi}_f\rangle$  calculated with the use of  
a phenomenological potential,  $V^{\text{phenom}}$   
"Realistic potential"  $\rightarrow$  reasonably reliable

(ii)  $\tilde{T}$ : transition operator controlled by  
 $\chi$ PT



MEEFT (More effective EFT)  
or EFT\*

$\mathcal{J}^{\text{EFT}}$  often involves (an) unknown  
LEC(s).

How to handle this problem?

Can be explained better with the use  
of concrete examples.

We consider here

$\nu d$ ,  $pp$  fusion and  $hep$  fusion.

### 3. Applications

3-1

$\sigma(vd)$

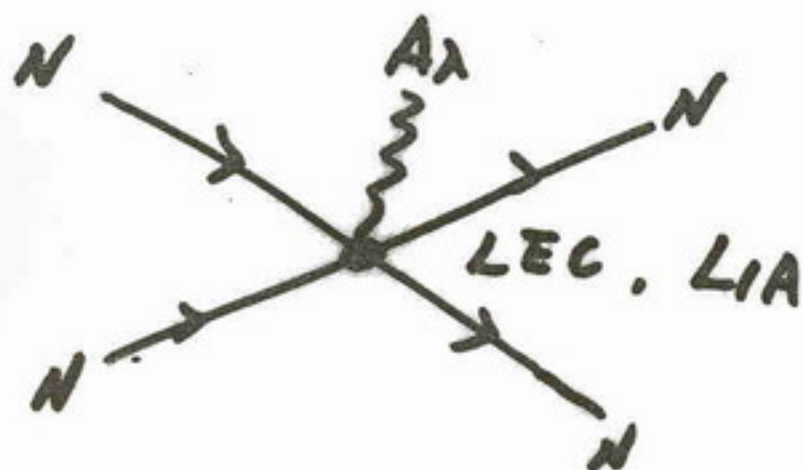
SNPA calc.

S. Nakamura et al., Phys. Rev. C 63 ('01) 034617

EFT calc. (power divergence subtraction, PDS)

Butler, Chen and Kong, Phys. Rev. C 63 ('01) 035507

"LIA" is unknown



BCK optimized LIA to reproduce  $\sigma(vd)$ : SNPA

→ Agreement within 1% for all the channels  
and for all energies up to 20 MeV

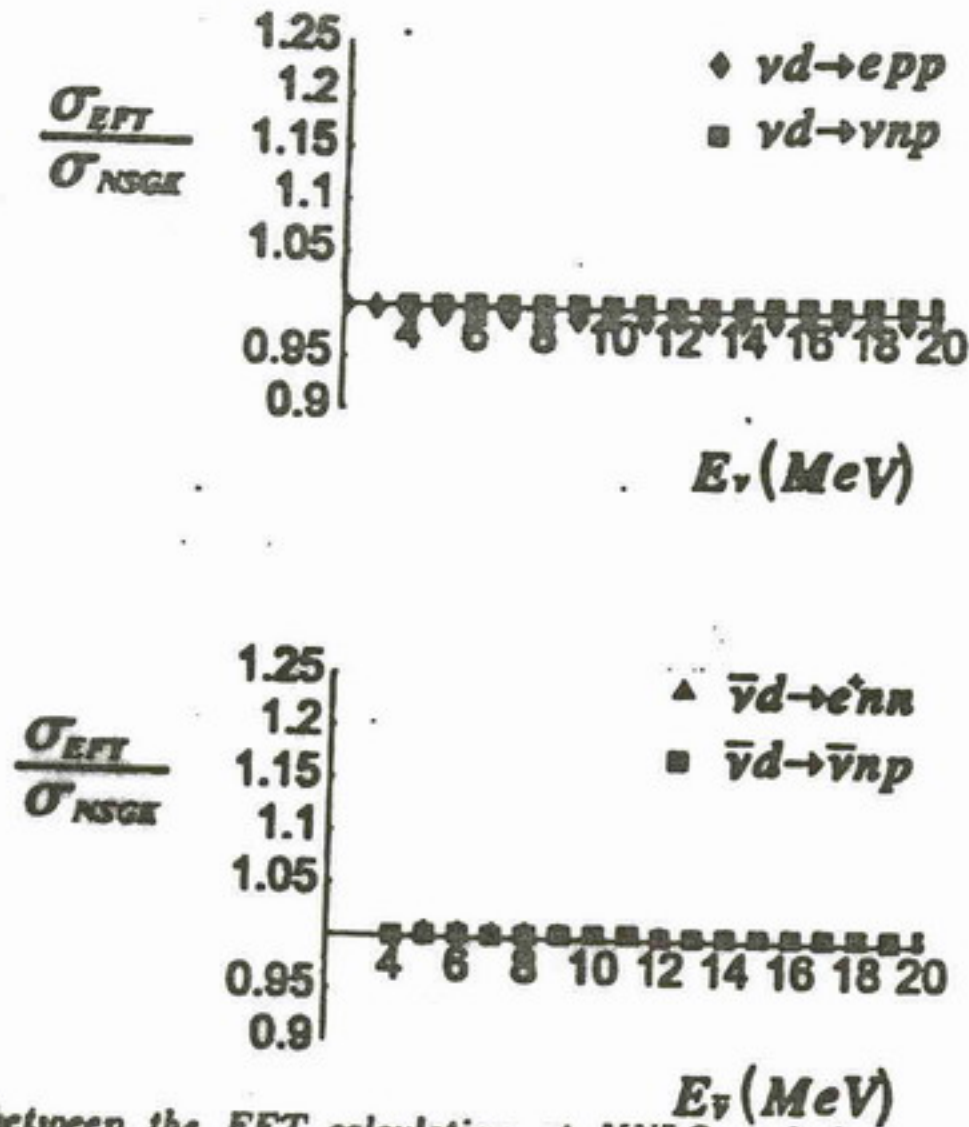


FIG. 11. Ratios between the EFT calculation at NNLO and the potential model result of NSGK [12]. The upper graph compares the two channels of  $\nu$ -d scattering, and the lower graph the two channels of  $\bar{\nu}$ -d scattering. Agreement is better than 1% over the whole range of energies shown, for a single value of  $\bar{L}_{1,\Lambda} = 5.6 \text{ fm}^3$ .

|           | $a^1S_0,pp \text{ (fm)}$ | $r_0^1S_0,pp \text{ (fm)}$ | $a^1S_0,nn \text{ (fm)}$ | $r_0^1S_0,nn \text{ (fm)}$ | $a^1S_0,np \text{ (fm)}$ | $r_0^1S_0,np \text{ (fm)}$ | $\rho_d \text{ (fm)}$ |
|-----------|--------------------------|----------------------------|--------------------------|----------------------------|--------------------------|----------------------------|-----------------------|
| This Work | -7.82                    | 2.79                       | -18.5                    | 2.80                       | -23.7                    | 2.73                       | 1.764                 |
| NSGK      | -7.815                   | 2.78                       | -18.5                    | 2.83                       | -23.73                   | 2.69                       | 1.767                 |

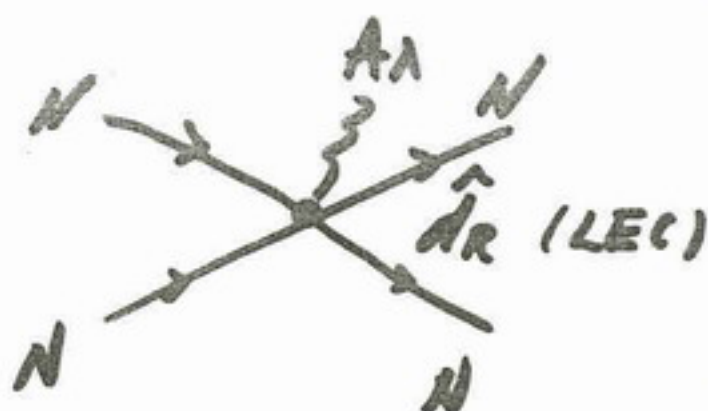
TABLE III. Effective range parameters as used in our work and NSGK [12].

# EFT\* or MEEFT (More effective ...)

3-3

T.-S. Park et al., nucl-th/0106025, arXiv:0107012

Phys. Rev. C 67 (2003) 055206



Based on hybrid  $\chi$ PT, but  
the LEC,  $\hat{d}_R$ , controlled by  
nuclear data.

The same vertex features in

$$t \rightarrow {}^3\text{He} + e^- + \bar{\nu}_e$$

⊙  $\Gamma_i^A$  well known.

⊙ High-precision wave functions  
 available for  $A=3$

→ Parameter-free calc. of  $\sigma(vd)$   
 and  $\sigma(pp)$ ,  $\sigma(He p)$ .

① Cut-off independence,

a measure of the reliability of the formalism

$$\frac{\overline{\vec{g}} \downarrow \vec{g}}{\Gamma} \quad \left\{ \vec{J}_\lambda \right. \quad |\vec{g}| < 1 \quad \text{cut-off}$$

2-body and 3-body systems share the same short-range physics

3-5 ~~24~~

$\sigma(Vd)_{EFT^*}$

Nakamura et al., Nucl. Phys. A 707 ('02) 561

Ando et al., Phys. Lett. B 555 ('03) 49.

$$\sigma(Vd)_{EFT^*} = \sigma(Vd)_{SNPA}$$

within  $\sim 1\%$

$\Lambda$ -dependence of  $\sigma(Vd)_{EFT^*} \sim 0.1\%$

for  $\Lambda = 500 \text{ MeV} \sim 800 \text{ MeV}$

$T_{\frac{1}{2}}^B \sim 0.5\% \text{ uncertainty}$

3-6   
The same approach applied to pp

$$p+p \rightarrow e^+ + d + \nu_e$$

Park et al., Phys. Rev. C 67 (1993) 055206

Astrophysical S-factor

$$S_{pp}(0) = 3.94 \times (1 \pm 0.004) \times 10^{-25} \text{ MeV}\cdot\text{b}$$

improvement in accuracy  
by a factor of  $\sim 10$

hep and her

3-7 ~~3-7~~ ~~3-7~~



$$\overline{E}_\nu^{\text{max}} = 18.8 \text{ MeV} !!$$

But  $\Phi_\nu(\text{hep})$  is tiny.

→ diagnostic purposes

Anomaly (?) seen in the  $\Phi_\nu({}^8\text{B})$  spectrum  
at Super-K ?

see 3.4'

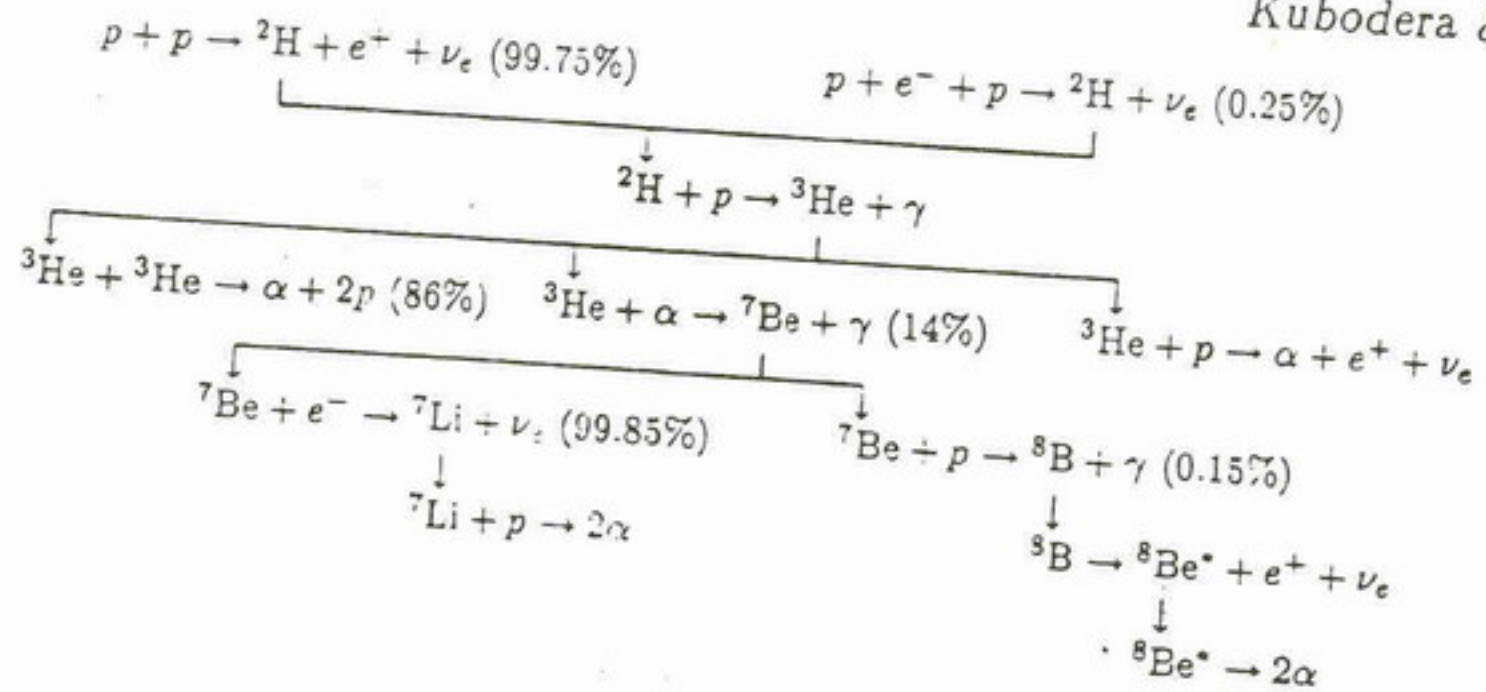


Figure 1: Solar thermonuclear reactions in the *pp*-chain and their branching ratios. The *Hep* branching ratio is of the order of 0.01% or less.

3-9  
3.9  
3.9

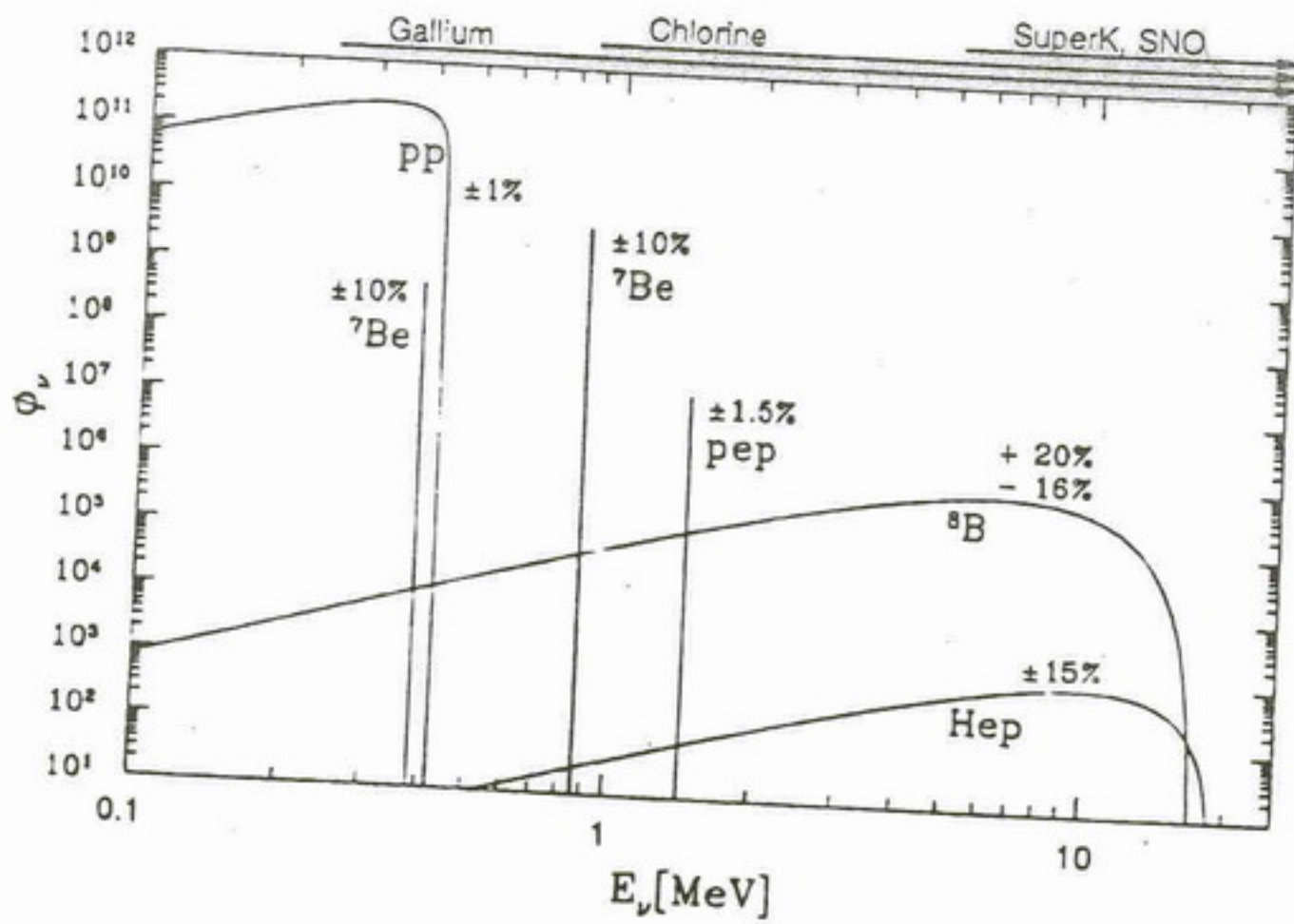
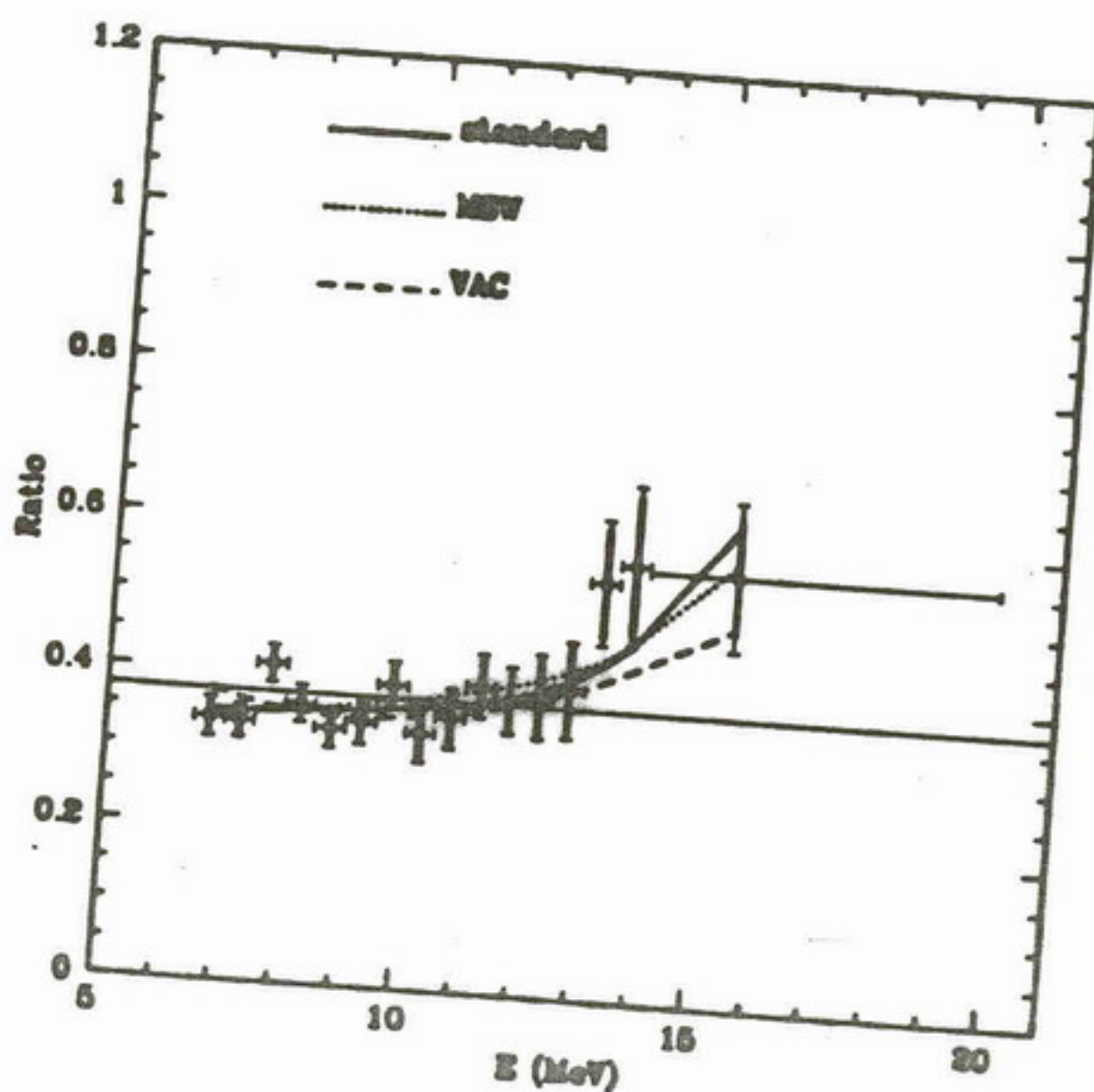


Figure 2: Solar neutrino spectrum  $\phi_\nu$  vs the neutrino energy  $E_\nu$ . The neutrino fluxes from continuum sources are given in units of  $\text{cm}^{-2}\text{s}^{-1}\text{MeV}^{-1}$ , and the line fluxes in units of  $\text{cm}^{-2}\text{s}^{-1}$ . The arrows at the top indicate the ranges of  $E_\nu$  covered by the experiments mentioned in the text.

~~3-10~~  
3-10 ~~3-10~~



Bahcall and Krastev (1981)  
A new analysis of SNO data  
in progress (E. Beier)

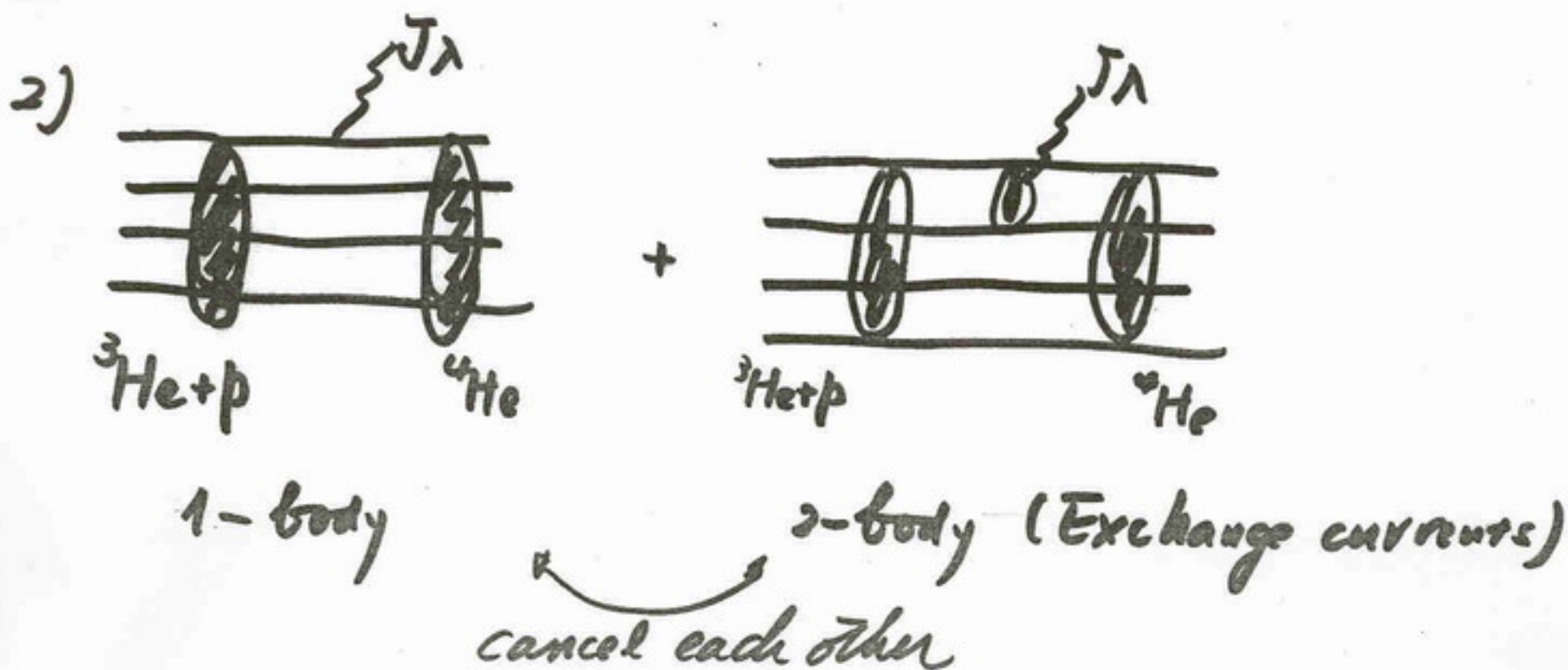
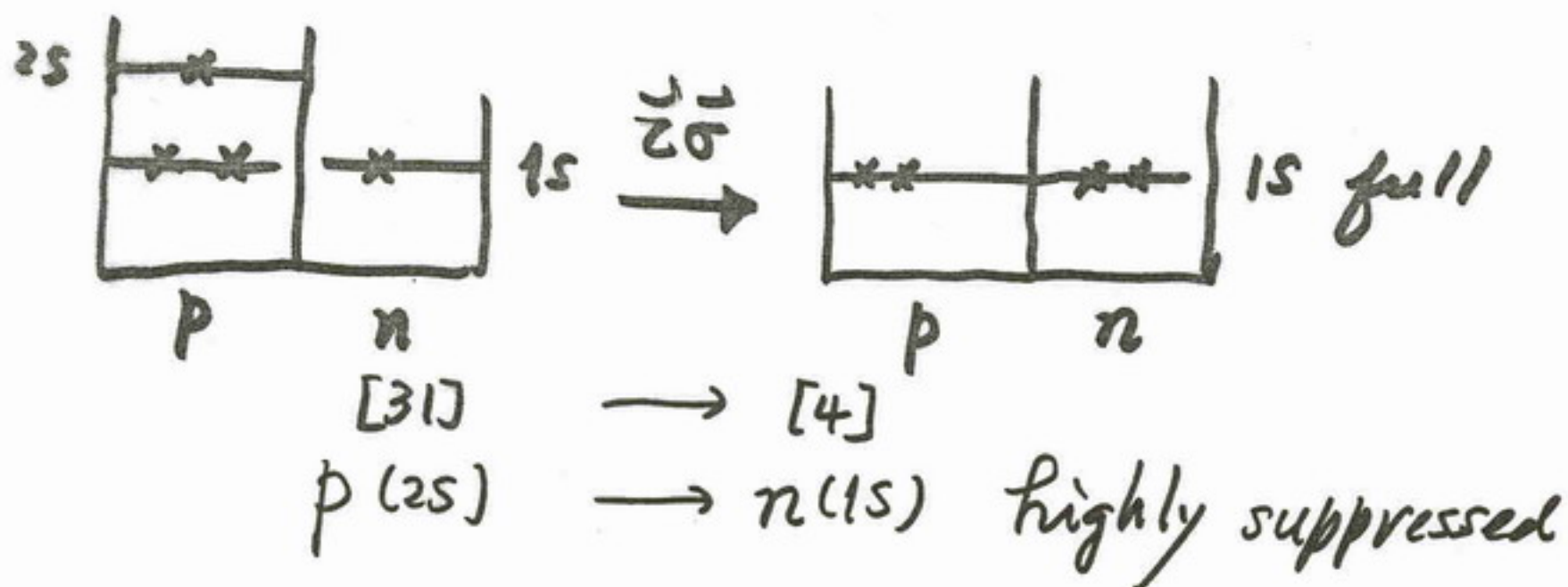
Table 1

Calculated values of  $S_0(\text{hep})$ . The table lists all the published values with which we are familiar of the low energy cross section factor for the hep reaction shown in Eq. 1.

| $S_0(\text{hep})$<br>( $10^{-20}$ kev b) | Physics                                                         | Year | Reference                     |
|------------------------------------------|-----------------------------------------------------------------|------|-------------------------------|
| 630                                      | single particle                                                 | 1952 | [10] <i>Salpeter</i>          |
| 3.7                                      | forbidden; $M_\rho \propto M_\gamma$                            | 1967 | [11] <i>Worty-Brennan</i>     |
| 8.1                                      | better wave function                                            | 1973 | [12] <i>Worty-Brennan</i>     |
| 4-25                                     | D-states + meson exchange                                       | 1983 | [13] <i>Tegner-Ruybelty</i>   |
| $15.3 \pm 4.7$                           | measured ${}^3\text{He}(n, \gamma){}^4\text{He}$                | 1989 | [14] <i>Wolf et al.</i>       |
| 57                                       | measured ${}^3\text{He}(n, \gamma){}^4\text{He}$<br>shell model | 1991 | [15] <i>Wardman et al.</i>    |
| 1.3                                      | destructive interference,<br>detailed wavefunctions             | 1991 | [16] <i>Carlson</i>           |
| 1.4-3.1                                  | $\Delta$ -isobar current                                        | 1992 | [17] <i>Schiavilla et al.</i> |

Why so unstable?

1) Pauli principle



→ 2-body contributions need to be calculated with very high precision

→ EFT\*

T.-S. Park et al.,

3-13 ~~2-129~~  
~~3-3-7~~

Phys. Rev. C 67, 055206 (2003)

4-body w.f. provided by

Schiavilla and the Pisa group

$$S_{\text{hep}}(0) = (8.6 \pm 1.3) \times 10^{-2} \text{ keV-b}$$

$\sim 15\%$  error

$\Rightarrow$

$\Phi_{\nu}(\text{hep})$  can now be predicted  
with a finite ( $\sim 15\%$ ) error.

cf. Bahcall's challenge

Possibility to check the reliability  
of the whole argument  
using data ?

~~3-14-20~~  
3-14-20

"hen"



Same type of suppression of 1-body  
contribution

→ dominance of EXC currents

→ stringent test of MEEFT

Y.-H. Song & T.-S. Park, nucl-th/0311055

$$\sigma = (60.1 \pm 3.2 \pm 1.0) \mu\text{b}$$

$$\sigma^{\text{exp}} = (54 \pm 6) \mu\text{b} \quad \text{or} \quad \sigma^{\text{exp}} = (55 \pm 3) \mu\text{b}$$

#### 4. Further Examination of MEEFT-EFT\*

4-1

$$T^{\text{EFT}^*} \sim \langle \Psi^{\text{SNPA}} | O^{\text{EFT}} | \Psi_i^{\text{SNPA}} \rangle$$

possible mismatch?

- ⊙ Construction of  $V_{ij}^{\text{EFT}}$  realistic enough, and its application to electroweak transitions yet to be done.

cf. e.g., Epelbaum, Gölke, U.G. Meißner et al.

- $V_{ij}^{\text{EFT}}$  derived by Epelbaum et al., uses a cut-off parameter  $\Lambda$ , which is consistent with the value used by Park et al.

→ We concentrate here on discussing issues within MEEFT.

© Big help from Stony Brook  
 cf. Bogner, Kuo & Schwenk,  
 Phys. Rep. 386 (03) 1.

$V^{\text{phenom}}$  is not unique for short distance  
 $\leftrightarrow$  off-shell ambiguity  
 model dependence

$V^{\text{phenom}}$  eliminate  $k \gtrsim 2.1 \text{ fm}^{-1}$   $V_{\text{low-}k}$

$V_{\text{low-}k}$  is unique !!

In EFT\*, there is a cut-off attached  
 to  $T$ .

$\rightarrow$  similar to the use of  $V_{\text{low-}k}$

off-shell ambiguity expected to be  
 reduced

\* The uniqueness of  $V_{low-k}$

4-100

~~FB~~  $\Lambda$ -independence of HOT

half-off-shell  $T$ -matrix

$\Omega = V^{-1}T$  : wave operator

$$|\psi^{dist}\rangle = \Omega |\psi_0\rangle$$

Criticism of " $V_{low-k}$ " à la Bogner et al.

- ~~of~~ HOT : unphysical

Why do we have to impose  $\Lambda$ -independence on HOT?

- Related problem

Are wave functions a physical observable?

"Integrating out" high-energy physics  
in nuclear physics

4-4

$$H = H_0 + V$$

$$H|\Psi\rangle = E|\Psi\rangle$$

$$\left\{ \begin{array}{l} P = \sum_{i=1}^d |i\rangle\langle i| \quad : \text{"model" space : low-energy regime} \\ Q = \sum_{i=d+1}^{\infty} |i\rangle\langle i| \end{array} \right.$$

$$\begin{pmatrix} PHP & PHQ \\ QHP & QHQ \end{pmatrix} \begin{pmatrix} P|\Psi\rangle \\ Q|\Psi\rangle \end{pmatrix} = E \begin{pmatrix} P|\Psi\rangle \\ Q|\Psi\rangle \end{pmatrix}$$

Solve for  $Q|\Psi\rangle$  and insert it in the equation controlling  $P|\Psi\rangle$

Bloch-Horowitz eq

$$H_{\text{eff}}^{B-H}(E) = H + H \frac{1}{E - QHQ} H$$

Energy-dependent !!!

Avoid energy dependence

Lee-Suzuki method

Phys. Lett. B 91 ('80) 173

Similarity transformation  $\mathbb{H}$

$$\mathbb{H}^{-1} H \mathbb{H} \equiv \mathcal{H}_{lowK}^{LS} = \begin{pmatrix} PHP & PHQ \\ 0 & QHQ \end{pmatrix}$$

block-triangular matrix

$$\mathbb{H} = 1 + W = \begin{pmatrix} 1 & 0 \\ W & 1 \end{pmatrix}$$

$$W = QCP$$

Bogner et al's  $V_{lowK}^{LS}$  corresponds to  $\mathcal{H}_{lowK}^{LS}$

→ No  $E$ -dependence

but

Non-hermitian

Unitary transformation methodS. Okubo, Prog. Theor. Phys. 12 (1954) 603.

Epelbaum et al.,

a recent reference

S. Fujii et al., PRC 70, 024003 (2004)

$$H\psi = E\psi$$

$$H' = U^\dagger H U, \quad \psi' = U^\dagger \psi$$

choose  $U$  in such a manner that

$$QH'P = 0$$

Energy-independent and Hermitian  $H'$ 

$$U = \begin{pmatrix} P(1 + \omega^\dagger \omega)^{-1/2} P & -P\omega^\dagger(1 + \omega\omega^\dagger)^{-1/2} Q \\ Q\omega(1 + \omega^\dagger \omega)^{-1/2} P & Q(1 + \omega\omega^\dagger)^{-1/2} Q \end{pmatrix}$$

$$V^{\text{low-}k} = P V_{\text{bare}}' P$$

This corresponds to imposing the  $\Lambda$ -independence condition only on on-shell  $T$ -matrix (phase shift, d-binding energy).

Bogner et al. report

4-7

$$\underline{V_{l=0-k}^{LS} \approx V_{l=0-k}^{\text{Hermitian}} \text{ numerically}}$$

Fujii et al report,

with  $\Lambda \approx 2 \text{ fm}^{-1}$  (used by the Stony Brook group),

$V_{l=0-k}^{LS}$  outbids  ${}^3\text{H}$  and  ${}^4\text{He}$

significantly. (by  $\sim 3 \text{ MeV}$ )

( $\Lambda \approx 5 \text{ fm}^{-1}$  needed to avoid this difficulty.)

Effects for  $T$  to be seen.

4-8  
● Is the  $\Lambda$  appearing in the derivation of  $V_{low-k}$   
something extraneous to  $\Lambda$  of EFT?

- In MEEFT,  $\Lambda$  is practically identified with  $\Lambda_{\text{EFT}}$ .

↔ Introduction of  $\Lambda$  does not ~~induce~~ induce additional transition operators.

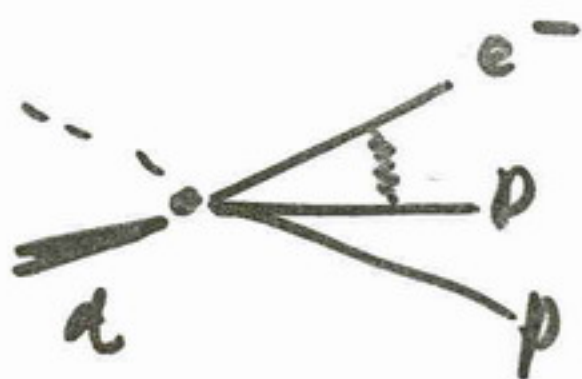
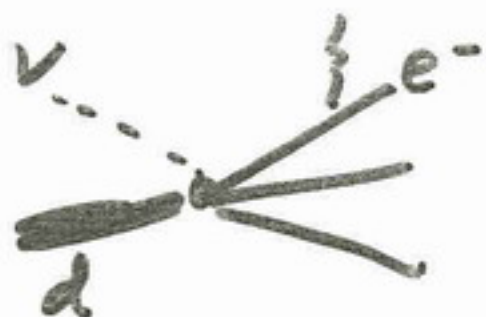
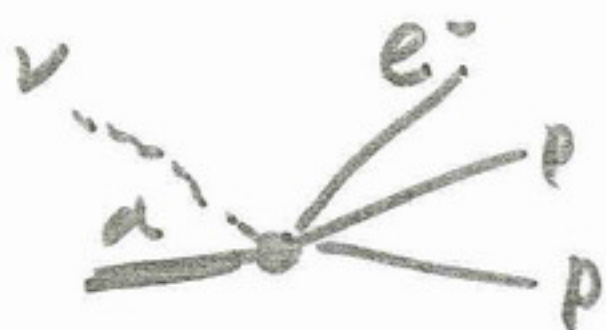
$$O \rightarrow O' \equiv U^\dagger O U$$

- Epelbaum et al. propose to consider possible differences between  $O$  and  $O'$

It is hoped that the use of "renormalization" by fitting to observables reduces the significance of this controversy.

# Radiative corrections

4-9



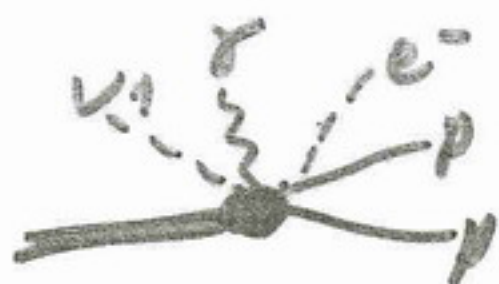
$\sigma(\nu d)$  may be affected  
up to 2~3 %

Similar effects expected  
for  $pp \rightarrow d + e^+ + \nu_e$ , etc.

• "Conventional approach"

4-10 (4)

Hadronic structure effects



$$\bar{u} [f_V/g^2] \gamma_\mu + f_A/g^2] \gamma_\mu \gamma_5 + \dots] u$$

$$j_\mu \rightarrow j_\mu e A_\mu$$

model dependence  
not well controlled

• Quark-lepton approach

At the current quark level

everything is well controlled



must be embedded in  
hadronic states

model dependence !!

# Application of EFT

S. Ando et al.,  
Phys. Lett. B 595 (2004) 250.

"Warm-up" problem

(In fact, important on its own right.  
CKM unitarity check.

SNS experiments )

$$1 \rightarrow (1 + \frac{\alpha}{2\pi} e_V^R)$$

$$g_A \rightarrow \tilde{g}_A [1 + \frac{\alpha}{4\pi} (e_A^R - e_V^R)]$$

Two unknown LEC's,  
 $e_V^R$  &  $e_A^R$

→ Application to  $nd$ ,  $pp$ , etc.

Acknowledgement :

4-12 #3

The work reported here is based on collaborations with :

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| Toru Sato        | (Osaka)        |
| V. Gudkov        | (USC)          |
| F. Myhrer        | (USC)          |
| H. Fearing       | (TRIUMF)       |
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My deep thanks to all of them.

- 57/10/19
- The Current status of MEEFT-EFT<sup>2</sup> has been reviewed
  - Its strengths and the formal problems involved in it discussed
  - The practical utility of MEEFT is hoped to survive these problems.
  - Further studies needed