A Diquark Effective Theory and Exotic Hadrons

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1. Introduction

- In 60's the naive quark model has been extremely successful. (Gell-Mann and Zweig 1964)
  1. Hadrons are made of either three constituent quarks, qqq ($\bar{q}\bar{q}\bar{q}$) or constituent quark and antiquark $q\bar{q}$.
  2. Static properties like the magnetic moments or mass differences of hadrons agree with the experimental data more than 10% accuracy.
- QCD tells us why the quark model works. (Georgi and Manohar 1984)
  1. Large scale-separation between confinement and χSB:

$$\Lambda_{\mathrm{QCD}} \simeq 150 \sim 200 \text{ MeV} \ll \Lambda_{\chi\mathrm{SB}} \simeq 4\pi f_\pi \simeq 1 \text{ GeV}.$$

  2. The explicit symmetry breaking terms are quite small:

$$\frac{\mu\, M}{\Lambda^2_{\chi\mathrm{SB}}} \ll 1.$$

- Discovery of Exotic Hadrons (2003)

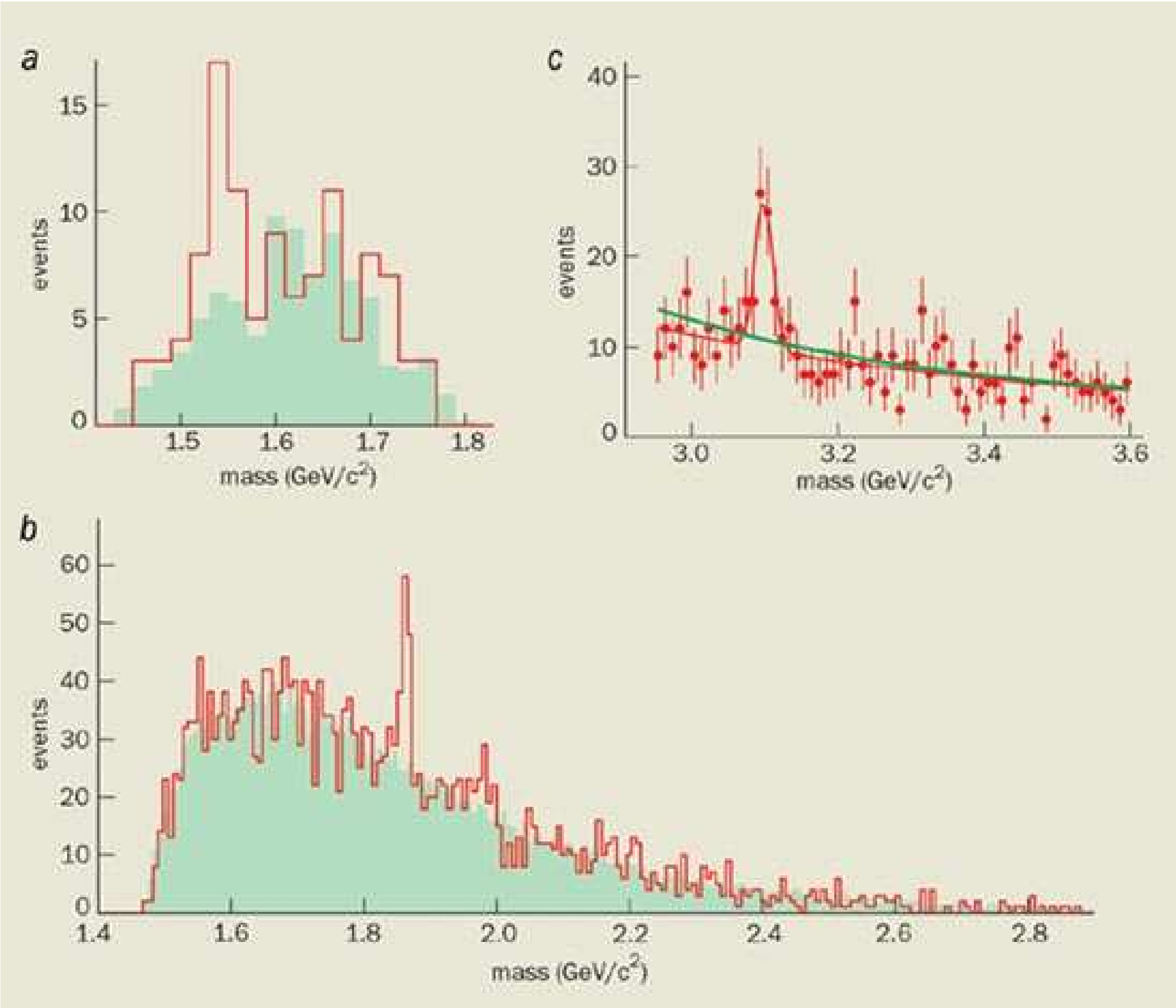


Figure 1: Experiments on Pentaquarks

- Recent discovery of exotic baryons is well explained by a naive diquark model of Jaffe-Wilczek which assumes strong quark-quark correlation in $(\bar{3}_c, \bar{3}_f, S=0)$ channel.
- For instance, its prediction on low dimensional multiplets and the mass spectrum of $\overline{10}_f$ were partially confirmed by NA49 data.

$$(\bar{3}_f \otimes \bar{3}_f)_S \otimes \bar{3}_f \; = \overline{10}_f \oplus 8_f$$

- The naive diquark model should be explained by QCD.
- The success of JW model can be explained by the diquark chiral effective theory, which describes the exotic hadrons from QCD.

DχET	$\simeq$	QCD with Diquarks

- Predictions of JW model

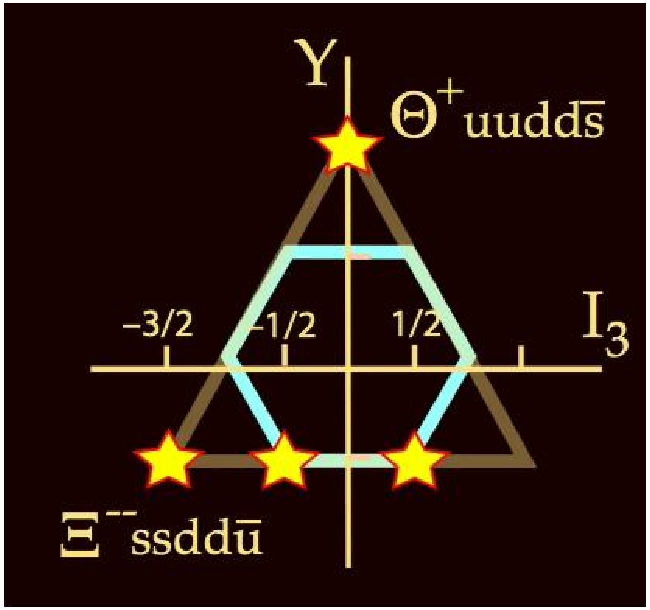


Figure 2: Degenerate $\overline{10}_f$ and 8_f.

- Competitor 1: Chiral Soliton Model (= Theory of mesons)
  1. Inconsistency with large N_C QCD. (T. Cohen; M. Praszalowicz)
  2. Large decay width ~ 30 MeV (Jaffe '04). (Recent calculation shows $\Gamma = 11$ MeV)
- Competitor 2: Uncorrelated Quark Model: Narrow width??

2. Diquark Chiral Effective Theory

- The diquark field is described by a scalar field,

$$\varphi^i_\alpha(x) = \lim_{y\to x} \frac{|y-x|^{\gamma_m}}{\kappa} \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \bar{\psi}^\beta_{cj}(x) i\gamma_5 \psi^\gamma_k(y), \quad (1)$$

where $\psi_c \equiv C\bar{\psi}^T$ is a charge conjugated field. (cf. Pseudo scalar diquarks are too heavy!)

- To account for the resonance energy of exotic baryons, JW assumed the diquark correlator has a strong peak around 4 - 500 MeV.

Flavor	Color	Spin	ΔE
$\bar{3}$ (A)	$\bar{3}$ (A)	0 (A)	-8
$\bar{3}$ (A)	6 (S)	1 (S)	-4/3
6 (S)	$\bar{3}$ (A)	1 (S)	8/3
6 (S)	6 (S)	0 (A)	4

Figure 3: MAC analysis

- The relevant energy scale of diquark field is $\Lambda_{\rm QCD} < E < \Lambda_{\chi{\rm SB}}$.
- The effective Lagrangian for diquarks, quarks, and pions is given as

$$\mathcal{L} \;=\; |(D_\mu + V_\mu)\,\varphi|^2 - M_0^2\,|\varphi_i|^2 - h_A {\rm Tr}\left(\varphi^*\varphi\, A_\mu^2\right) - h'_A\varphi\varphi^*\,{\rm Tr}\left(A_\mu^2\right)$$
$$-\;\left(g\,\varphi_i^{\alpha}\,\epsilon^{ijk}\,\epsilon_{\alpha\beta\gamma}\,\bar{\psi}^{\beta}_{cj}\,i\gamma_5\psi^{\gamma}_k + {\rm h.c.}\right) + \mathcal{L}'(\phi,\psi,\bar{\psi}) + \mathcal{L}_{\chi {\rm Q}}\,,$$

where $D_\mu\,\varphi = \partial_\mu\,\varphi + ig_s A_\mu^a\,T^{a*}\,\varphi$.

- We have introduced to implement the chiral symmetry, consistently with QCD,

$$V_\mu = \frac{1}{2}\left(\xi^\dagger\partial_\mu\xi\,+\,\xi\partial_\mu\xi^\dagger\right), \quad A_\mu = \frac{i}{2}\left(\xi^\dagger\partial_\mu\xi\,-\,\xi\partial_\mu\xi^\dagger\right) \tag{2}$$

and

$$\xi = e^{i\pi/f_\pi}\,, \tag{3}$$

where $\pi = \pi_a\,T_a$ and T_a are $SU(3)$ generators with a normalization ${\rm tr}\;(T_aT_b) = 1/2\,\delta_{ab}$.

- Under $SU(3)_L \times SU(3)_R$,

$$\begin{aligned} \xi &\mapsto & \xi' &= L\xi U^\dagger = U\xi R^\dagger \\ \varphi &\mapsto & \varphi' &= U^\dagger \varphi U. \end{aligned} \tag{4}$$

- The diquark effective theory describes how diquarks interact with quarks and NG bosons consistently with QCD.
- It incorporates systematically the symmetry breaking terms.
- We can also introduce a pseudoscalar diquark:

$$\varphi^i_{P\alpha}(x) = \lim_{y \to x} \frac{|y-x|^{\gamma_P}}{\kappa_P} \epsilon^{ijk} \epsilon_{\alpha\beta\gamma} \bar{\psi}^\beta_{cj}(x) \psi^\gamma_k(y) \tag{5}$$

$$\varphi_{R,L} = \frac{1}{2} \left(\varphi_S \pm i \varphi_P \right) \tag{6}$$

- The mass term becomes

$$\begin{aligned} \mathcal{L}_m &= -\Delta^2 \left|\varphi_R\right|^2 + \frac{1}{2} g_\pi f_\pi \varphi_R \xi^2 \varphi_R + (R \leftrightarrow L) \\ &= -\frac{1}{2} M_0^2 \left|\varphi\right|^2 - \frac{1}{2} M_P^2 \left|\varphi_P\right|^2 + \cdots \end{aligned} \tag{7}$$

- From the generalized Goldberg-Treiman relation, we get

$$M_P^2 - M_0^2 = g_\pi f_\pi$$

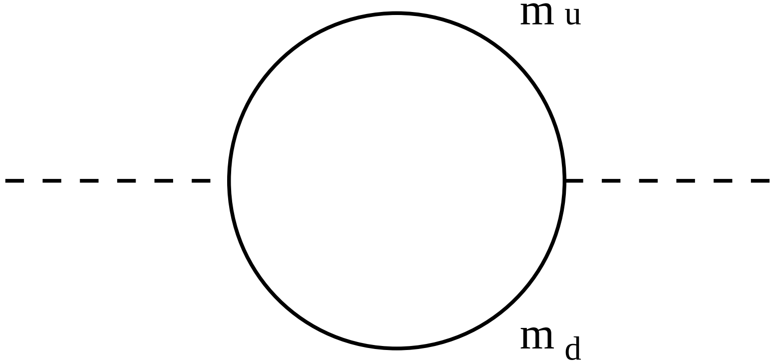
or

$$M_P - M_0 = g_{\pi D} f_\pi \simeq \frac{2}{3} g_{\pi N} f_\pi \simeq \frac{2}{3} M_N,$$

which is consistent with recent observation of chiral doubler in exotic baryons by H1 experiment.

3. Static properties of Exotic Baryons

- The couplings, g, $M_0, \cdots$ are to be determined by experimental data.
- The diquark mass at one-loop:



$$M_{jk}^2 = M_0^2 + \frac{4g^2}{\pi^2}\left[\left(m_j\, m_k - m_j^2 - m_k^2\right) + \frac{m_j^3 \ln\left(m_j^2/\mu^2\right) + m_k^3 \ln\left(m_j^2/\mu^2\right)}{m_j + m_k}\right]$$

- The best data is light nonet scalars, which are assumed to be bound states of diquark and anti-diquark.

- We find a mass formula

$$\begin{aligned} M(a_0) = \quad M(f_0) \quad &= M_{us} + M_{ds} - B \\ M(\kappa) \quad &= \quad M_{us} + M_{ud} - B \\ M(\sigma) \quad &= \quad 2M_{ud} - B, \end{aligned} \tag{8}$$

- Sum rule:

$$2M(\kappa) = \frac{1}{2}\left[M(a_0) + M(f_0)\right] + M(\sigma), \tag{9}$$

which works very well with the experimental values, $M(a_0) = M(f_0) =$ 980 MeV, $M(\kappa) = 800$ MeV, and $M(\sigma) = 500 \sim 600$ MeV.

- Taking the renormalization point $\mu \simeq 200$ MeV, we get in units of GeV

M_0	$M_{us}\,, M_{ds}$	M_{ud}	B	g^2
0.35	0.56	0.38	0.15	2.64
0.40	0.61	0.43	0.25	2.92
0.42	0.63	0.45	0.28	3.03
0.45	0.66	0.48	0.34	3.19
0.48	0.69	0.51	0.40	3.36

(10)

1. The coupling is not too large and the higher-order corrections are less than 10%.
2. The smallness of the higher-order corrections indicates that indeed the diquark picture captures the correct physics of QCD around $\Lambda_{\rm QCD} < E < \Lambda_{\chi {\rm SB}}$.
3. Diquark mass agrees with the random instanton model by Schafer-Shuryak-Verbaarschot.

- From nonet mass $M_{us} - M_{ud} \simeq 180 \sim 250$ MeV we get

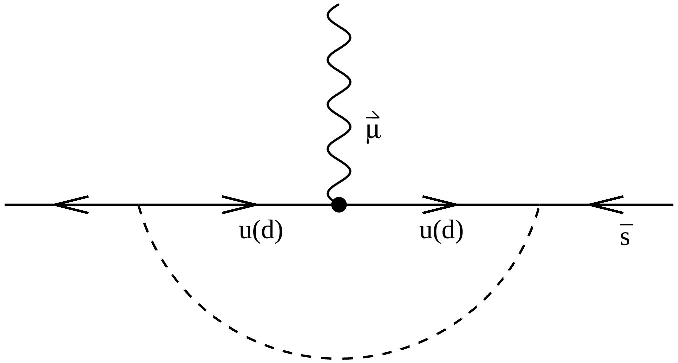
$$
\begin{aligned}
M(\Xi^{--}_{3/2}) - M(\Theta^{+}) &= (2M_{ds} + m_u) - (2M_{ud} + m_s) \\
&= 2\,(250 \sim 180)\ \text{MeV} - (540 - 360)\ \text{MeV} \\
&= 320 \sim 180\ \text{MeV} \qquad (11)
\end{aligned}
$$

which works reasonably well with the NA49 data.

- Magnetic Moments of Θ^+, which is important in photo-production :

$$\vec{\mu}_m(\Theta^+) = \vec{\mu}_{\bar{s}} + \vec{\mu}_L + \delta\vec{\mu}_m, \quad (\mu_{\bar{s}} = 0.75 \text{ n.m.}) \tag{12}$$

- At the leading order the correction becomes (other diagrams vanish)



$$\delta\mu_m = \mu_u \frac{g^2}{16\pi^2} \frac{m_u^2}{M_{ds}^2 - m_u^2} \left[1 - \frac{M_{ds}^2}{M_{ds}^2 - m_u^2} \ln\left(\frac{M_{ds}^2}{m_u^2}\right) \right] + (u \leftrightarrow d)$$

- Since diquarks are in p-wave, for $400\ \mathrm{MeV} \leq M_0 \leq 450\ \mathrm{MeV}$,

$$\mu_L = \frac{e}{3}\frac{1}{M_{ud}} \simeq 1.31 \sim 1.46\ \mathrm{n.m.}. \tag{13}$$

- We find $\mu_m(\Theta^+) = 0.71 \sim 0.56\ \mathrm{n.m.}$ for $J^P = \frac{1}{2}^+$, if we take $m_u \simeq m_d \simeq 360\ \mathrm{MeV}$ and $\mu_u = 1.98\,\mathrm{n.m.}$, $\mu_d = -1.10\,\mathrm{n.m.}$.
- For $J^P = \frac{3}{2}^+$, we get $\mu_m(\Theta^+) = 2.21 \sim 2.06\ \mathrm{n.m.}$.
- It is slightly bigger that the values obtained by QCD sum rule and by Chiral soliton model.
- The corrections, $\delta\mu_m \simeq 5 \times 10^{-3}$ n.m., are quite small.

4. Decay Width of Pentaquarks

- Puzzle: The decay width is unusually narrow!

Experiments	Results Mass (MeV)	Width (Mev)	Significance (σ)
LEPS	$1540 \pm 10 \pm 5$	$\Gamma < 25$	4.6 ± 1
DIANA	$1539 \pm 2 \pm$ "few"	$\Gamma < 8$	4.4
CLAS	$1542 \pm 2 \pm 5$	FWhM < 21	5.3 ± 0.5
SAPHIR	$1540 \pm 4 \pm 2$	$\Gamma < 25$	4.8
ITEP (ν's)	1533 ± 5	$\Gamma < 29$	6.7
HERMES	$1526 \pm 2 \pm 2.5$	$\Gamma < 20$	5.6
World Average	1535 ± 2.5		
Prediction	1530	$\Gamma < 15$	I=0 S=+1 $J^P = \frac{1}{2}^+$

- Decay of $\Theta^+ \mapsto K^+ + n$. The decay width is given as

$$\Gamma = \lim_{v \to 0} \sigma(\bar{s} + \phi_{ud} + \phi_{ud} \to K^+ + n)\, v\, |\psi(0)|^2 , \tag{14}$$

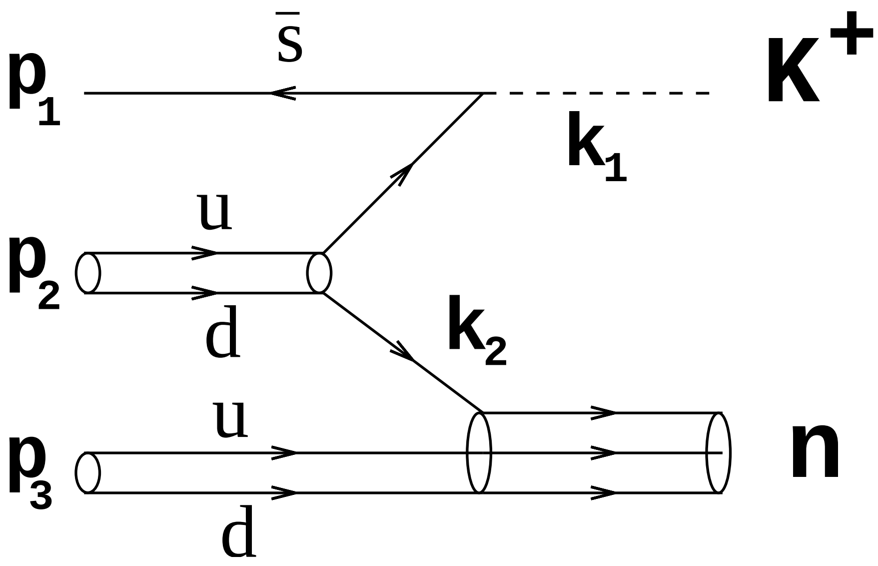


Figure 4: Decay process of $\Theta^+ \to K^+ + n$. Gluons are suppressed.

- Since $M_{jk} < m_j + m_k$, the decay of exotic baryons should occur through tunneling.

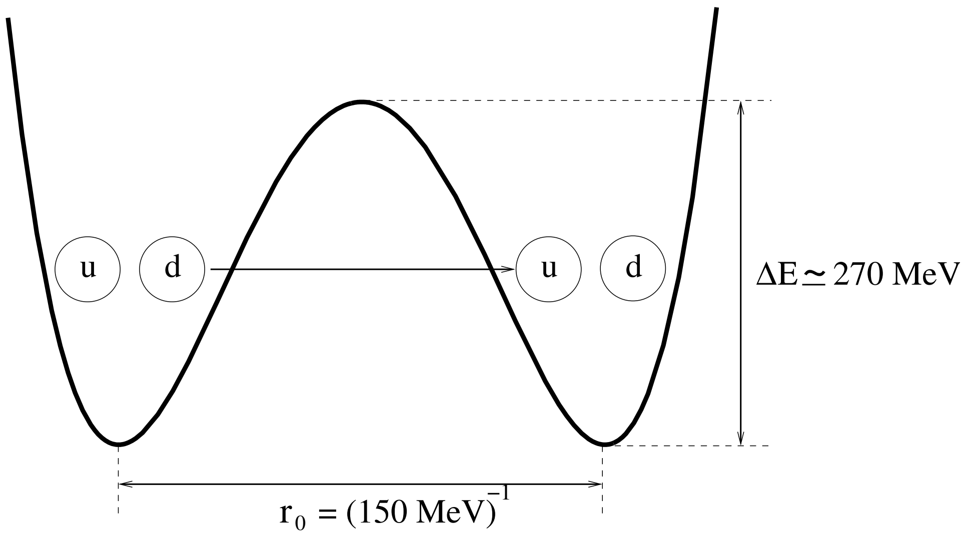


Figure 5: Decay as a tunnelling process

- Here d quark tunnels from one diquark φ_{ud} to the other diquark to form a neutron and an off-shell u quark. (If u were to tunnel, the decay is to $K^0 p$ with a comparable decay width.)

- The differential cross section is given as

$$d\sigma = \frac{(2\pi)^4 \, |\mathcal{M}|^2}{4\sqrt{(p_3 \cdot p_2)^2 - m_s^2 M_{ud}^2}} \, 4\, e^{-2S_0} \, d\Phi(p_3 + p_2; k_1, k_2), \qquad (15)$$

- Assuming factorization or small corrections by gluons, the amplitude becomes

$$\mathcal{M} = \frac{g_A \, g}{\sqrt{2} f_K} \, \bar{v}_s(p_1) \, \not{k}_1 \gamma_5 \, \frac{1}{\not{k}_1 - \not{p}_1 - m_u} \, \gamma_5 \, v_d(k_2), \qquad (16)$$

- Integrating over the phase space and taking $v \to 0$, we find the decay width

$$\Gamma_{\Theta^+} \simeq 5.0 \, e^{-2S_0} \, \frac{g^2 g_A^2}{8\pi f_K^2} \, |\psi(0)|^2 \, . \qquad (17)$$

- $e^{-S_0} = \langle n, \, u| \, H_{\rm int} \, |\varphi_{ud} \, , \varphi_{ud}\rangle$ is the tunnelling amplitude.

- Using the WKB approximation, we get the tunnelling amplitude,

$$e^{-S_0} = \langle n \,|\, T\, e^{i\int \mathrm{d}^4x\, \mathcal{L}_{\rm int}} \,|d\,,\varphi_{ud}\rangle \approx e^{-\Delta E\, r_0}, \qquad (18)$$

where $\Delta E = (m_u + m_d) - M_{ud} \simeq 270$ MeV is the diquark binding energy and r_0 is the distance between two diquarks.

- By the naive (di)-quark model,

$$M_{\Theta^+} = 2M_{ud} + m_{\bar{s}} + \frac{2}{M_{ud}\, r_0^2}, \qquad (19)$$

we find $r_0 = (150 \text{ MeV})^{-1} = 1.3$ fm, using the empirical mass $M_{\Theta^+} = 1540$ MeV.

- The tunneling amplitude becomes $e^{-1.8} \simeq 0.17$.

- The $1S$ wave function of the quark-diquark at the origin is

$$\psi(0) = \frac{2}{a_0^{3/2}} \frac{1}{\sqrt{4\pi}}, \qquad a_0 \simeq (2\overline{m}\, B)^{-1/2} \qquad (20)$$

- Taking $B = 100 \sim 200$ MeV, comparable to the pentaquark binding energy, $g^2 = 3.03$ and $g_A = 0.75$ from the quark model, we find

$$\Gamma_{\Theta^+} \simeq 2.5 \sim 7.0 \text{ MeV}. \tag{21}$$

- Since the binding energy the us (or ds) diquarks $\Delta E_1 \simeq 270$ MeV, for the $\Xi^{--}_{3/2}$ the average separation of P-wave strange-diquarks $r_1 \simeq (270 \text{ MeV})^{-1}$.

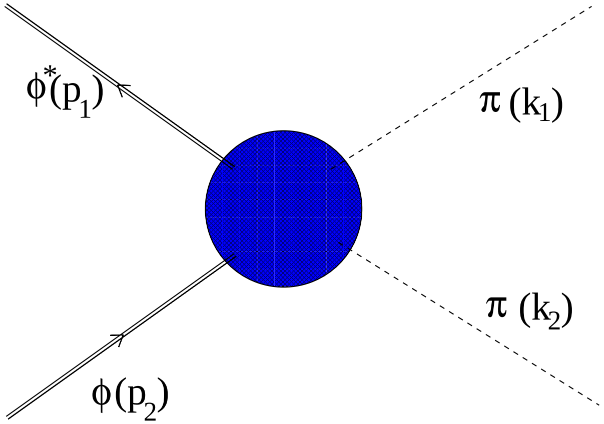
$$\Gamma_{\Xi^{--}_{3/2}} \simeq 0.51\, e^{-2\Delta E_1\, r_1} \cdot \frac{g^2 g_A^2}{8\pi f_\pi^2}\, |\psi_u(0)|^2 \simeq 1.7 \sim 4.8 \text{ MeV}, \tag{22}$$

- The relative partial decay width is

$$\frac{\Gamma_{\Xi^{--}_{3/2}}}{\Gamma_{\Theta^+}} \simeq 0.1 \left[\frac{f_K\, |\psi_u(0)|}{f_\pi\, |\psi_s(0)|}\right]^2 e^{2(\Delta E\, r_0 - \Delta E_1\, r_1)} \simeq 0.7\,. \tag{23}$$

5. Applications to Scalar Nonet

- Decay process of scalar nonet



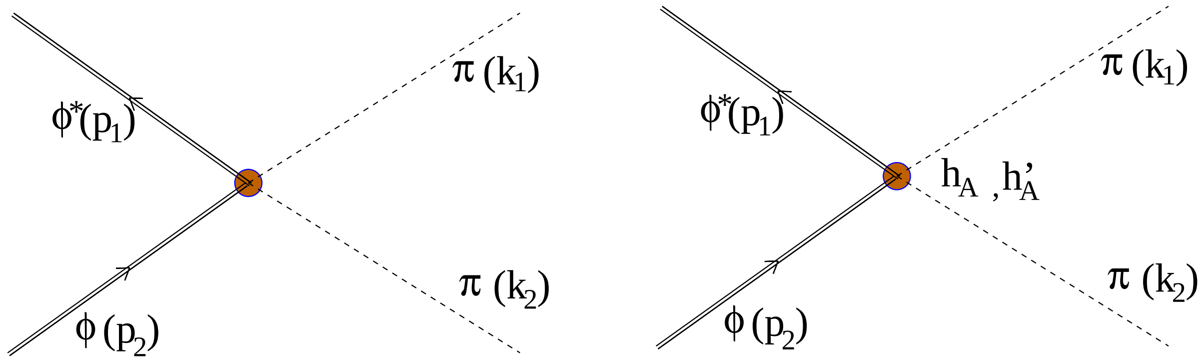
$$\Gamma_{S\to\pi\pi} = \lim_{v_2\to 0} v_2\,\sigma\,(\varphi+\varphi^* \to \pi+\pi)\,|\Psi(0)|^2 + O(v_2^2) \qquad (24)$$

- Integrating over the phase space

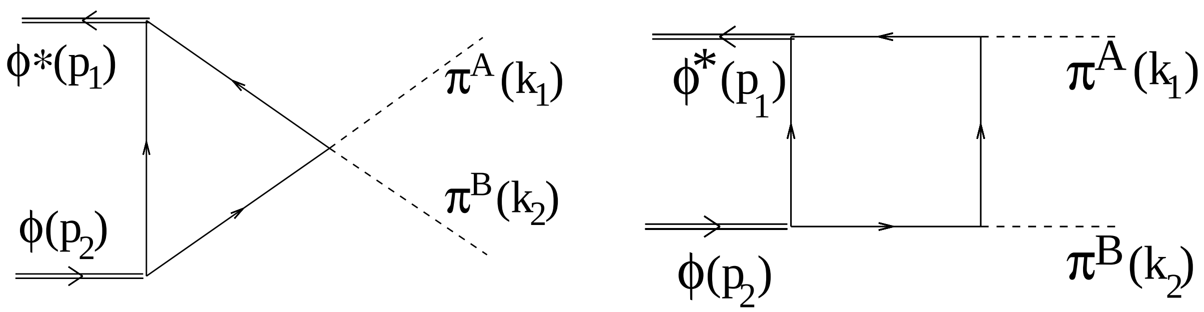
$$\Gamma_{S\to\pi\pi} = \frac{1}{16\pi}\frac{|\mathcal{M}|^2}{M_1 M_2}\frac{|\vec{k}_1|}{\sqrt{\vec{k}_1^2+m_A^2}+\sqrt{\vec{k}_1^2+m_B^2}}|\Psi(0)|^2, \qquad (25)$$

- To the order we are interested in

$$\mathcal{M} = \mathcal{M}_{\rm tree} + \mathcal{M}_{\rm 1\,loop}. \qquad (26)$$



(a) (b)



(a) (b)

- Fitting the couplings with data

set	h_A	h'_A	$\Gamma_{\sigma\to 2\pi}$(MeV)	$\Gamma_{f_0\to 2\pi}$(MeV)	$\Gamma_{a_0\to \eta\pi}$(MeV)
1	1.46	-1.66	317	67	83
2	1.20	-1.38	234	71	57

- Postdiction for subdominant modes: $\Gamma^{\rm exp}_{f_0\to 2K} = 5.6 \sim 32$ MeV

$$\Gamma_{S\to\pi^A\pi^B} = N \int_{s_{min}}^{s_{max}} \frac{{\rm d}s}{4\pi^2\, s} \frac{|\vec{k}_f|\,|\mathcal{M}|^2}{\sqrt{\vec{k}_f^2 + m_A^2} + \sqrt{\vec{k}_f^2 + m_B^2}} \frac{m_S\,\Gamma_S\,|\Psi_s(0)|^2}{(s-m_S^2)^2 + m_S^2\Gamma_S^2}$$

- We obtain 75 MeV with set 1 and 59 MeV with set 2 for $\Gamma_{f_0\to 2K}$.

6. Conclusion and Outlook

- Diquark effective theory captures a correct physics of QCD.

$$\frac{g^2}{16\pi^2} \simeq 0.2$$

- It provides a systematic approach for exotics:

1. Magnetic moment:

$$\begin{aligned}\mu_m(\Theta^+) &= 0.71 \sim 0.56 \text{ n.m. for } J^P = \frac{1}{2}^+, \\ &\quad 2.21 \sim 2.06 \text{ n.m. for } J^P = \frac{3}{2}^+\end{aligned}$$

2. Mass spectrum in $\overline{10}_f$:

$$M(\Xi^{++}_{3/2}) - M(\Theta^+) = 320 \sim 180 \text{ MeV}$$

3. Decay widths

$$\Gamma_{\Theta^+} \simeq 5.0\, e^{-2S_0} \frac{g^2 g_A^2}{8\pi f_K^2} |\psi(0)|^2 \simeq 2.5 \sim 7.0 \text{ MeV}$$

$$\Gamma_{\Xi^{++}} \simeq 0.51\, e^{-2\Delta E_1 r_1} \cdot \frac{g^2 g_A^2}{8\pi f_\pi^2} |\psi_u(0)|^2 \simeq 1.7 \sim 4.8 \text{ MeV}.$$

which is quite small due to the tunneling, $e^{-2S_0} \simeq 0.03$.

⋆ N.B. We need to know more about the diquark potential.

- Rigorous QCD calculations like lattice QCD or others are needed to estimate the diquark correlation.
- In reasonably good agreement with the experimental data on the decay width of scalar nonet. $h_A = 1.20, \quad h'_A = -1.38$
- Production of pentaquarks in DχQET (with Song and Sohn)