Introduction to Cosmology

Basic units

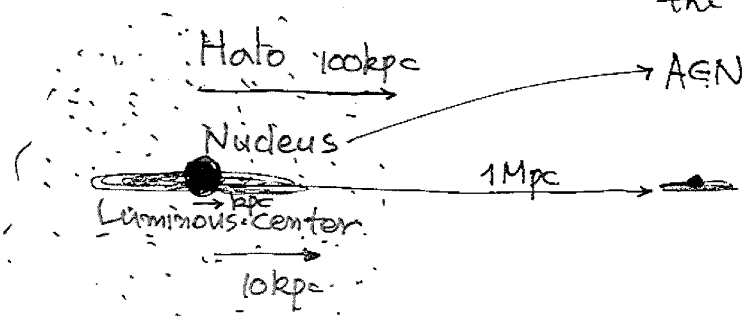
$\hbar = c = k_B = 1$

$1\,pc = 3.26$ light years $= 3.09 \times 10^{16}\,m$

$M_\odot = 1.99 \times 10^{33}\,g$

Building blocks

- Stars : $1 - 10\,M_\odot$
- Galaxies : $10^6\,M_\odot - 10^{12}\,M_\odot$ basic building blocks of the universe.



→ AGN : Seyfert galaxies
quasars, blazers, ...

- Clusters (Group) of galaxies : 2 – 1000 galaxies
- Superclusters : ~ 100 Mpc
  sheets, filaments & voids
- Above 100 Mpc : Homogeneous distribution of matter
- Observable universe : $\sim 10^4$ Mpc
- Beyond the observable universe

The universe observed

Observational basis for the standard cosmology

- The expansion of the universe
  - Hubble constant H_0
  - Deceleration parameter q_0
- The age of the universe t_0
- The matter content of the universe
  - Ω_i , $i = M(B, DM), \Lambda, R, \cdots$
- CBR : CMB, IR, UV, RW, X, γ, ν, CR, $\cdots$
- The abundances of light elements : D, ^3He, ^4He, ^7Li
- The distribution of galaxies.

① The expansion

- luminosity distance - red shift
- angular diameter - "
- galaxy count - "

"known" absolute luminosity

Luminosity distance $d_L = \left(\frac{\mathcal{L}}{4\pi F_i} \right)^{1/2}$

"measured" flux

Red shift $z = \frac{\lambda_m}{\lambda_0} - 1$

measured wavelength.
original "

$$H_0 d_L = z + \frac{1}{2}(1 - q_0) z^2 + \cdots$$

Hubble constant — Deceleration parameter

$H_0 = 100\,h$ Km/s/Mpc — $\rightarrow$ Matter content

$H_0^{-1} \rightarrow$ cosmic scale

$$q_0 = \sum_i \Omega_i \frac{1 + 3w_i}{2}$$

Hubble time $9.78\,h^{-1} \times 10^9$ yr
Hubbledistance $3000\,h^{-1}$ Mpc

② Large scale isotropy & homogeneity

- Uniformity of CMB temperature

  $T_0 = 2.726 \pm 0.01\,K$

  $\Delta T_{dipole} = 3.365 \pm 0.027\ mK$

  $\Delta T/T \approx 10^{-5}$

  $\Rightarrow$ At last scattering of CMB, the universe was highly isotropic & homogeneous.

- Distribution of galaxies
- Isotropy of X-ray background
- Peculia velocity field

③ The age of the universe

- The expansion rate of the universe
- Dating the oldest stars in globular clusters
- Dating the radio active elements
- The cooling of white dwarf stars
- The cooling of hot gas in clusters

$10\ \mathrm{Gyr} \lesssim t_0 \lesssim 20\ \mathrm{Gyr}$

$t_0 = H_0^{-1} f(\Omega_i)$

$H_0^{-1} = 9.78 h^{-1}\,\mathrm{Gyr}$

Model	$f(\Omega_i)$
$\Omega_T = 0$	1
$\Omega_T = \Omega_M = 1$	$\frac{2}{3}$
$\Omega_M = 0.2\ \Omega_\Lambda = 0.8$	

④ CMB

- The last scattering surface

  $z \sim 1100$, $t \sim 180{,}000\,(\Omega_0 h^2)^{-1/2}$ yr

- Spectrum : Blackbody, with $T = 2.726 \pm 0.01$ K
- Anisotropy : Variation in the CMB temperature in different directions.
  - the motion of our local reference frame (the earth) w.r.t the cosmic rest frame.
  - rotation of the universe
  - anisotropic universe
  - <u>the presence of density inhomogeneity</u>

$\Delta T_{dipole} = 3.365 \pm 0.027$ mK

$\rightarrow v_{local} = 627 \pm 22$ km/s to $\begin{cases} RA = 166^\circ \pm 3^\circ \\ DEC = -27.1 \pm 3^\circ \end{cases}$

$\Delta T_Q = 11 \pm 3\,\mu K$

$(\Delta T/T)_{rms} \simeq 10^{-5}$

$\rightarrow$ Powerful test of structure formation theories.

⑤ Light element abundances

- Nucleosynthesis

  $t \simeq 0.01 - 100$ sec, $T \simeq 10$ MeV $-$ 0.1 MeV

  D : D/H = few $\times 10^{-5}$ ← No known astrophysical processe can account for these values.

  ^3He : $^3He/H \simeq$ few $\times 10^{-5}$

  ^4He : $Y = 4He/N \simeq 0.25$ ⇒ only in "Nucleosynthesis".

  ^7Li : $^7Li/H \simeq 1\text{-}2 \times 10^{-10}$

$\rightarrow$ Determination of baryon density

$\eta = (4-7) \times 10^{-10}$, $0.015 \lesssim \Omega_B h^2 \lesssim 0.026$

- Constraint on the light species.

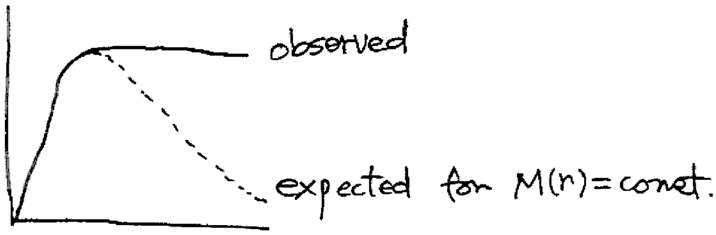
⑥ The matter density

$\Omega_{Lum} \lesssim 0.01$

$\Omega_{Halo} \gtrsim 0.1$ ← rotation curve

• rotation curve

$$\frac{GM(r)m}{r^2} = \frac{mv^2}{r}, \qquad v^2 = \frac{GM(r)}{r}$$



• Ω problem

$\Omega_{LUM} < \Omega_B < \Omega_M$

⑦ Large scale structure

• Hierarchy : stars – nebula – Lyα absorber
galaxies – clusters – superclusters, voids, walls,

• Galaxy catalogs

• structure formation

One page review of General Relativity

Newtonian gravity : Gravitational potential = Matter (mass)

General relativity : Geometry = Matter (energy)

Geometry: metric $g_{\mu\nu}$ — Matter: EM tensor $T_{\mu\nu}$

$\rightarrow$ Einstein equation $G_{\mu\nu} = \kappa^2 T_{\mu\nu}$, with $\kappa^2 = 8\pi G$

Einstein T $\quad G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}$

Ricci T $\quad R_{\mu\nu} = R^{\lambda}{}_{\mu\lambda\nu}$

Riemann T (curvature) $\quad R^{\rho}{}_{\sigma\mu\nu} = \partial_\mu \Gamma^{\rho}_{\nu\sigma} - \partial_\nu \Gamma^{\rho}_{\mu\sigma} + \Gamma^{\rho}_{\mu\lambda}\Gamma^{\lambda}_{\nu\sigma} - \Gamma^{\rho}_{\nu\lambda}\Gamma^{\lambda}_{\mu\sigma}$

RC symbols (Connection) $\quad \Gamma^{\sigma}{}_{\mu\nu} = \frac{1}{2} g^{\sigma\rho} \left(\partial_\mu g_{\nu\rho} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} \right)$

Einstein equation can be derived from the EH action

$$S = \int d^4x \sqrt{-g} \left(\kappa^{-2} R + \mathcal{L}_M \right)$$

> Prob. 1. Derive Einstein equation from the EH action.

- Geodesic equation : $\dfrac{d^2 x^\mu}{d\tau^2} + \Gamma^{\mu}_{\rho\sigma} \dfrac{dx^\rho}{d\tau} \dfrac{dx^\sigma}{d\tau} = 0$

  Path of freely falling particle.

- Isometry and Killing vector

  Symmetry of manifold (spacetime) $\quad \xi_{\mu;\nu} + \xi_{\nu;\mu} = 0$.

  maximally symmetric space : $\frac{n(n+1)}{2}$ Killing vectors

  2D example : E^2, S^2, H^2

FRW universe

Copernican principle : the universe is pretty much the same "everywhere".

Observational facts

- The distribution of matter and radiation in the "observable universe" is homogeneous & isotropic.
- The universe is "not" static : distant galaxies are receeding from us.

→ Our local Hubble volume during Hubble time
~ spacetime with homogeneous and isotropic spatial sections.

$$M = \mathbb{R} \times \Sigma \quad \leftarrow \text{Maximally symmetric, 3D}$$

RW metric

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d^2\theta + \sin^2\theta\, d\phi^2)\right]$$

$k=-1$	open	H^3
$k=0$	flat	E^3
$k=+1$	closed	S^3

Note: The assumption of local homogeneity and isotropy only implies that the spatial metric is locally S^3, H^3 or R^3 and can have ~~global~~ different global properties.

Kinematics of RW metric

- **Particle horizon**

Consider the light propagation in RW metric, $ds^2=0$.

$$\int_0^t \frac{dt}{a(t)} = \int_0^{r_H} \frac{dr}{\sqrt{1-kr^2}}$$

The proper distance to the horizon

$$d_H(t) = \int_0^{r_H} \sqrt{g_{rr}}\, dr = a(t)\int_0^t \frac{dt'}{a(t')}$$

- **Freely falling particle**

$$u^\mu \equiv \frac{dx^\mu}{ds}$$ "Peculiar velocity"
4-velocity w.r.t. the comoving frame

$$\frac{du^\mu}{ds} + \Gamma^\mu_{\nu\lambda} u^\nu u^\lambda = 0$$

$$\mu=0: \frac{du^0}{ds} + \Gamma^0_{\mu\lambda} u^\nu u^\lambda = \frac{du^0}{ds} + \frac{\dot a}{a}|\vec u|^2 = 0.$$

> **Prob. 2.** For RW metric, verify
> $$\Gamma^0_{ij} = \frac{\dot a}{a} g_{ij}, \quad \Gamma^i_{0j} = \Gamma^i_{j0} = \frac{\dot a}{a}\delta^i_j, \quad \Gamma^i_{jk} = ??$$

$$u^2 = (u^0)^2 - |\vec u|^2 = 1, \quad u^0 du^0 = |\vec u|\, d|\vec u|$$

$$(u^0)^{-1}|\vec u| \frac{d|\vec u|}{ds} + \frac{\dot a}{a}|\vec u|^2 =$$

$$\frac{dt}{ds} \qquad \frac{d|\vec u|}{dt}$$

$$\frac{|\dot{\vec u}|}{|\vec u|} = -\frac{\dot a}{a} \longrightarrow |\vec u| \propto a^{-1}$$

The magnitude of the 3-momentum of a freely falling particle red shifts as a^{-1}.

This is also true for massless particles.

$$\boxed{\frac{\lambda_0}{\lambda_e} = \frac{a(t_0)}{a(t_e)} \equiv 1+z \leftarrow \text{red shift parameter}}$$

- Hubble's law

A source (e.g. galaxy) with absolute luminosity $\mathcal{L}$

$r=0$ $\qquad$ $r=r_1$

detection at t_0 $\quad$ emission at t_1

Luminosity distance

$$d_L^2 \equiv \frac{\mathcal{L}}{4\pi\mathcal{F}}$$ ← measured flux

$$\mathcal{F} = \frac{\mathcal{L}}{4\pi\,(a(t_0)r_1)^2\underbrace{(1+z)^2}}$$

- red shift of photon
- increased time interval between detection

$$d_L^2 = [a(t_0)r_1]^2(1+z)^2$$

With the knowledge of $a(t)$, we can express d_L i.t.o. z.
Taylor expansion gives

$$H_0 d_L = z + \frac{1}{2}(1-q_0)z^2 + \cdots$$

$$H_0 = \frac{\dot{a}_0}{a_0}, \quad q_0 = -\frac{a_0\ddot{a}_0}{\dot{a}_0^2} = \Omega \sum_i \left(\frac{1+3\omega_i}{2}\right)$$

- Galax count – red shift relationship
- Angular diameter – red shift relationship

Dynamics of RW metric

Einstein equation

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}$$

LHS : RW metric

Connection $\Gamma^0_{ij} = a\dot{a}\,\tilde{g}_{ij}$, $\Gamma^i_{0j} = \frac{\dot{a}}{a}\delta^i_j$, $\Gamma^i_{jk} = \tilde{\Gamma}^i_{jk}$

Ricci tensor $R_{00} = -3\frac{\ddot{a}}{a}$, $R_{ij} = (a\ddot{a} + 2\dot{a}^2 + 2k)\,\tilde{g}_{ij}$

Ricci scalar $R = 6\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right)$

RHS : Matter content of the universe.

To be consistent with the symmetries of metric

$T_{\mu\nu}$ must be diagonal and $T_{11} = T_{22} = T_{33}$

$\rightarrow$ Energy-momentum tensor of perfect fluid.

$$T_{\mu\nu} = (p+\rho)\,U_\mu U_\nu + p\,g_{\mu\nu}$$

p, ρ : functions of t only

$U_\mu = (1,0,0,0)$ fluid at rest in comoving frame.

(00) $3\left(\frac{\dot{a}^2}{a^2} + \frac{k}{a^2}\right) = \kappa^2 \rho$ "Friedmann equations"

(ii) $2\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = -\kappa^2 p$

> Prob. 3 Verify Ricci tensor and Friedmann equations.

> Prob 4. Derive the conservation equation $T_{\mu\nu}{}^{;\nu} = 0$ and discuss its physical meaning.

$T_{\mu\nu;}{}^{\nu} = 0 \longrightarrow \dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$

We have two independent equations for three unknowns a, ρ & p. So we need one more equation.

- Matter dynamics
- The equation of state enters here. $p = p(\rho)$

Basic equations for dynamical cosmology

① Friedmann eq. $\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\kappa^2}{3}\rho$

② EM conservation eq. $\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0.$

③ Equation of state $p = p(\rho)$

Most kinds of matter content of our interest can be described by the simple equation of state $p = \omega\rho$.

EM conservation eq. $\longrightarrow \rho \propto a^{-3(1+\omega)}$

$\omega = \frac{1}{3}$, $p = \frac{1}{3}\rho$ Radiation $\rho \propto a^{-4}$

$\omega = 0$, $p = 0$ Matter $\rho \propto a^{-3}$

$\omega = -1$, $p = -\rho$ Vacuum energy $\rho = \text{const.}$ (Cosmological constant)

Note 1. Particle physics enters cosmology through matter dynamics, the matter content of the universe.

Note 2. The existence of Big Bang.

$$\frac{\ddot{a}}{a} = -\frac{\kappa^2}{6}(\rho + 3p)$$

$\underline{\rho + 3p > 0} \Longrightarrow a = 0$ must have reached at some finite time in the past.

(Weak energy condition)

The expansion rate of the universe

<u>Hubble parameter</u> $H(t) \equiv \dfrac{\dot{a}(t)}{a(t)}$

Note: The Hubble constant is the present value of Hubble parameter.

Note: H^{-1}, Hubble time and length
time scale for the expansion.

<u>The critical density</u> $\rho_c \equiv \dfrac{3H^2}{\kappa^2}$

the ratio of density to the critical density.

$$\Omega_i \equiv \frac{\rho_i}{\rho_c} \quad \text{Density parameter}$$

Correspondence between the signs of k and $\Omega - 1$

$$\frac{\dot{a}^2}{a^2} + \frac{k}{a^2} = \frac{\kappa^2}{3}\rho \rightarrow \frac{k}{H^2a^2} = \frac{\rho}{\rho_c} - 1 = \Omega - 1.$$

$\Omega > 1 \longleftrightarrow k = +1$ closed

$\Omega = 1 \longleftrightarrow k = 0$ flat

$\Omega < 1 \longleftrightarrow k = -1$ open

Note: At early times, the curvature term is negligible.

$$\frac{k}{a^2} \propto a^{-2} \ll \frac{\kappa^2}{3}\rho \propto a^{-3} \text{ or } a^{-4} \text{ as } a \to 0.$$

So, the Friedmann equation at early time is

$$H^2 = \frac{\kappa^2}{3}\rho$$

Note: Behavior of $\Omega - 1$

$$\Omega - 1 = \frac{k}{H^2a^2} \propto \frac{1}{\rho a^2} \propto \begin{cases} a & \text{for MD} \\ a^2 & \text{for RD} \end{cases}$$

$$|\Omega - 1| \simeq \begin{cases} (1+z)^{-1} & \text{MD} \\ 10^4 (1+z)^{-2} & \text{RD} \end{cases}$$

spatial curvature. $^3R^{(3)} = \frac{6k}{a^2} = 6H^2(\Omega-1)$

radius of curvature $R^{(3)}_{cur} \equiv a|k|^{-1/2} = \frac{H^{-1}}{|\Omega-1|^{1/2}}$

- At earlier epoch, the universe was nearly critical.
- The universe close to critical density is very flat.

Integration of Friedmann equation.

$$\left(\frac{\dot a}{a}\right)^2 + \underbrace{\frac{k}{a^2}}_{H_0^2(\Omega_0-1)} = \frac{\kappa^2}{3}\sum_i \rho_i = \frac{\kappa^2}{3}\sum_i \rho_{i0}\left(\frac{a}{a_0}\right)^{-3(1+\omega_i)}$$

$$p_i = \omega_i \rho_i \rightarrow \left(\frac{\rho_i}{\rho_{i0}}\right) = \left(\frac{a}{a_0}\right)^{-3(1+\omega)}$$

$$\left(\frac{\dot a}{a_0}\right)^2 = H_0^2\left[1-\Omega+\sum_i \Omega_i\left(\frac{a}{a_0}\right)^{-(1+3\omega_i)}\right]$$

$$H_0 t = \int_0^{a/a_0}\left[1-\Omega+\sum_i \Omega_i x^{-1-3\omega_i}\right]^{-1/2} dx.$$

The age of the universe

$$H_0 t_0 = \int_0^1 \left[1-\Omega+\sum_i \Omega_i x^{-1-3\omega_i}\right]^{-1/2} dx \equiv f(\Omega_i)$$

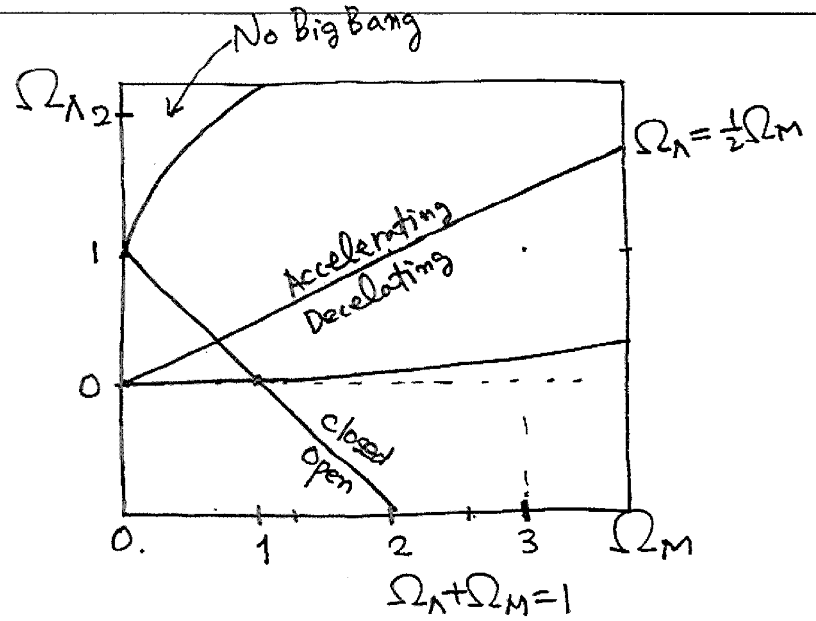
Scale factor as a function of time can be obtained by solving $H_0 t$ for a/a_0. $a \propto t^{2/3(1+\omega)}$ $(\omega \neq -1)$, $e^{H_0 t}$ $(\omega = -1)$

$t^{2/3}$ (MD), $t^{1/2}$ (RD)

Currently favored values

$\Omega = 1$, $\Omega_\Lambda = 0.7$-0.8, $\Omega_M = 0.2$-0.3

$(\Omega_R \simeq 10^{-4})$



Prob. 5. Identify the lines in the figure and draw the contour lines for constant t_0.

Note. The age of the present universe provides a very powerful constraint on Ω.

Prob. 6. Determine the relationship between the luminosity distance d_L and redshift z, as a function of the cosmological parameters Ω_M, Ω_Λ. To what order in z should we go to determine independently those parameters

Prob 7. Calculate the horizon distance $d_H(t)$ as a function of the cosmological parameters.

Equilibrium thermodynamics

- Direct evidence for a hot early universe — CMB
  isotropic, accurate blackbody spectrum with $T \approx 3K$.

  $\longrightarrow$ The early universe
  Hot. ideal gas in thermal equilibrium

- Distribution function in thermal equilibrium
$$f(\vec{p}) = \frac{1}{e^{(E-\mu)/T} \mp 1} \qquad \begin{matrix} +\text{FD} \\ -\text{BE} \end{matrix}$$
μ: chemical potential, T: Temperature

Number density
$$n = \int \frac{g\, d^3\vec{p}}{(2\pi)^3} f(\vec{p}) = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2-m^2)^{1/2} E}{e^{(E-\mu)/T} \pm 1} dE$$
Energy density
$$\rho = \int \frac{g\, d^3\vec{p}}{(2\pi)^3} E(\vec{p}) f(\vec{p}) = \frac{g}{2\pi^2}\int_m^\infty \frac{(E^2-m^2)^{1/2} E^2}{e^{(E-\mu)/T} \pm 1} dE$$
Pressure
$$P = \int \frac{g\, d^3\vec{p}}{(2\pi)^3} \frac{|\vec{p}|^2}{3E} f(\vec{p}) = \frac{g}{6\pi^2}\int_m^\infty \frac{(E^2-m^2)^{3/2}}{e^{(E-\mu)/T} \pm 1} dE$$

Relativistic, Non-degenerate $T \gg m,\ T \gg \mu$
$$n = \left[\tfrac{3}{4}\right] \frac{\zeta(3)}{\pi^2} g T^3, \quad \rho = \left[\tfrac{7}{8}\right] \frac{\pi^2}{30} g T^4, \quad P = \tfrac{1}{3}\rho$$

Non-relativistic, $m \gg T$
$$n = g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-(m-\mu)/T}$$
$$\rho = mn + \tfrac{3}{2}P$$
$$P = nT \ll \rho$$

The ~~total~~ energy density : The energy density of nonrelativistic species is exponentially smaller than that of relativistic species

$$\rho_R = \frac{\pi^2}{30} g_* T^4$$

$$P_R = \frac{1}{3}\rho_R$$

$$g_* = \sum_{\text{bosons}} g_b \left(\frac{T_i}{T}\right)^4 + \frac{7}{8}\sum_{\text{fermions}} g_f \left(\frac{T_i}{T}\right)^4 \quad \text{if } T_i \neq T.$$

- Entropy

The entropy in a comoving volume is conserved in thermal equilibrium.

→ Entropy is a useful fiducial quantity during expansion

$$s = \frac{\rho + P}{T}$$

Entropy in the early universe is dominated by relativistic p'les.

$$s = \frac{2\pi^2}{45} g_* T^3 \qquad \left(\text{cf. relation to } n_\gamma ; s = \frac{\pi^4}{45\zeta(3)} g_* n_\gamma\right)$$

Uses of entropy conservation

① Number density $n \propto a^{-3}$, $s \propto a^{-3}$

$Y \equiv \frac{n}{s}$ is convenient to represent the abundances for decoupled particle.

② The evolution of temperature of the universe

$$S \propto g_* T^3 a^3 = \text{const.}$$

$$\underline{T \propto g_*^{-1/3} a^{-1}}$$

Thermal history of the universe

- Thermal equilibrium : The universe has for much of its history been very nearly in thermal equilibrium.
- Departure from thermal equilibrium Makes fossil records of the universe.

- Rule of thumb for thermal equilibrium.

  Interaction rate $\qquad$ Expansion rate

$$\Gamma_{int} \quad > \quad H$$

$$\Gamma_{int} \equiv n \langle \sigma |v| \rangle = \Gamma_{int}(T)$$

Note: For $\Gamma_{int} = aT^n \; (n>2)$

$$N_{int} = \int_t^\infty \Gamma_{int}(t')dt' = \frac{(\Gamma/H)|_t}{n-2}$$

A particle interacts less than once after $\Gamma = H$

- Rough understanding of decoupling

① interaction mediated by a massless gauge boson

$$\sigma \sim \frac{\alpha^2}{s}, \quad \alpha = \frac{g^2}{4\pi}$$

$$\Gamma \sim n\langle\sigma|v|\rangle \sim T^3 \cdot \frac{\alpha}{T^2} = \alpha^2 T$$

$$H \sim T^2/M_P$$

$$\Gamma/H \sim \alpha^2 M_P/T$$

$$T > \alpha^2 M_P \sim 10^{16}\,\text{GeV}, \quad \Gamma/H \gtrsim 1$$

$$T < \alpha^2 M_P, \quad \text{frozen}$$

② interaction mediated by a massive gauge boson

$$\sigma \sim G_X^2 s \,, \quad G_X \sim \frac{\alpha}{m_X}$$

$$\Gamma \sim n\langle\sigma|v|\rangle \sim T^3 \cdot G_X^2 T^2 = G_X^2 T^5$$

$$\Gamma/H \sim G_X^2 M_P T^3$$

$$m_X \gtrsim T \gtrsim G_X^{-2/3} M_P^{-1/3} \sim \left(\frac{m_X}{100\,\text{GeV}}\right)^{4/3} \text{MeV} \to \Gamma/H \gtrsim 1$$

$$T \lesssim \;〃\; \text{freez out.}$$

• Brief thermal history.

	t	T	Thermal Bath
Today	15Gyr	3K	
Life on earth			
Solar system			
Quasars			
Galaxy formation			
Recombination	10^{13} sec	10^{-1} eV	p, e, γ
Matter domination	10^{11} sec		
Nucleosynthesis	1 sec	1 MeV	
			$e^{\pm}, \gamma, \nu$
QCD phase transition χSB,	10^{-5} sec	100–300 MeV	
EW phase transition	10^{-12} sec	300 GeV	
⋮			
GUT PT	10^{-35} sec	10^{15} GeV	
Planck epoch	10^{-43} sec	10^{19} GeV	

- The electroweak phase transition

  SU(2)xU(1) symmetry is restored above $T_c \sim 100\,\mathrm{GeV}$.

  Around T_c, phase transition occurs $\langle\phi\rangle = 0 \longrightarrow \langle\phi\rangle \neq 0$.

  Consquences of PT.

  1st order — bubble creation
  2nd order — smooth transition
  Topological defects.

  SM : 2nd order
  MSSM : ?
  Electroweak baryogenesis

- The quark-hadron transition

  $T \sim \Lambda_{QCD} \sim 100\,\mathrm{MeV}$

  thought to be 2nd order

- Neutrino decoupling

  interaction $e^+e^- \longleftrightarrow \nu\bar{\nu}$

  $\sigma \simeq G_F^2 s$

  $\Gamma_{int} = n\langle\sigma|v|\rangle \simeq G_F^2 T^5$

  $$\frac{\Gamma_{int}}{H} \simeq \left(\frac{G_F^2 T^5}{T^2/M_P}\right) \simeq \left(\frac{T}{1\,\mathrm{MeV}}\right)^3$$

- Neutrino temperature, $e^\pm$ annihilation

  $S = g_* T^3 = \mathrm{const}$

  before $e^\pm$ annihilation: $g_* = \frac{11}{2}$ after $e^\pm$ annihilation: $g_* = 2$

  $$\underset{\downarrow}{\frac{11}{2} T_b^3} = 2\,\underset{\downarrow}{T_f^3}$$

  $T_b \to T_\nu$, $T_f \to T_\gamma$

  $$\left(\frac{T_\gamma}{T_\nu}\right) = \left(\frac{11}{4}\right)^{1/3} \simeq 1.40.$$

interaction $n \leftrightarrow p e \bar{\nu}$

$$\frac{n}{p} = e^{-\Delta m/T} \quad , \quad \Delta m = m_n - m_p = 1.3\,\mathrm{MeV}$$

$\simeq 1/6$ at decoupling

$\to 1/7$ at $T \simeq 0.1\,\mathrm{MeV}$ due to neutron decay

- Nucleosynthesis

Equilibrium number density of nuclear species

A : mass number

Z : charge

Kinetic equilibrium

$$n_A = g_A \left(\frac{m_A T}{2\pi}\right)^{3/2} e^{(\mu_A - m_A)/T}$$

μ_A ↑ chemical potential.

chemical equilibrium

$$\mu_A = Z \underbrace{\mu_p}_{\text{proton chemical potential}} + (A-Z) \underbrace{\mu_n}_{\text{neutron chemical potential}}$$

⋮

$$n_A^{EQ} = \frac{g_A A^{3/2}}{2^A} \left(\frac{2\pi}{m_N T}\right)^{3(A-1)/2} n_p^Z n_n^{(A-Z)} e^{B_A/T}$$

mass fraction

$$X_A^{EQ} = \frac{g_A \zeta(3)^{A-1} 2^{(3A-5)/2}}{\pi^{(A-1)/2}} A^{5/2} \eta^{A-1} \left(\frac{T}{m_N}\right)^{3(A-1)/2} X_p^Z X_n^{A-Z} e^{B/T}$$

$$\eta = \frac{n_N}{n_\gamma} = 2.68 \times 10^{-8} (\Omega_B h^2)$$

$$\sim 10^{-10} - 10^{-9}$$

To determin whether a given set of nuclei will actually be in thermal equilibrium at a given epoch in the early universe, one has to take the rates of the relevant nuclear reactions from accelerator experiments.

$T \gtrsim 0.3\,\mathrm{MeV}$: the lightest few nuclei in thermal equilibrium.

$T \sim 0.1\,\mathrm{MeV}$: neutrons bind into ${}^4He$

$$X = \frac{2n}{n+p} \approx 22\% \quad \text{for } n/p = 1/7$$

accurate calculations $\longrightarrow$ computer.

- Photon decoupling

$e^- \gamma \longleftrightarrow e^- \gamma$ (Thomson scattering)

$\Gamma_{int} = n_e \sigma_T \longleftarrow$ Thomson cross section

$\sigma_T = 6.65 \times 10^{-25}\,\mathrm{cm}^2$

n_e: electron number density

$\rightarrow$ Equilibrium abundance of free electron

$pe^- \longleftrightarrow H\gamma$ (H: Hydrogen atom)

n_H, n_p, n_e, n_γ : # densities of H-atom, proton, free el. γ

charge neutrality : $n_p = n_e$

Baryon conservation : $n_B = n_p + n_H$ (we neglect neutrons in ${}^4He$)

Equilibrium # density : $n_i = g_i \left(\frac{m_i T}{2\pi}\right)^{3/2} e^{(\mu_i - m_i)/T}$

Due to $pe^- \leftrightarrow H\gamma$, $\mu_p + \mu_e = \mu_H$

$$n_H = g_H \left(\frac{m_H T}{2\pi}\right)^{3/2} e^{(\mu_H - m_H)/T}$$
$$n_p = g_p \left(\frac{m_p T}{2\pi}\right)^{3/2} e^{(\mu_p - m_p)/T}$$
$$n_e = g_e \left(\frac{m_e T}{2\pi}\right)^{3/2} e^{(\mu_e - m_e)/T}$$

$$\frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e}\left(\frac{2\pi}{T}\frac{m_H}{m_p m_e}\right)^{3/2} e^{[\underbrace{(\mu_H-\mu_p-\mu_e)}_{=0} - \underbrace{(m_H-m_p-m_e)}_{=-B=13.6eV}]/T}$$

$$n_H = \frac{g_H}{g_p g_e} n_p n_e \left(\frac{m_e T}{2\pi}\right)^{-3/2} e^{B/T}$$

$$g_p = g_e = 2, \quad g_H = 4, \quad n_B = \eta n_\gamma$$

Fractional ionization $X_e \equiv \dfrac{n_p}{n_B}$

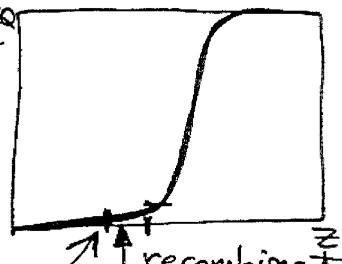
$$n_H = n_B - n_p, \quad n_e = n_p$$

$$\frac{1-n_p/n_B}{n_p^2/n_B^2} = n_B\left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B/T} \qquad \frac{2\zeta(3)}{\pi^2}T^3$$

$$= \frac{1-X_e^{EQ}}{(X_e^{EQ})^2} = \eta n_\gamma \left(\frac{2\pi}{m_e T}\right)^{3/2} e^{B/T}$$

$$\frac{1-X_e^{EQ}}{(X_e^{EQ})^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\,\eta\left(\frac{T}{m_e}\right)^{3/2} e^{B/T}$$

$$\eta = (\Omega_B h^2)\times(2.68\times 10^{-8})$$
$$T = (1+z)\times(2.7K)$$



$\Gamma_{int}(ep \leftrightarrow H\gamma) \gtrsim H$
for $(1+z) > 1100$ $\longrightarrow$ freeze-in of free electron

recombination, $X_e \simeq 0.1$
$T_{rec} \approx 0.3 eV \ll B \simeq 13.6 eV$

Residual ionization fraction $X_\infty \simeq 3\times 10^{-5}\,\Omega_0/\Omega_B h$

Decoupling of photon

$$\Gamma_{int}(e\gamma \leftrightarrow e\gamma) = X_e \eta n_\gamma \sigma_T \simeq H \text{ for } z \simeq 1100$$

- Nucleosynthesis continued.

Begining of nucleosynthesis chain

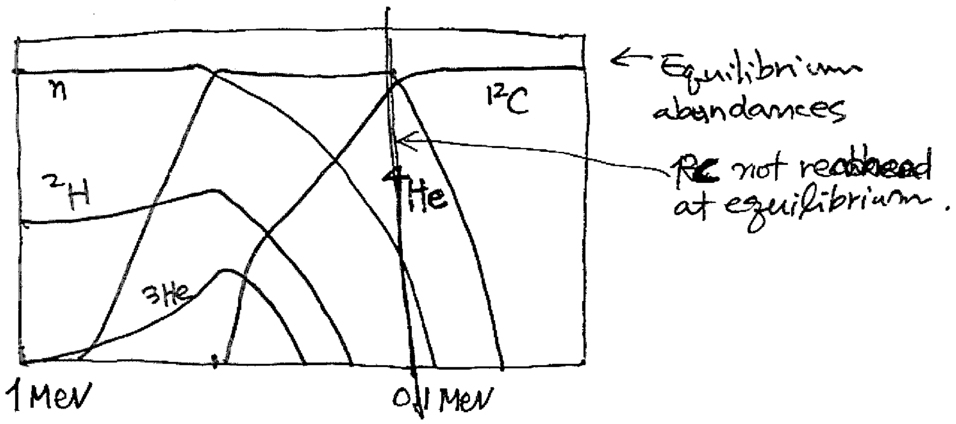
$p + n \longrightarrow D + \gamma$ deuterium production

$E_B = 2.2$ MeV but due to the smallness of η deuterium production begins at $T \sim 0.1$ MeV

$D(D,p)T$, $D(n,\gamma)T$, ${}^3He(n,p)T$, $D(D,n){}^3He$ $D(p,\gamma){}^3He$

$D(D,\gamma){}^4He$ ...

${}^3He({}^4He,\gamma){}^7Be \longrightarrow {}^7Li + e^+ + \nu_e$, $T({}^4He,\gamma){}^7Li$



Dominant product of big bang nucleosynthesis is ^4He.

$$Y_p = \frac{2(n/p)}{1+(n/p)} \simeq 0.25$$

General concordance between theoretical predictions and the observational data.

$\rightarrow$ $4.7 < \eta < 6.2$

$\Omega_B h^2 = 0.017 - 0.023$

$\rightarrow$ $N_\nu \lesssim 4.2$

The Boltzmann equation

Most of the constituents of the universe were in thermal equilibrium.

But, Departure from T.E. $\Longrightarrow$ Important Relics

- Neutrino decoupling
- CMB decoupling
- Nucleosynthesis
- Inflation
- Baryogenesis
- Decoupling of relic WIMP, ...

Boltzmann equation

$$\hat{L}[f] = \hat{C}[f]$$

Distribution function

Liouvill op. — free streaming

Collision op. — collision

$$\frac{dn_a}{dt} + 3Hn_a = -\sum_{b\cdots}\sum_{i,j\cdots}\int d\Pi_a d\Pi_b \cdots d\Pi_i d\Pi_j \cdots (2\pi)^4 \delta^4(p_a + p_b + \cdots - p_i - p_j \cdots)$$
$$\times \left[|m|^2_{a+b\cdots\to i+j\cdots} f_a f_b \cdots (1 \pm f_i)(1 \pm f_j)\cdots - |m|^2_{i+j\cdots\to a+b\cdots} f_i f_j \cdots (1 \pm f_a)(1 \pm f_b)\cdots \right]$$

Simplifying assumptions

- All but one species have equilibrium f^{EQ}
- CP invariance : $|m|^2_{a+b\to i+j} = |m|_{i+j\to a+b} \equiv |m|^2$
- MB statistics instead of FD or BE.

Annihilation process : $\psi\bar{\psi} \longleftrightarrow \chi\bar{\chi}$ in thermal equilibrium

$$\frac{dn_\psi}{dt} + 3Hn_\psi = -\langle \sigma(\psi\bar{\psi}\to\chi\bar{\chi}) v \rangle \left[n_\psi^2 - (n_\psi^{EQ})^2 \right]$$

The same equation in terms of $Y \equiv \frac{n}{s}$, $x \equiv \frac{m}{T}$

$$\frac{dY}{dx} = -\left(\frac{\pi}{45G}\right)^{1/2} \frac{g_*^{1/2} m}{x^2} \langle \sigma v \rangle \left(Y^2 - Y_{EQ}^2\right)$$

or

$$\frac{x}{Y_{EQ}} \frac{dY}{dx} = -\frac{\Gamma}{H}\left[\left(\frac{Y}{Y_{EQ}}\right)^2 - 1\right]$$

Decoupling $\Gamma_A(x_f) = H(x_f)$

Rough solution

$$Y(x) \simeq Y_{EQ}(x) \quad \text{for } x \lesssim x_f$$

$$Y(x) \simeq Y_{EQ}(x_f) \quad \text{for } x \gtrsim x_f$$

- Hot relic

$x_f \ll 1$ $(x_f \lesssim 3)$

$$Y_0 = Y_{EQ}(x_f) = 0.278 \frac{g_{eff}}{g_*(x_f)}$$

$$\Omega_\psi h^2 = 7.83 \times 10^{-2} \left(\frac{g_{eff}}{g_*(x_f)}\right)\left(\frac{m_\psi}{eV}\right)$$

Example: light neutrino

$T_f \sim 1$ MeV , $g_* = 10.75$, $g_{eff} = \frac{3}{2}$

$$\Omega_\nu h^2 = \left(\frac{m_\nu}{92 eV}\right)$$

- Cold relic

$x_f \gtrsim 3$

$$\langle \sigma_A v \rangle \propto T^n \quad \rightarrow \sigma_0 x^{-n}$$

$$x_f = \ln\left[0.038(n+1)\frac{g}{g_*^{1/2}} M_P m \sigma_0\right] - \left(n+\frac{1}{2}\right) \ln\left\{\ln\left[0.038(n+1)\frac{g}{g_*^{1/2}} M_P m \sigma_0\right]\right\}$$

$$Y_\infty = \frac{3.79(n+1) x_f^{n+1}}{(g_{*s}/g_*^{1/2}) M_P m \sigma_0}$$

Note: Simple solution, $\Gamma(x_f)=H(x_f)$, $Y_\infty=Y(x_f)$ gives good result.

Example
- Neutralino $\longrightarrow$ Dark matter.
- Relic baryons in baryon symmetric universe

$$\langle\sigma_A v\rangle = c_1 m_\pi^{-2}, \quad m_\pi = 135\,\text{MeV}, \quad c \sim \mathcal{O}(1)$$

$$x_f \simeq 42 + \ln c_1$$

$$T_f \simeq 22\,\text{MeV}$$

$$Y_\infty \simeq 7\times10^{-20} c_1^{-1}$$

Our universe has $Y_B = \frac{n_B}{s} \simeq \frac{\eta}{7} \sim (6-10)\times10^{-11}$

$\Rightarrow$ Necessity of Baryon asymmetry

Prob. 8. Explain why the addition of new light neutrino species change the BBN results.

Prob. 9 Explain why the recombination occurs at a temperature much lower than the Hydrogen binding energy.

Baryogenesis

- observed baryon asymmetry

  Nucleosynthesis : $\eta \simeq (4-7) \times 10^{-10}$

- Baryogenesis problem

  How this curious number η could arise dynamically in a universe that is initially baryon symmetric or possibly even irrespective of any initial baryon asymmetry present?

- 3 basic ingredients

  ① $\not{B}$

  ② $\not{C}$, $\not{CP}$

  ③ Non-equilibrium

  - If B is conserved, the present B asymmetry is just asymmetric initial condition
  - If C & CP conserved, $\not{B}$ will produce B and $\bar{B}$ excesses at the same rate.
  - In Equilibrium, $\not{B}$ interaction forces $n_B = n_{\bar{B}}$.

SM : $\not{B}$, $\not{C}$ & $\not{CP}$, EWPT

GUT : $\not{B}$, $\not{C}$ & $\not{CP}$, Out of Equil. decay.

GUT illustration

$$X \longrightarrow qq, \quad \bar{q}\bar{l}$$

X: leptoquark, $B=0$; qq: $B = \frac{2}{3}$; $\bar{q}\bar{l}$: $B = -\frac{1}{3}$

Particle		final state	BR	B
X	$\longrightarrow$	qq	r	$2/3$
X	$\longrightarrow$	$\bar{q}\bar{l}$	$1-r$	$-1/3$
$\bar{X}$	$\longrightarrow$	$\bar{q}\bar{q}$	$\bar{r}$	$-2/3$
$\bar{X}$	$\longrightarrow$	ql	$1-\bar{r}$	$1/3$

CPT : $\begin{cases} m_X = m_{\bar{X}} \\ \Gamma_X = \Gamma_{\bar{X}} \end{cases}$ $\quad$ $\not{C}$ & $\not{CP}$: $r \neq \bar{r}$

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symmetric initial condition $n_X = n_{\bar X}$

OEQ : $X, \bar X$ froze out.

Net baryon number per decay of $X, \bar X$ pair

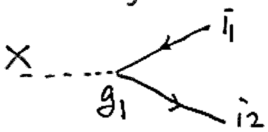
$$\varepsilon \equiv B_X + B_{\bar X} = r - \bar r \quad \left(\tfrac{2}{3}r - \tfrac{1}{3}(1-r)\right) + \left(-\tfrac{2}{3}\bar r + \tfrac{1}{3}(1-\bar r)\right)$$

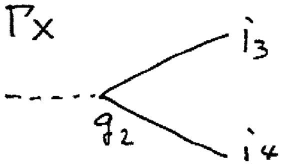
$$n_B = \epsilon n_X \longrightarrow \frac{n_B}{s} \sim \frac{\epsilon n_X}{g_* n_\gamma} \sim \frac{\epsilon}{g_*}$$

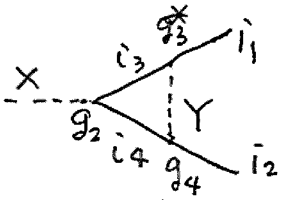
Toy model

$$\mathcal{L} = g_1 X i_2^\dagger i_1 + g_2 X i_4^\dagger i_3 + g_3 Y i_1^\dagger i_3 + g_4 Y i_2^\dagger i_4$$

$$\epsilon_X = \sum_f B_f \frac{\Gamma(X \to f) - \Gamma(\bar X \to \bar f)}{\Gamma_X}$$







$$m = g_1 \bigcirc + g_2 g_3^* g_4 \,(\text{loop integral})$$

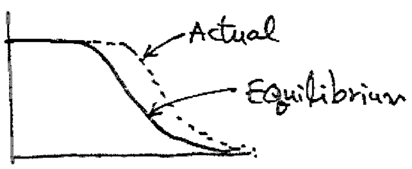
$$\Gamma(X \to \bar i_1 i_2) = |g_1|^2 I_X + \underline{g_1 g_2^* g_3 g_4^* I_{XY}} + h.c. + |g_2^* g_3^* g_4|^2 \bigcirc$$

$$\Gamma(\bar X \to i_1 \bar i_2) = |g_1^*|^2 I_{\bar X} + g_1^* g_2 g_3^* g_4 I_{XY} + h.c.$$

$$\Gamma(X \to \bar i_1 i_2) - \Gamma(\bar X \to i_1 \bar i_2) = 4\, \mathrm{Im}\, I_{XY}\, \mathrm{Im}(g_1^* g_2 g_3^* g_4)$$

$$\epsilon_X = \frac{4}{\Gamma_X} \mathrm{Im}\, I_{XY} \underbrace{\mathrm{Im}(g_1^* g_2 g_3^* g_4)}_{\not{C}\ \&\ \not{CP}} \underbrace{[B_{i_4} - B_{i_3} - B_{i_2} + B_{i_1}]}_{\not{B}}$$

Departure from thermal equilibrium.



$$K = \left(\frac{\Gamma_D}{2H}\right)_{T = m_X} = \frac{\alpha M_P}{3.3\, g_*^{1/2} m_X}$$

- Electroweak baryogenesis.

  ① $\not{B}$: SU(2) instanton effect.

  ② $\not{C}$ & $\not{CP}$: CKM seems too weak, need new source of $\not{CP}$

  ③ $\not{Eq}$: 1st order EW phase transition

  Baryon production near bubble walls

  SM : ruled out

  MSSM : marginal

- Affleck-Dine mechanism
- Leptogenesis

  Lepton asymmetry $\xrightarrow{\text{Sphaleron}}$ Baryon asymmetry.
  B−L

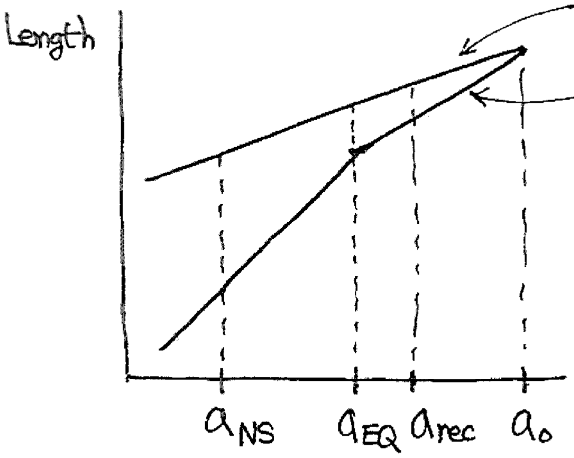
  <u>Note</u>: Sphaleron process eraise the baryon asymmetry previously produced in thermal equilibrium.

Prob. 10. Discuss about the observational evidences of baryon asymmetry of our universe.

Prob 11. Explain briefly the idea of leptogenesis. What is the physics responsible for it. Do we have any hints from particle physics that may support it?

Shortcomings of Big Bang Cosmology

- Large scale smoothness – The horizon problem.



$\lambda_{HOR} \propto a$

particle horizon

$$H^{-1} \sim t \propto \begin{cases} a^{3/2} & MD \\ a^2 & RD \end{cases}$$

The size of particle horizon $d_H \sim t$.
The entropy within a horizon volume

$$S_{HOR} = \frac{4\pi}{3} d_H^3 s \simeq \begin{cases} 0.05\, g_*^{-1/2} \left(\frac{M_P}{T}\right)^3 & RD \\ 3\times 10^{87} (\Omega_0 h^2)^{-3/2} (1+z)^{-3/2} & MD \end{cases}$$

$S_{HOR}(t=t_0) \simeq 10^{88}$
$S_{HOR}(t=t_{rec}) \simeq 10^{83} \longrightarrow 10^5$ Hubble volumes
$S_{HOR}(t=t_{NS}) \simeq 10^{63} \longrightarrow 10^{25}$ Hubble volumes

The present Hubble volume consists of 10^5 causally disconnected region at recombination
$H^{-1}(t_{rec}) \rightarrow 0.8^\circ$ on the sky today
But CMB is smooth for $10'' - 180^\circ$.

- Small scale inhomogeneity
  - very smooth on large scales
  - plethora of structure on smaller scales

    galaxies, clusters, voids, sheets, superclusters

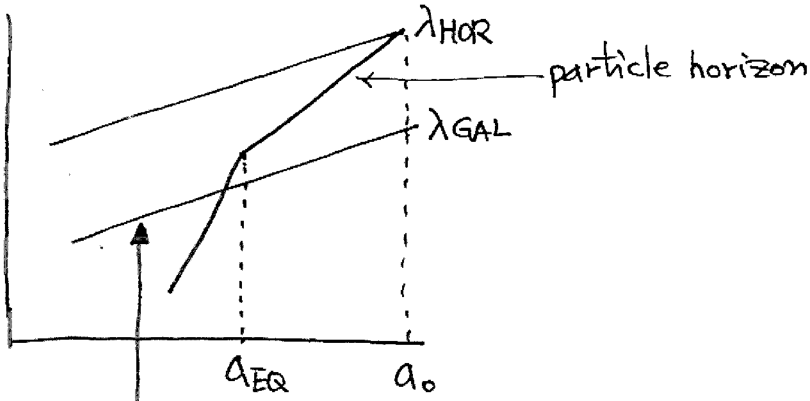
    1Mpc ———————— 100 Mpc

small, primeval inhomogeneities

$\downarrow$ Matter dominated
Gravitational instability

large scale structures, CMB anisotropy

$$\frac{\Delta T}{T} \propto \frac{\delta \rho}{\rho} \simeq 10^{-5}$$



Causal microphysical processes cannot make $\left(\frac{\delta\rho}{\rho}\right)_{\lambda_{GAL}}$ because λ_{GAL} is outside the horizon

• Spatial flatness, oldness

physical radius of curvature $R_{curv} = a(t)|k|^{-1/2} = \dfrac{H^{-1}}{|\Omega-1|^{1/2}}$

Ω as a function of a

$$\Omega - 1 = \frac{k}{H^2 a^2} \propto \frac{1}{\rho a} \propto \begin{cases} a, & \text{MD} \\ a^2, & \text{RD} \end{cases}$$

$|\Omega(1\text{sec}) - 1| \lesssim 10^{-16}$, $R_{curv}(1\text{sec}) \gtrsim 10^{8} H^{-1}$

$|\Omega(10^{-43}\text{sec}) - 1| \lesssim 10^{-60}$, $R_{curv}(10^{-43}\text{sec}) \gtrsim 10^{30} H^{-1}$

Big Bang universe requires very special initial condition. (at Planck time).

Note: If $\Omega \sim 1$, $R_{curv} \sim H^{-1}$ at planck time

$k > 0$: recollapse within few $\times 10^{-43}$ sec

$k < 0$: 3K reached at $t = 10^{-11}$ sec

Note: The natural time scale for cosmology is Planck time $= 10^{-43}$ sec.

But our FRW universe age $\sim 10^{60} t_p$.

• Cosmological constant.

$T_{\mu\nu} = \Lambda g_{\mu\nu}$

Eq. of state $w = \dfrac{p}{\rho} = -1$, $\Lambda = \rho_{vac}$.

$\rightarrow$ de Sitter solution

$a(t) \propto e^{Ht}$, $H = \left(\dfrac{\kappa^2}{3}\rho_{vac}\right)^{1/2} = \text{const.}$

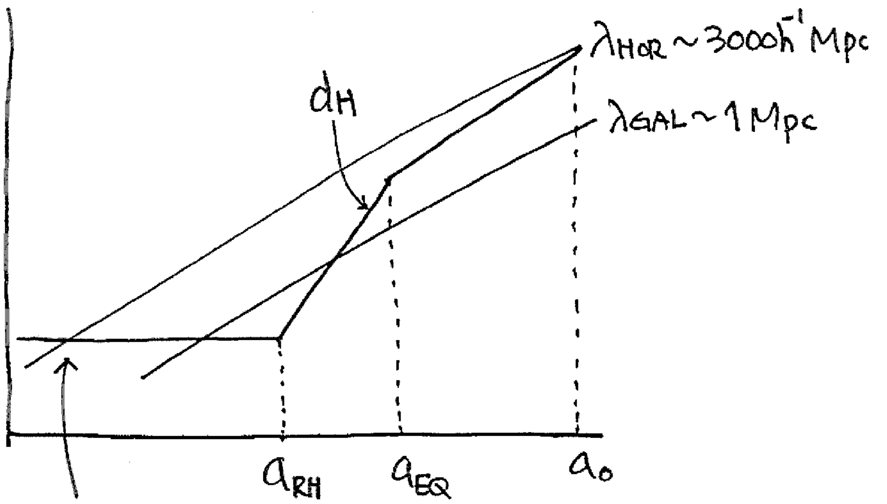
Observation: $\rho_{vac} \sim \rho_c \sim (3 \times 10^{-3} \text{eV})^4$

Cosmological constant problem: The smallness of the present value of vacuum energy relative to any fundamental scales in physics. $\rho_{vac} \sim M_p^4 \sim 10^{123} \rho_c$

- Inflation – Basic picture

There was an epoch when the vacuum energy was the dominant component of the energy density.

$\Rightarrow$ Scale factor grows exponentially



λ_{HOR}, causally connected $\rightarrow$ Horizon problem is solved.

$$\Omega - 1 = \frac{1}{\rho a^2} = |\Omega_i - 1|\, e^{-2Ht}$$

$\rightarrow 0$ as $t \rightarrow$. Solution to flatness problem.

- Scalar field dynamics

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - V(\phi) = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

Homogeneous scalar field

$$T^{\mu\nu} = \partial^\mu\phi\,\partial^\nu\phi - g^{\mu\nu}\mathcal{L}$$

$$\rho_\phi = \frac{1}{2}\dot{\phi}^2 + V(\phi) + \frac{(\vec{\nabla}\phi)^2}{2a^2}$$

$$P_\phi = \frac{1}{2}\dot{\phi}^2 - V(\phi) - \frac{(\vec{\nabla}\phi)^2}{6a^2}$$

rapidly damped as $a \sim e^{Ht}$.

The classical equation of motion

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad H^2 = \frac{k^2}{3}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

① Slow roll regime

3H$\dot{\phi}$ dominate, ϕ rolls at terminal velocity.

② Rapid oscillation regime

Damped harmonic oscillator

①: $3H\dot{\phi} = -V'(\phi), \quad H^2 = \frac{k^2}{3} V(\phi)$

Consistency condition

$$\frac{1}{2}\dot{\phi}^2 \ll V(\phi) \longrightarrow \frac{1}{2}\left(\frac{V'}{3H}\right)^2 \ll V, \quad \frac{1}{2}\frac{V'^2}{9 \cdot \frac{8\pi}{3M_p^2} V} \ll V$$

$$\left(\frac{M_p V'}{V}\right)^2 \ll 48\pi$$

$$\boxed{\epsilon(\phi) \equiv \left| \frac{M_p V'(\phi)}{(48\pi)^{1/2} V(\phi)} \right| \ll 1.}$$

Vacuum energy dominates.

$$\ddot{\phi} \ll 3H\dot{\phi} \longrightarrow 3H\ddot{\phi} + 3\dot{H}\dot{\phi} = -V''\dot{\phi}$$

$$\ddot{\phi} = -\frac{3\dot{H} + V''}{3H}\dot{\phi} \ll 3H\dot{\phi}$$

$$3\dot{H} + V''(\phi) \ll 9H^2 = 24\pi V / M_p^2$$

$$\boxed{\eta(\phi) \equiv \left| \frac{M_p^2 V''(\phi)}{(24\pi) V(\phi)} \right| \ll 1}$$

Friction dominates.
→ enough inflation

The number of e-fold

$$\ln(a_2/a_1) \equiv N(\phi_1 \to \phi_2) \equiv \int_{t_1}^{t_2} H\,dt$$

$$= \int_{\phi_1}^{\phi_2} H \frac{d\phi}{\dot{\phi}} = \int_{\phi_1}^{\phi_2} -\frac{3H^2}{V'(\phi)}\,d\phi$$

$$= -\frac{8\pi}{M_p^2} \int_{\phi_1}^{\phi_2} \frac{V(\phi)}{V'(\phi)}\,d\phi$$

Reheating

- Basic picture

① After inflation, the inflaton field begins to oscillate about the true vacuum state.

↳ QM: A coherent state of $k=0$ inflaton particles

② Through the interaction with other particles, the coherent state will decay into quanta of elementary particles

③ Produced particles rapidly thermalize.

- Toy model

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\lambda(\phi^2-\sigma^2)^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \underbrace{\frac{1}{2}g^2\phi^2\chi^2}_{\text{Interaction}}$$

$$m_\phi = \lambda^{1/2}\sigma$$

$$\Gamma_\phi = \frac{g^2\sigma^2}{8\pi m_\phi^2}$$

$$\ddot{\phi} + 3H\dot{\phi} + \underline{\Gamma_\phi\dot{\phi}} + V'(\phi) = 0.$$

$H > \Gamma_\phi$: particle production is negligible

$H \simeq \Gamma_\phi$: Reheating starts, energy density of ϕ is transfered to χ.

Reheating temperature

$$T_R \simeq (\Gamma_\phi M_p)^{1/2}$$

E.g. $g^2 \sim \lambda \lesssim 10^{-12} \longrightarrow T_R \lesssim 10^{10}\,\text{GeV}$

- Modern theory of reheating

An essential point is missing in the above analysis.
Look at χ equation

$$\ddot{\chi} + 3H\dot{\chi} - \left[\left(\frac{\nabla}{a}\right)^2 - m_\chi^2 - 2g^2\sigma\phi \right]\chi = 0.$$

$$\ddot{\chi}_k + 3H\dot{\chi}_k + \left[\underline{k_p^2 + m_\chi^2 + 2g^2\sigma\phi} \right]\chi_k = 0.$$

ϕ is oscillating

$\Rightarrow$ parametric resonance.

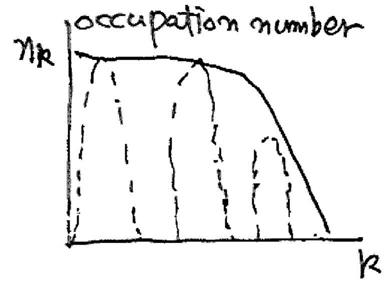
Creation of χ quanta

Without expansion $\omega_k^2 = k_p^2 + m_\chi^2 = \left(\frac{n}{2}\omega\right)^2$, $n = 1, 2, \cdots$

instability band

$\Delta\omega_k \sim g\sigma^{1/2}\phi_0^{1/2}$

Expansion $\rightarrow$ ω_k becomes time-dependent.



$\Rightarrow$ explosive production of χ
"preheating"

Note : Generically, reheating in chaotic inflation models are explosive.

Note : The state of χ after parametric resonance is not a thermal state.

- The amount of inflation required to solve the horizon and flatness problem.

$$N_{TOT} \gtrsim 53 + \frac{2}{3}\ln\left(\frac{M}{10^{14}\,GeV}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{10}\,GeV}\right) + \frac{1}{2}\ln|\Omega_i - 1| \sim \text{"}60\text{"}$$

- Classical density perturbation from quantum fluctuations in the de Sitter phase of an inflationary universe.

sub horizon size, quantum fluctuations

$$\text{Inflation} \;\Big\downarrow\; \text{horizon exit - freeze in}$$

classical metric perturbation

$$\text{RD, MD} \;\Big\downarrow\; \text{horizon reentry}$$

density perturbation

Note: The number of e-folds between horizon crossing and the end of inflation.

$$N_\lambda = 45 + \ln\left(\frac{\lambda}{\mathrm{Mpc}}\right) + \frac{1}{3}\ln\left(\frac{M}{10^{14}\,\mathrm{GeV}}\right) + \frac{1}{3}\ln\left(\frac{T_R}{10^{10}\,\mathrm{GeV}}\right)$$

During the slow roll : $|V''| \ll H^2 \rightarrow \phi$ is massless

QM fluctuations of massless, minimally coupled scalar field in dS space

$$(\Delta\phi)_k^2 \equiv \frac{k^3|\delta\phi_k|^2}{2\pi^2 V} = \left(\frac{H}{2\pi}\right)^2, \quad \delta\phi_k = \int d^3x\, e^{i\vec{k}\cdot\vec{x}}\,\delta\phi(x)$$

Evolution of $\delta\phi_k$ with $k \ll aH$ (superhorizon size)

$$\delta\ddot{\phi}_k + 3H\delta\dot{\phi}_k + k^2\delta\phi_k/a^2 = 0 \longrightarrow \delta\phi_k = \text{const}.$$

$$\delta\phi \longrightarrow \underbrace{\delta\rho_\phi}_{\text{Density perturbation}} = V'(\phi)\,\delta\phi$$

Evolution of density perturbation, $\zeta = \dfrac{\delta\rho}{\rho + p}$

For superhorizon size $\zeta = \text{const}$.

At horizon reentry $\zeta \sim \delta\rho/\rho$

$$\left(\frac{\delta\rho}{\rho}\right)_{\substack{\text{Hor}\\ \text{Reentry}}} = \zeta_\lambda = \left(\frac{\delta\phi\, V'}{\dot{\phi}^2}\right)_\lambda \simeq \left(\frac{H^2}{\dot{\phi}}\right)_\lambda$$

$\rho_\phi + p_\phi \simeq \dot{\phi}^2$ during inflation

During inflation, $H, \dot{\phi}$ vary very slowly

→ scale invariant spectrum (Harrison-Zeldovich)

Nature of quantum fluctuation — Gaussian.

Note: Amplitude

$$\left(\frac{\delta\rho}{\rho}\right)_\lambda \sim \left(\frac{H^2}{\dot{\phi}}\right)_\lambda \lesssim 10^{-5} \text{ from CMBA.}$$

⇒ Most restrictive constraint on inflationary potential.

E.g. chaotic inflation

$$V(\phi) = \frac{1}{2}m^2\phi^2 \longrightarrow m \sim 10^{13}\,\text{GeV}$$

$$V(\phi) = \frac{1}{4}\lambda\phi^4 \longrightarrow \lambda \sim 10^{-12}$$

• Gravitational instability — Newtonian theory

Density perturbation produced from inflation grows after MD → large scale structures.

From
$$\frac{\partial\rho}{\partial t} + \vec{\nabla}\cdot(\rho\vec{v}) = 0$$
$$\frac{\partial\vec{v}}{\partial t} + (\vec{v}\cdot\vec{\nabla})\vec{v} + \frac{1}{\rho}\vec{\nabla}p + \vec{\nabla}\phi = 0$$
$$\nabla^2\phi = 4\pi G\rho$$

Equation for $\delta\rho/\rho \equiv \delta$

$$\ddot{\delta}_k + 2\frac{\dot{a}}{a}\dot{\delta}_k + \left[\frac{c_s^2 k^2}{a^2} - 4\pi G\rho\right]\delta_k = 0.$$

$c_s^2 = \frac{\partial p}{\partial\rho}$ speed of sound

$$\lambda_J = \frac{2\pi}{k_J}, \quad k_J = \left(\frac{4\pi G\rho}{c_s^2}\right)^{1/2}. \quad \left(\begin{array}{l}\lambda < \lambda_J : \text{oscillating} \\ \lambda > \lambda_J : \text{growing}\end{array}\right.$$

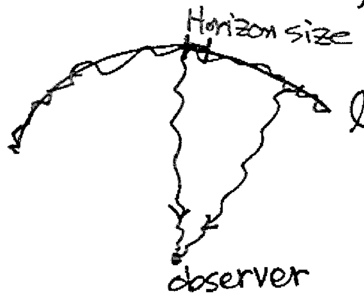
$$\left(\begin{array}{l}\text{MD} : c_s \simeq 0 \\ \text{RD} : c_s \simeq O(1)\end{array}\right.$$

- CMB anisotropy

what determines CMBA?

① Gravity — Gravitational red/blue shift
② pressure — Acoustic oscillation
③ Velocity of baryons/fluid. – Doppler shift

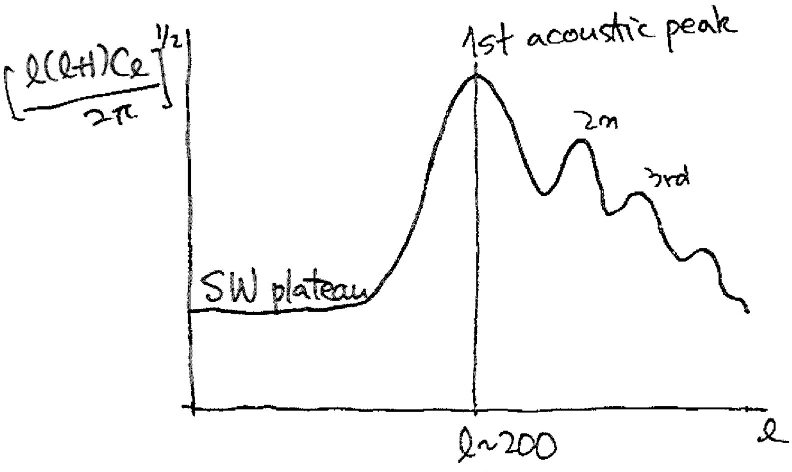
$$\frac{\delta T}{T}(\vec{r}) = \Phi(\vec{r}, t_{dec}) + 2\int_{t_{dec}}^{t_0} \dot{\Phi}(\vec{r}, t)\,dt + \frac{1}{3}\frac{\delta\rho}{\rho} - \underbrace{\frac{\vec{n}\cdot\vec{v}}{c}}_{\text{Doppler shift due to fluid motion}}$$



Sachs-Wolfe effect

$$\frac{l(l+1)C_l^{(s)}}{2\pi} = \frac{A_s^2}{25}, \ldots$$

scale larger than horizon size at last scattering
Sachs-Wolf effect dominate.
Below horizon size – acoustic oscillation.



↳ $\Omega_T = 1$.

• **Summary**

The history of the universe up to BBN is firmly established once the proper initial conditions are set.

① Thermal bath, $T \gtrsim 10\,\mathrm{MeV}$

Homogeneous & isotropic / Small density perturbation.

② Matter content

Baryon $\eta = (4-7)\times 10^{-10}$

Dark matter

Dark energy

Today: Expansion of the universe

Light element abundances* $\Omega_\Lambda, \Omega_M, \Omega_B$

CMB*

CMB anisotropy

Large scale structure

BIG BANG UNIVERSE

In the begining: Thermal bath, $T \gtrsim 10\,\mathrm{MeV}$, small seed density perturbation

$\Omega_B, \Omega_M, \Omega_\Lambda, \Omega_T = 1$

Baryogenesis Dark matter ? Dark energy

Inflation