

# Multi-field inflation: Formulation, effective theory and phenomenology

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Based on

- A. Achúcarro, [JG](#), S. Hardeman, G. A. Palma and S. P. Patil, arXiv:1005.3848 [hep-th]
- A. Achúcarro, [JG](#), S. Hardeman, G. A. Palma and S. P. Patil, JCAP1101:030 (2011)  
[arXiv:1010.3693 [hep-ph]]
- [JG](#) and T. Tanaka, JCAP1103:015 (2011) [arXiv:1101.4809 [astro-ph.CO]]
- A. Achúcarro, [JG](#), G. A. Palma and S. P. Patil, in preparation

# Outline

## 1 Introduction

## 2 Formulation of perturbations

- Issue of mapping
- General matter Lagrangian
- Gravity

## 3 Effective single field theory

- 2-field case
- Effective single field theory: quadratic action
- Reduction of cubic action

## 4 Distinctive phenomenology

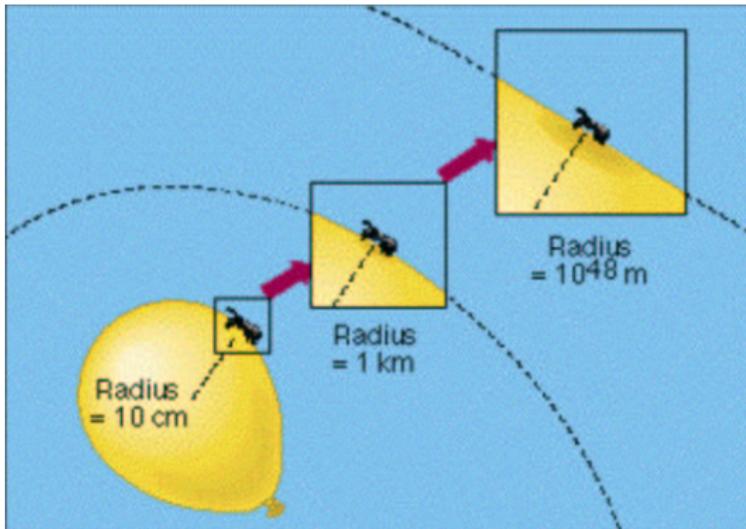
- Curved trajectory
  - Power spectrum
  - Bispectrum
- Field space geometry

## 5 Conclusions

# What is inflation?



# What is inflation?



- Accelerated phase of expansion:  $\ddot{a} > 0$
- Special form of matter:  $p < -\rho/3$
- (Quasi) de Sitter expansion:  $H \approx \text{constant}$

# Why inflation?

## Hot big bang

- Horizon problem
- Flatness problem
- Monopole problem
- Initial perturbations

## Inflation

- Single causal patch
- Locally flat
- Diluted away
- Quantum fluctuations

Currently, inflation

- ➊ can **dynamically** provide **initial conditions** for hot big bang
- ➋ is strongly supported by **observations**: WMAP, SDSS, etc

# Why non-linear perturbations?

## Non-linear perturbations

- Higher order contributions to observable quantities
- Observationally accessible: Planck, CMBPol, ACT, Euclid...
- Invaluable information
  - ① Inflation models and the physics behind
  - ② Nature of non-linear evolution

A crucial probe to the early universe

# Multi-field inflation

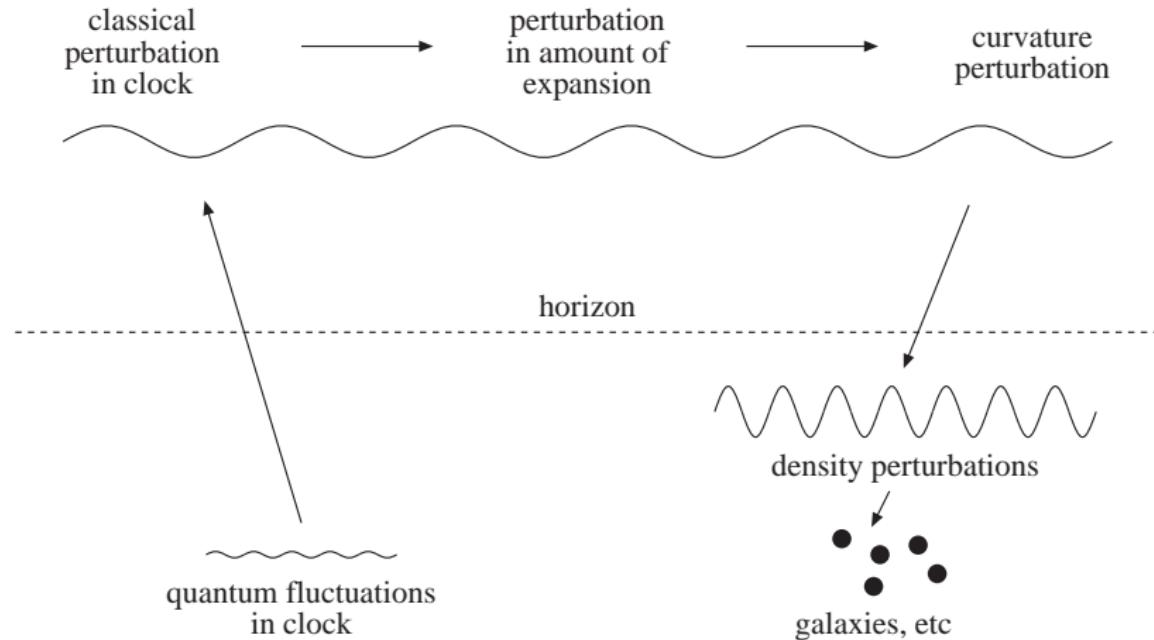
## Why multi-field inflation?

- ① No inflaton candidate in SM: **inflation = BSM**  
 cf. F. L. Bezrukov and M. Shaposhnikov, Phys. Lett. B **659**, 703 (2008): **SM Higgs = inflaton?**
- ② Plenty of scalar fields = *inflaton candidates*
- ③ Rich phenomenology : isocurvature pert, non-Gaussianity...
- ④ Both **theoretical** and **phenomenological** motivations

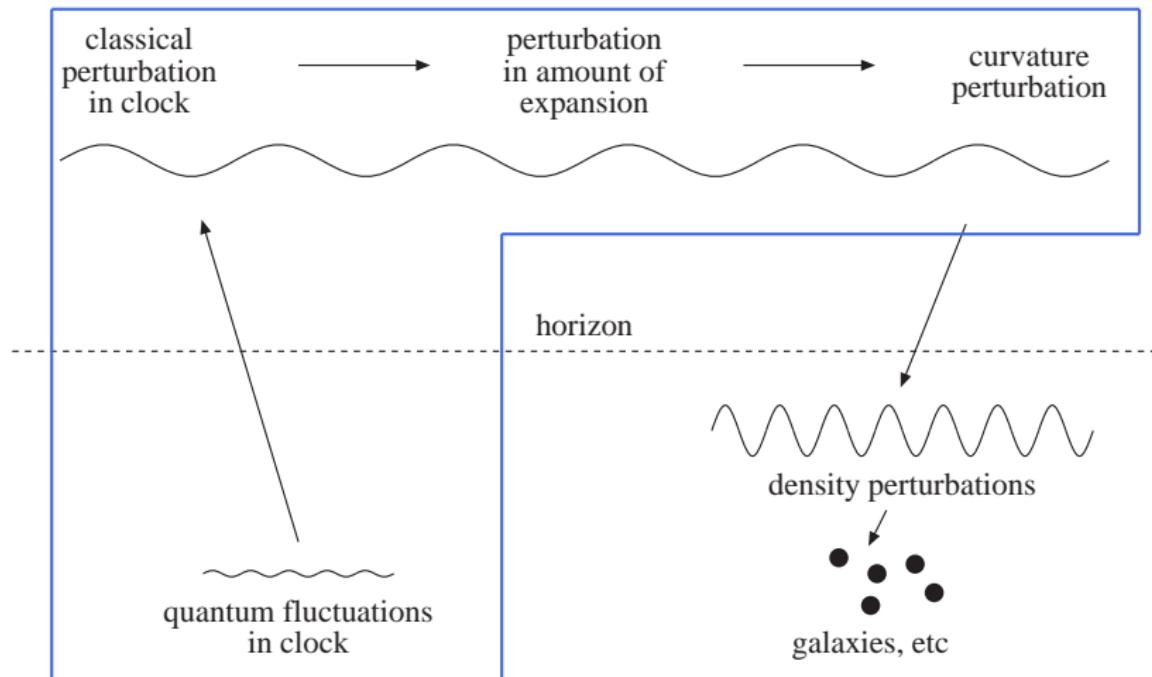
## In this presentation

- Fully covariant, easily extendable formulation of perturbation
- Reduction to effective single field theory
- Associated observational signatures

# Generation and evolution of perturbations



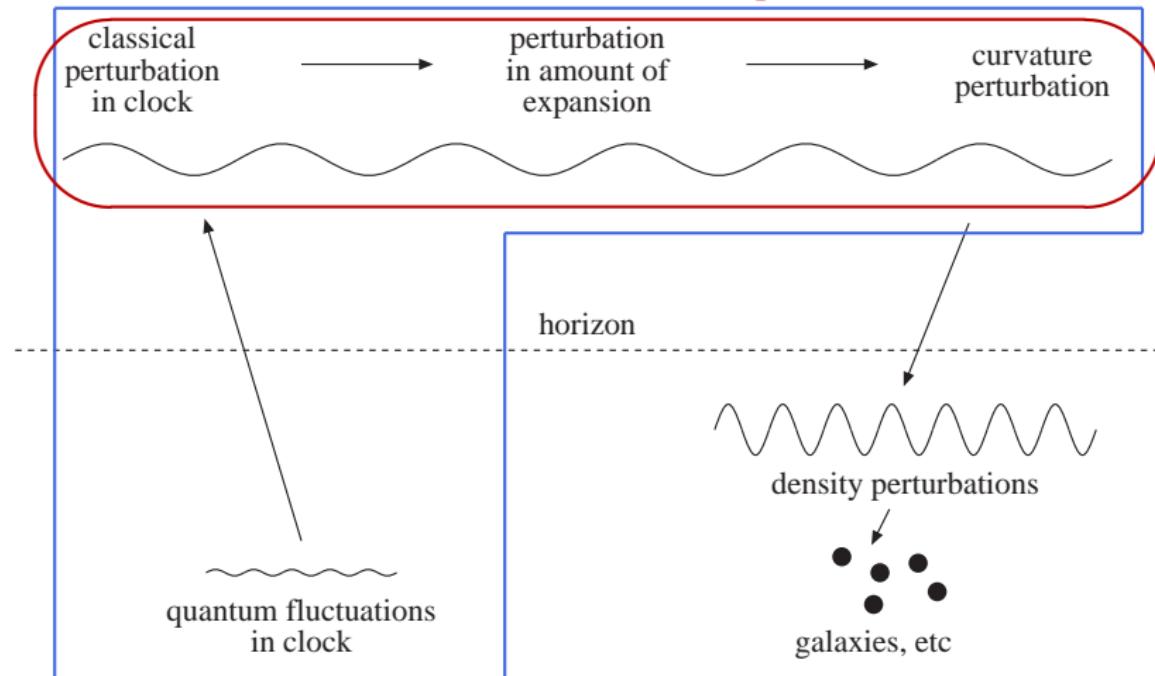
# Generation and evolution of perturbations



## Action formulation

# Generation and evolution of perturbations

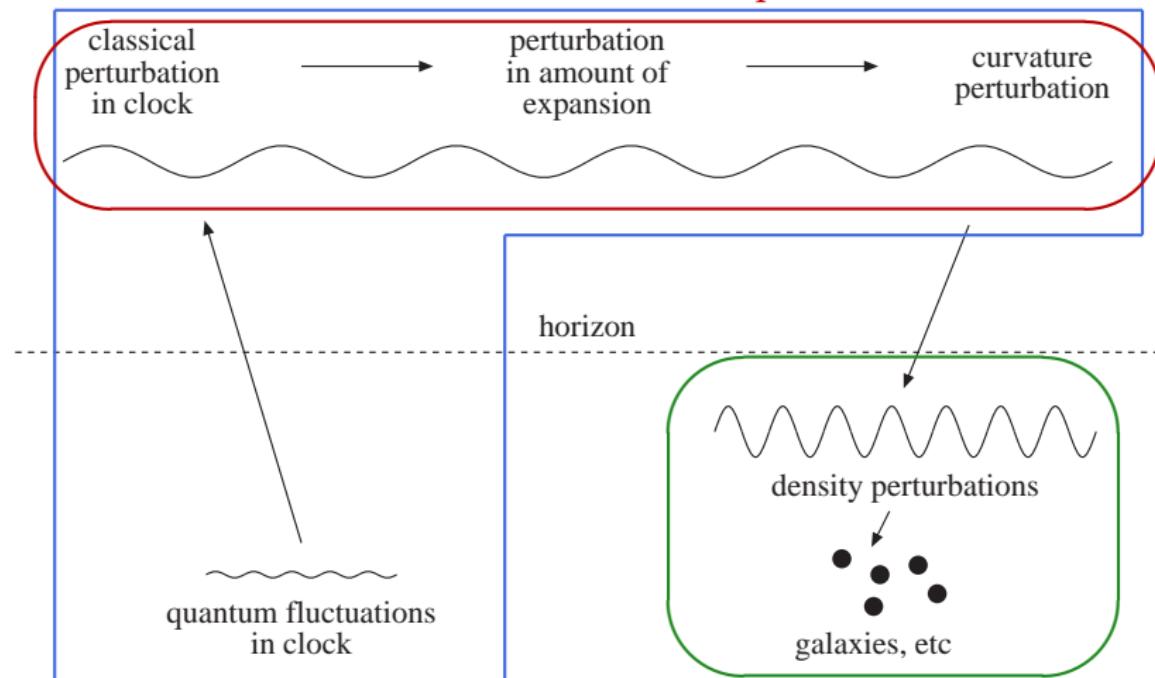
## $\delta N$ formalism, Einstein equation



## Action formulation

# Generation and evolution of perturbations

## $\delta N$ formalism, Einstein equation

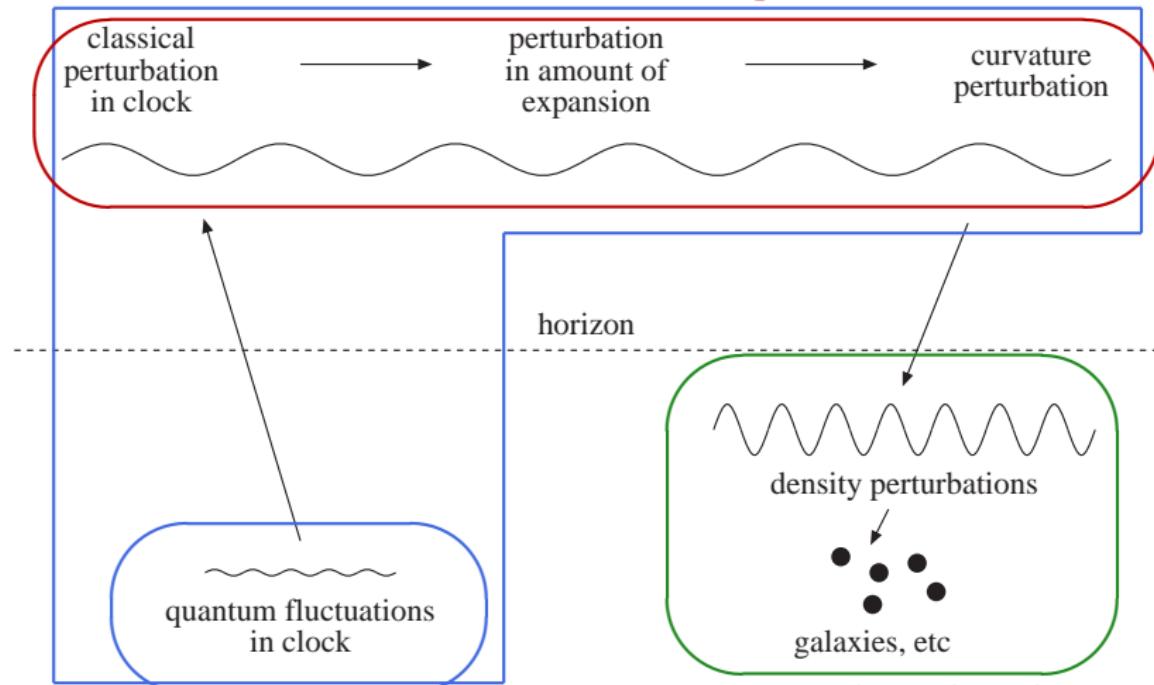


Action formulation

Newtonian, Einstein

# Generation and evolution of perturbations

## $\delta N$ formalism, Einstein equation



Action formulation

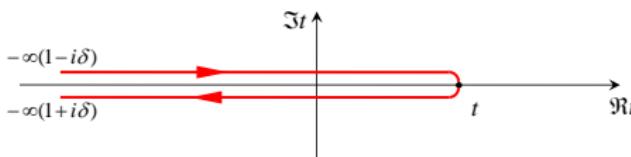
Newtonian, Einstein

# Higher order action and correlation functions

Free & interaction Hamiltonian:  $H = H_0 + H_{\text{int}}$

$$\begin{aligned} \langle \hat{\mathcal{O}}(t) \rangle &= \left\langle \Omega \left| \left( e^{-i \int_{t_{\text{in}}}^t H_0(t') dt'} \right)^\dagger \hat{\mathcal{O}} \left( e^{-i \int_{t_{\text{in}}}^t H_0(t'') dt''} \right) \right| \Omega \right\rangle \\ &= \lim_{t_{\text{in}} \rightarrow -\infty(1-i\delta)} \langle 0 | U_{\text{int}}^\dagger(t, t_{\text{in}}) \hat{\mathcal{O}}(t) U_{\text{int}}(t, t_{\text{in}}) | 0 \rangle \\ &= \sum_{n=0}^{\infty} i^n \int_{t_{\text{in}}}^t dt_n \int_{t_{\text{in}}}^{t_n} dt_{n-1} \cdots \int_{t_{\text{in}}}^{t_2} dt_1 \langle 0 | [H_{\text{int}}(t_1), [H_{\text{int}}(t_2), \cdots [H_{\text{int}}(t_n), \hat{\mathcal{O}}(t)] \cdots]] | 0 \rangle \end{aligned}$$

“In-in”, or “closed time path” formalism

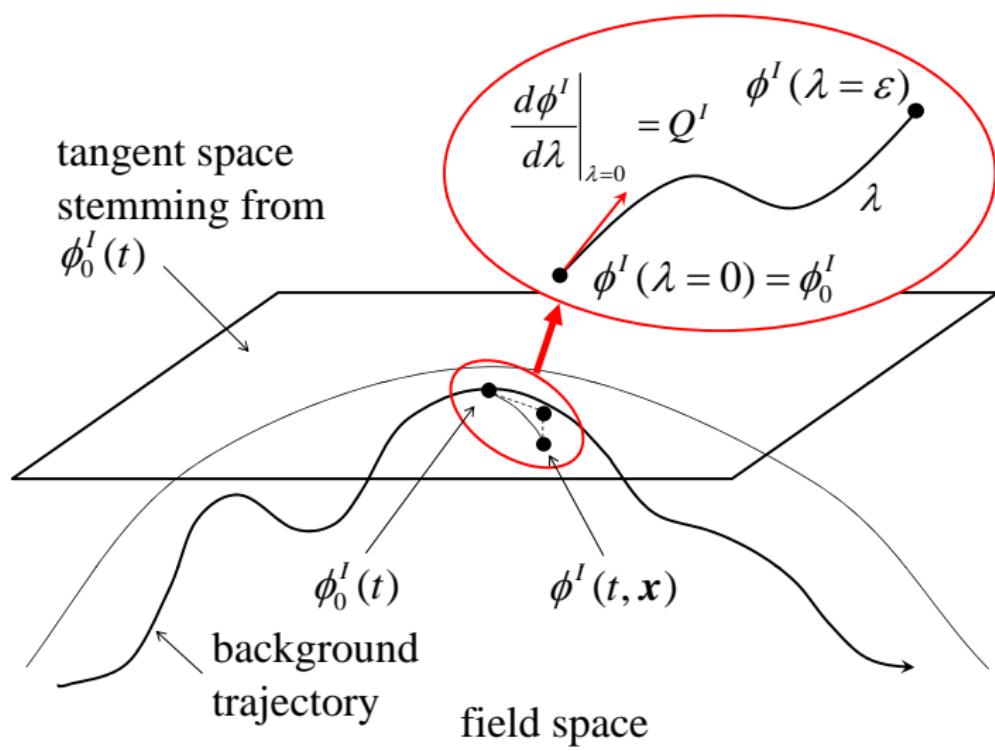


$\mathcal{R}$  = free field: expansion using cre / ann ops  $\mathcal{R}_k = a_k \hat{\mathcal{R}}_k + a_k^\dagger \hat{\mathcal{R}}_k^*$

$$\langle \mathcal{R} \mathcal{R} \mathcal{R} \rangle \sim \langle 0 | \mathcal{R} \mathcal{R} \mathcal{R} \exp(\mathcal{R} \mathcal{R} \mathcal{R}) | 0 \rangle$$

Even numbers of cre / ann ops: non-zero 3-point correlation fct

# Field fluctuations in field space



# Expansion near the background trajectory

Geodesic equation :  $D_\lambda^2 \phi^I = \frac{d^2 \phi^I}{d\lambda^2} + \Gamma_{JK}^I \frac{d\phi^J}{d\lambda} \frac{d\phi^K}{d\lambda} = 0$

Initial conditions :  $\phi^I|_{\lambda=0} = \phi_0^I, D_\lambda \phi^I|_{\lambda=0} = \frac{d\phi^I}{d\lambda}\Big|_{\lambda=0} = Q^I$

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Expansion of  $\phi^I(\lambda = \epsilon)$  from  $\lambda = 0$

$$\phi^I(\lambda = \epsilon) = \phi^I|_{\lambda=0} + \frac{d\phi^I}{d\lambda}\Big|_{\lambda=0} \epsilon + \frac{1}{2} \frac{d^2 \phi^I}{d\lambda^2}\Big|_{\lambda=0} \epsilon^2 + \frac{1}{3!} \frac{d^3 \phi^I}{d\lambda^3}\Big|_{\lambda=0} \epsilon^3 + \dots$$

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Setting  $\epsilon = 1$ ,

$$\phi^I - \phi_0^I \equiv \delta\phi^I = Q^I - \frac{1}{2} \Gamma_{JK}^I Q^J Q^K + \frac{1}{6} (\Gamma_{LM}^I \Gamma_{JK}^M - \Gamma_{JK;L}^I) Q^J Q^K Q^L + \dots$$

# First try

$$P = P\left(G_{IJ}, X^{IJ} \equiv -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi^I\partial_\nu\phi^J, \phi^I\right)$$

e.g. k-inflation, DBI inflation

$$P = P|_{\lambda=0} + D_\lambda P|_{\lambda=0}\epsilon + \frac{1}{2!} D_\lambda^2 P|_{\lambda=0}\epsilon^2 + \frac{1}{3!} D_\lambda^3 P|_{\lambda=0}\epsilon^3 + \dots$$

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- ➊ No problem with the linear term

$$D_\lambda P = \underbrace{\frac{\partial P}{\partial X^{IJ}}}_{\equiv P_{\langle IJ\rangle}} D_\lambda X^{IJ} + \underbrace{\frac{\partial P}{\partial \phi^I}}_{\equiv P_{,I}} D_\lambda \phi^I \quad \left( \frac{\partial P}{\partial X^{IJ}} \rightarrow P_{\langle IJ\rangle} : \text{symmetrization} \right)$$

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- ② Problem with the quadratic term

$$D_\lambda^2 P = D_\lambda^2 X^{IJ} P_{\langle IJ\rangle} + D_\lambda X^{IJ} (D_\lambda P_{\langle IJ\rangle}) + D_\lambda \phi^I (D_\lambda P_{,I})$$

- Covariant and ordinary derivatives mixed
- Not commute, not covariant:  $P_{\langle IJ\rangle;K} = P_{;K\langle IJ\rangle} - \Gamma_{IK}^L P_{\langle LJ\rangle} - \Gamma_{JK}^L P_{\langle IL\rangle}$

# Second try

Assumption:  $P$  depends on  $\phi^I$  only through tensors  $f_a^{J_1 \cdots J_{n_a}}(\phi^I)$

$$P = P \left[ G_{IJ}, X^{IJ}, f_a^{J_1 \cdots J_{n_a}}(\phi^I) \right]$$

e.g. potential  $V(\phi^I)$ : field space scalar

$$D_\lambda P = D_\lambda X^{IJ} P_{\langle IJ \rangle} + \sum_a f_a^{J_1 \cdots J_{n_a}; I} D_\lambda \phi^I P_{\{J_1 \cdots J_{n_a}\}_a} \quad \left( P_{\{J_1 \cdots J_{n_a}\}_a} \equiv \frac{\partial P}{\partial f_a^{J_1 \cdots J_{n_a}}} \right)$$

Up to cubic order ( $f_a^{J_1 \cdots J_{n_a}}$  are all scalars for simplicity)

$$\begin{aligned} P = & P|_{\lambda=0} + P_{\langle IJ \rangle} \delta X^{IJ} + P_a \delta f_a \\ & + \frac{1}{2!} P_{\langle IJ \rangle \langle KL \rangle} \delta X^{IJ} \delta X^{KL} + P_{\langle IJ \rangle a} \delta X^{IJ} \delta f_a + \frac{1}{2!} P_{ab} \delta f_a \delta f_b \\ & + \frac{1}{3!} P_{\langle IJ \rangle \langle KL \rangle \langle MN \rangle} \delta X^{IJ} \delta X^{KL} \delta X^{MN} + \frac{1}{2!} P_{\langle IJ \rangle \langle KL \rangle a} \delta X^{IJ} \delta X^{KL} \delta f_a \\ & + \frac{1}{2!} P_{\langle IJ \rangle ab} \delta X^{IJ} \delta f_a \delta f_b + \frac{1}{3!} P_{abc} \delta f_a \delta f_b \delta f_c + \dots \end{aligned}$$

## Gauge choice

Einstein gravity + total  $n$  scalar fields

- ➊ Eliminate 2 out of  $n+4$ : from 1 temporal and 1 spatial gauge transformations
  - ➋ Eliminate 2 out of  $n+2$ : from the Hamiltonian and momentum constraint eqs
  - ➌ Total  $n$  d.o.f. → all contained in  $n$  scalar fields

ADM form:  $ds^2 = -N^2 dt^2 + \gamma_{ij} (N^i dt + dx^i) (N^j dt + dx^j)$ ,  $\gamma_{ij} = a^2(t) \delta_{ij}$

## “Flat gauge”

# Including metric perturbations

$N, N^i \rightarrow \xi^\alpha$ : expand  $\xi^\alpha$  in  $\epsilon$  as

$$\xi^\alpha(\lambda = \epsilon) = \xi_0^\alpha + \xi_{(1)}^\alpha \epsilon + \xi_{(2)}^\alpha \epsilon^2 + \dots$$

Constraint equations

$$\frac{\delta S}{\delta \xi^\alpha} = 0$$

Action expansion with respect to  $\xi_{(n)}^\alpha$

$$S = S|_{\xi_{(n)}^\alpha = 0} + \underbrace{\left. \frac{\delta S}{\delta \xi_{(n)}^\alpha} \right|_{\xi_{(n)}^\mu = 0} \xi_{(n)}^\alpha + \frac{1}{2} \left. \frac{\delta^2 S}{\delta \xi_{(n)}^\alpha \delta \xi_{(n)}^\beta} \right|_{\xi_{(n)}^\mu = 0} \xi_{(n)}^\alpha \xi_{(n)}^\beta}_{+ \dots} + \dots$$

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Constraint equations

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Up to the cubic order in  $\epsilon$ ,  $\xi_{(2)}^\alpha$  not necessary: linear solutions only

# Linear, quadratic and cubic order actions

Metric perturbations in **flat FRW BG**:  $N = 1 + N_{(1)}$ ,  $N^i = N_{(1)}^i = \partial^i \chi$

- From linear action, we can obtain **BG equations**

$$S_1 = \int d^4x a^3 \left[ \left( 3m_{\text{Pl}}^2 H^2 + P_0 - P_{\langle IJ \rangle} \dot{\phi}_0^I \dot{\phi}_0^J \right) N_{(1)} + P_{\langle IJ \rangle} D_t Q^I \dot{\phi}_0^J + P_{af;I} Q^I \right]$$

- From quadratic action, we can find the **sols of metric perts**  $N_{(1)}, \chi$

$$\begin{aligned} S_2 = & \int d^4x a^3 \left\{ \frac{1}{2} \left[ P_{\langle IJ \rangle} \left( R^I_{\phantom{I}KLM} \dot{\phi}_0^J \dot{\phi}_0^K Q^L + D_t Q^I D_t Q^J - \gamma^{ij} \partial_i Q^I \partial_j Q^J \right) + P_{af;IJ} Q^I Q^J \right. \right. \\ & + P_{\langle IJ \rangle \langle KL \rangle} D_t Q^I \dot{\phi}_0^J D_t Q^K \dot{\phi}_0^L + 2P_{\langle IJ \rangle a} D_t Q^I \dot{\phi}_0^J f_{a;K} Q^K + P_{abf;a;Ifb;J} Q^I Q^J \Big] \\ & + N_{(1)} \left[ -P_{\langle IJ \rangle} D_t Q^I \dot{\phi}_0^J + P_{af;I} Q^I - \left( P_{\langle IJ \rangle \langle KL \rangle} D_t Q^K \dot{\phi}_0^L + P_{\langle IJ \rangle a} f_{a;K} Q^K \right) \right] \\ & + \frac{N_{(1)}^2}{2} \left( -6m_{\text{Pl}}^2 H^2 + P_{\langle IJ \rangle} \dot{\phi}_0^I \dot{\phi}_0^J + P_{\langle IJ \rangle \langle KL \rangle} \dot{\phi}_0^I \dot{\phi}_0^J \dot{\phi}_0^K \dot{\phi}_0^L \right) - 2m_{\text{Pl}}^2 H N_{(1)} N_{(1),i}^i \\ & \left. \left. - P_{\langle IJ \rangle} N_{(1)}^i \partial_i Q^I \dot{\phi}_0^J + \frac{m_{\text{Pl}}^2}{4} \left( N_{i,j}^{(1)} N_{(1)}^{i,j} + N_{i,j}^{(1)} N_{(1)}^{j,i} - 2N_{(1),i}^i N_{(1),j}^j \right) \right\} \right. \end{aligned}$$

- From cubic action, we can calculate **bispectrum**

$S_3 = \text{next page}$

# Cubic order action

$$S_3^{(G)} = \int d^4x a^3 \left\{ 3m_{\text{Pl}}^2 H^2 N_{(1)}^3 + 2m_{\text{Pl}}^2 H \frac{\Delta}{a^2} \chi N_{(1)}^2 - \frac{m_{\text{Pl}}^2}{2a^4} [\chi'^{ij} \chi_{,ij} - (\Delta \chi)^2] N_{(1)} \right\}$$

$$\begin{aligned} S_3^{(M)} = & \int d^4x a^3 \left[ (g_1)_{IJK} Q^I Q^J Q^K + (g_2)_{IJK} D_t Q^I Q^J Q^K + (g_3)_{IJK} D_t Q^I D_t Q^J Q^K + (g_4)_{IJK} D_t Q^I D_t Q^J D_t Q^K \right. \\ & \left. + (g_a)_{IJ} Q^I \partial_i Q^J N_{(1)}^i + (g_b)_{IJ} D_t Q^I \partial_i Q^J N_{(1)}^i + (g_c)_{IJK} Q^I \gamma^{ij} \partial_i Q^J \partial_j Q^K + (g_d)_{IJK} D_t Q^I \gamma^{ij} \partial_i Q^J \partial_j Q^K \right] \end{aligned}$$

Coefficients  $[\mathcal{N}_I \equiv P_{\langle IJ \rangle} \dot{\phi}_0^J / (2m_{\text{Pl}}^2 H)]$

$$\begin{aligned} (g_1)_{IJK} = & \frac{1}{6} \left( P_{\langle LM \rangle} R^L_{IJN;K} \dot{\phi}_0^M \dot{\phi}_0^N + P_{af;a;IJK} + 3P_{\langle LM \rangle} a R^L_{IJN} \dot{\phi}_0^M \dot{\phi}_0^N f_{a;K} + 3P_{abf;a;IJf_b;K} + P_{abcfa;If_b;Jf_c;K} \right) \\ & + \frac{1}{2} \mathcal{N}_K \left[ -P_{\langle LM \rangle} R^L_{IJN} \dot{\phi}_0^M \dot{\phi}_0^N + P_{af;a;IJ} + P_{abf;a;If_b;J} - \dot{\phi}_0^L \dot{\phi}_0^M \left( P_{\langle LM \rangle} \langle AB \rangle R^A_{IJC} \dot{\phi}_0^B \dot{\phi}_0^C + P_{\langle LM \rangle} af;a;IJ + P_{\langle LM \rangle} abf;a;If_b;J \right) \right] \\ & + \frac{1}{2} \mathcal{N}_J \mathcal{N}_K \left( P_{\langle AB \rangle} af;a;I \dot{\phi}_0^A \dot{\phi}_0^B + P_{\langle AB \rangle} \langle CD \rangle af;a;I \dot{\phi}_0^A \dot{\phi}_0^B \dot{\phi}_0^C \dot{\phi}_0^D \right) \\ & - \mathcal{N}_I \mathcal{N}_J \mathcal{N}_K \left( \frac{1}{2} P_{\langle AB \rangle} \dot{\phi}_0^A \dot{\phi}_0^B + P_{\langle AB \rangle} \langle CD \rangle \dot{\phi}_0^A \dot{\phi}_0^B \dot{\phi}_0^C \dot{\phi}_0^D + \frac{1}{6} P_{\langle AB \rangle} \langle CD \rangle \langle EF \rangle \dot{\phi}_0^A \dot{\phi}_0^B \dot{\phi}_0^C \dot{\phi}_0^D \dot{\phi}_0^E \dot{\phi}_0^F \right) \\ (g_2)_{IJK} = & \frac{1}{6} \left( P_{\langle LM \rangle} R^L_{JKI} + 3P_{\langle IL \rangle} R^L_{JKM} \right) \dot{\phi}_0^M + \frac{1}{2} P_{\langle IL \rangle} a \dot{\phi}_0^L f_{a;JK} + \frac{1}{2} P_{\langle IL \rangle} \langle AB \rangle R^A_{JKM} \dot{\phi}_0^B \dot{\phi}_0^L \dot{\phi}_0^M + \frac{1}{2} P_{\langle IL \rangle} ab \dot{\phi}_0^L f_{a;Jf_b;K} \\ & - \mathcal{N}_K \left( P_{\langle IL \rangle} a \dot{\phi}_0^L f_{a;J} + P_{\langle IL \rangle} \langle MN \rangle a \dot{\phi}_0^L \dot{\phi}_0^M \dot{\phi}_0^N f_{a;J} \right) + \mathcal{N}_J \mathcal{N}_K \left( P_{\langle IL \rangle} \dot{\phi}_0^L + \frac{5}{2} P_{\langle IL \rangle} \langle MN \rangle \dot{\phi}_0^L \dot{\phi}_0^M \dot{\phi}_0^N + \frac{1}{2} P_{\langle IL \rangle} \langle MN \rangle \langle AB \rangle \dot{\phi}_0^L \dot{\phi}_0^M \dot{\phi}_0^N \dot{\phi}_0^A \dot{\phi}_0^B \right) \\ (g_3)_{IJK} = & -\frac{1}{2} \mathcal{N}_K \left[ P_{\langle IJ \rangle} + 3 \left( P_{\langle IJ \rangle} \langle LM \rangle + P_{\langle IL \rangle} \langle JM \rangle \right) \dot{\phi}_0^L \dot{\phi}_0^M + P_{\langle IL \rangle} \langle JM \rangle \dot{\phi}_0^L \dot{\phi}_0^M \dot{\phi}_0^A \dot{\phi}_0^B \right] + \frac{1}{2} \left( P_{\langle IJ \rangle} a + P_{\langle IL \rangle} \langle JM \rangle a \dot{\phi}_0^L \dot{\phi}_0^M \right) f_{a;K} \\ (g_4)_{IJK} = & \frac{1}{2} P_{\langle IJ \rangle} \langle KL \rangle \dot{\phi}_0^L + \frac{1}{6} P_{\langle IL \rangle} \langle JM \rangle \dot{\phi}_0^L \dot{\phi}_0^M \dot{\phi}_0^N \quad (ga)_{IJ} = \mathcal{N}_I \left( P_{\langle JK \rangle} \dot{\phi}_0^K + P_{\langle JK \rangle} \langle LM \rangle \dot{\phi}_0^K \dot{\phi}_0^L \dot{\phi}_0^M \right) - P_{\langle JK \rangle} af;a;I \dot{\phi}_0^K \\ (gb)_{IJ} = & -P_{\langle IJ \rangle} - P_{\langle IK \rangle} \langle JL \rangle \dot{\phi}_0^K \dot{\phi}_0^L \quad (gc)_{IJK} = \frac{1}{2} \mathcal{N}_I \left( -P_{\langle JK \rangle} + P_{\langle JK \rangle} \langle LM \rangle \dot{\phi}_0^L \dot{\phi}_0^M \right) - \frac{1}{2} P_{\langle JK \rangle} af;a;I \quad (gd)_{IJK} = -\frac{1}{2} P_{\langle IL \rangle} \langle JK \rangle \dot{\phi}_0^L \end{aligned}$$

# “Effective” theory

Single field inflation  
in Einstein gravity

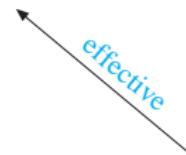
# “Effective” theory

Single (multi) field inflation

No gravity effect

Symmetry principles

(Senatore et al.)

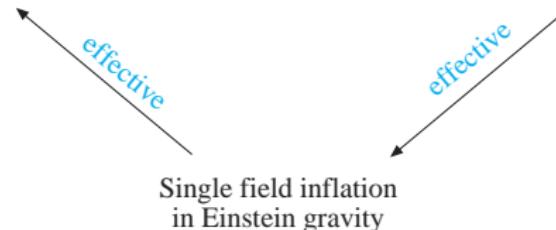


Single field inflation  
in Einstein gravity

# “Effective” theory

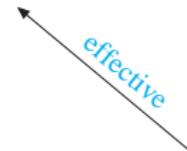
Single (multi) field inflation  
No gravity effect  
Symmetry principles  
(Senatore et al.)

Single field inflation  
in higher order gravity  
(Weinberg)



# “Effective” theory

Single (multi) field inflation  
No gravity effect  
Symmetry principles  
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Single field inflation  
in higher order gravity  
(Weinberg)

Single field inflation  
in Einstein gravity  
Strong adiabaticity



# “Effective” theory

Single (multi) field inflation  
No gravity effect  
Symmetry principles  
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Single field inflation  
in higher order gravity  
(Weinberg)

effective

effective

Single field inflation  
in Einstein gravity  
Strong adiabaticity

effective

Multi-field inflation  
in Einstein gravity  
Weaker or broken adiabaticity

# "Effective" theory

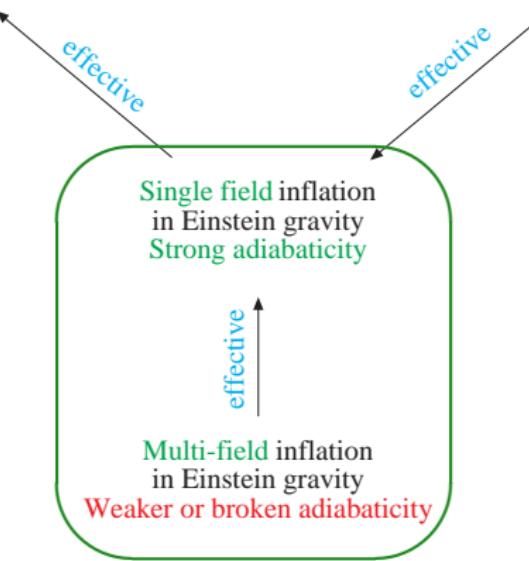
Single (multi) field inflation  
No gravity effect  
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Single field inflation  
in higher order gravity  
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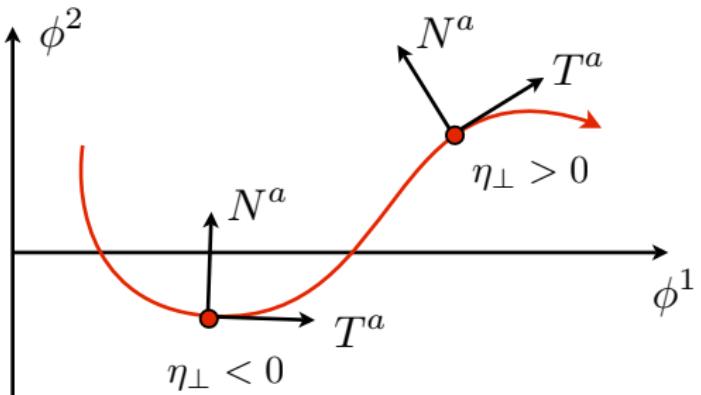
Single field inflation  
in Einstein gravity  
Strong adiabaticity

Multi-field inflation  
in Einstein gravity  
Weaker or broken adiabaticity

Our focus



# Simplest case: 2-field inflation in new basis



$$\gamma_{ab} \rightarrow \gamma_{ab} e_I^a e_J^b = \delta_{IJ} \text{ with } I, J = T, N$$

$$\left. \begin{aligned} T^a &\equiv \frac{\dot{\phi}_0^a}{\sqrt{\gamma_{ab}\dot{\phi}_0^a\dot{\phi}_0^b}} \\ N^a &\equiv s_N \left( \gamma_{bc} \frac{DT^b}{dt} \frac{DT^c}{dt} \right)^{-1/2} \frac{DT^a}{dt} \end{aligned} \right\} \rightarrow \eta_\perp \equiv \frac{N^a V_a}{H\dot{\phi}_0} \left( \frac{DT^a}{dt} = -H\eta_\perp N^a \right)$$

# Full reduction vs. perturbation

- Quadratic action = free theory

$$S = \underbrace{S_2[TT] + S_2[NN] + S_2[TN]}_{\text{free theory}} + \underbrace{S_3 + \dots}_{\text{interaction}}$$

- Quadratic **adiabatic** action = free theory

$$S = \underbrace{S_2[TT]}_{\text{free}} + \underbrace{S_2[NN] + S_2[TN] + S_3 + \dots}_{\text{interaction}}$$

Justifiable since expansion parameter  $\eta_\perp^2 H^2/M^2 \ll 1$

- Pros of 1st approach: we gain **full control**
- Pros of 2nd approach
  - ① **Quick** estimate
  - ② **Well-known** free solution is available
  - ③ Free from (or more suppression of) **higher derivatives**

# Quadratic action in new basis

Canonical 2-field Lagrangian:  $P = G_{ab}X^{ab} - V$

Conformal time  $d\tau = dt/a$ , rescaled field  $v_I = aQ_I$ ,  $\zeta \equiv aH\eta_\perp$

$$\begin{aligned} S_2 &= \int d^4x \frac{1}{2} \left[ {v_T}'^2 - (\nabla v_T)^2 - (\Omega_{TT} - \zeta^2) {v_T}^2 \right] && \text{function of } v_T \\ &+ \int d^4x \frac{1}{2} \left[ {v_N}'^2 - (\nabla v_N)^2 - (\Omega_{NN} - \zeta^2) {v_N}^2 \right] && \text{function of } v_N \\ &+ \int d^4x v_N (-\Omega_{TN} + \zeta' + 2\zeta \partial_\tau) v_N && \text{interaction} \end{aligned}$$

$$\Omega_{TT} = a^2 V_{TT} - a^2 H^2 (2 - \epsilon) - 2a^2 H^2 \epsilon (3 + \epsilon - 2\eta_\perp)$$

$$\Omega_{NN} = a^2 \left( \underbrace{V_{NN} + \epsilon m_{\text{Pl}}^2 H^2 \mathbb{R}}_{\equiv M^2} \right) - a^2 H^2 (2 - \epsilon) \quad \left( \mathbb{R} \equiv R_{abcd} T^a N^b T^c N^d \right)$$

$$\Omega_{TN} = a^2 V_{TN} + 2a^2 H^2 \epsilon \eta_\perp$$

# Effective single field quadratic action

Schematically

$$S_2 = \frac{1}{2} \int d^4x v_T \Delta_{TT} v_T + \frac{1}{2} \int d^4x v_N \Delta_{NN} v_N + \int d^4x v_N \mathcal{O} v_T$$

Evaluation of the Gaussian integral over  $v_N$  as

$$\begin{aligned} e^{S_{\text{eff}}[v_T]} &= \exp \left( \frac{1}{2} \int d^4x v_T \Delta_{TT} v_T \right) \int [D\psi] \exp \left( \int d^4x \frac{1}{2} v_N \Delta_{NN} v_N + v_N \mathcal{O} v_T \right) \\ &= \exp \left[ \frac{1}{2} \int d^4x (v_T \Delta_{TT} v_T - \mathcal{O} v_T \Delta_{NN}^{-1} \mathcal{O} v_T) \right] (\det \Delta_N)^{-1/2} \end{aligned}$$

Green's function in the Fourier space:  $M \rightarrow$  cutoff of the theory

$$\Delta_{NN}^{-1} = G(x; x') = \frac{1}{\square + \Omega_{NN} - \zeta^2} \rightarrow G(\tau, \tau'; k) = \frac{1}{k^2 + \Omega_{NN} - \zeta^2} (1 + \dots)$$

# Footprint of heavy isocurvature mode

Partial integration and redefinition  $u \equiv e^{\beta/2} v_T$

$$S_{\text{eff}} = \int d\tau d^3 k \frac{1}{2} \left[ u'^2 - e^{-\beta(\tau, k)} u^2 - \Omega(\tau, k) u^2 \right]$$

$$e^\beta = 1 + 4\eta_\perp^2 \left( \frac{M^2}{H^2} - 2 + \epsilon - \eta_\perp^2 + \frac{k^2}{a^2 H^2} \right)^{-1} \approx 1 + 4\eta_\perp^2 \frac{H^2}{M^2}$$

$$\Omega = \Omega_0 - \frac{\beta''}{2} - \left( \frac{\beta'}{2} \right)^2 - aH\beta' (1 + \epsilon - \eta_\parallel)$$

$$\Omega_0 = 2a^2 H^2 \left( 1 + \epsilon - \frac{3}{2}\eta_\parallel + \epsilon^2 - 2\epsilon\eta_\parallel + \frac{1}{2}\eta_\parallel\xi_\parallel \right) \rightarrow \text{single field}$$

Single field action with non-trivial speed of sound  $c_s^2 \equiv e^{-\beta}$

# Reduction of cubic action

Effective single field cubic order action

$$\begin{aligned} S[v_T, v_N] = & \frac{1}{2} \int v_T \Delta_{TT} v_T + \int v_T^2 K_{TTT} v_T + \frac{1}{2} \int v_N [\Delta_{NN} + 2v_T K_{TNN}] v_N \\ & + \int [\mathcal{O} v_T + v_T^2 K_{TTN}] v_N + \int v_N^2 K_{NNN} v_N \end{aligned}$$

Taking similar steps with the aid of external source term

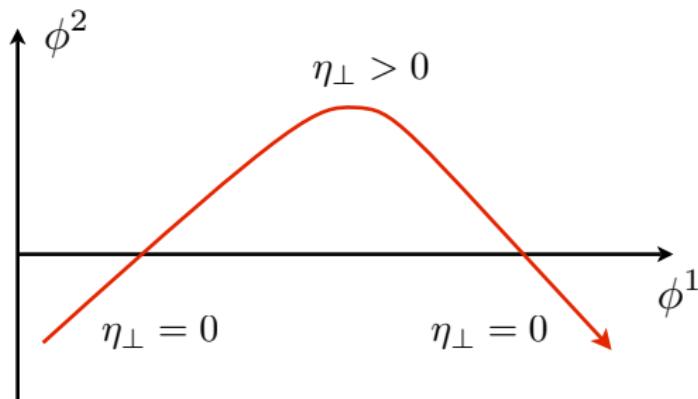
$$\begin{aligned} S[v_T] = & \frac{1}{2} \int v_T \Delta_{TT} v_T + \int v_T^2 K_{TTT} v_T \\ & - \frac{1}{2} \int [\mathcal{O} v_T + v_T^2 K_{TTN}] [\Delta_{NN} + 2v_T K_{TNN}]^{-1} [\mathcal{O} v_T + v_T^2 K_{TTN}] \\ & - \int \{[\Delta_{NN} + 2v_T K_{TNN}]^{-1} [\mathcal{O} v_T + v_T^2 K_{TTN}]\}^3 + \dots \\ \supset & \int d^4 x a^3 \frac{H\dot{\phi}_0}{2m_{\text{Pl}}^2} \eta_\perp^2 Q_T^3 = \mathcal{O}(\epsilon^2 \eta_\perp^2) \end{aligned}$$

# Example: A turn in the trajectory

$\eta_{\perp}$  is the **phenomenological parameter**

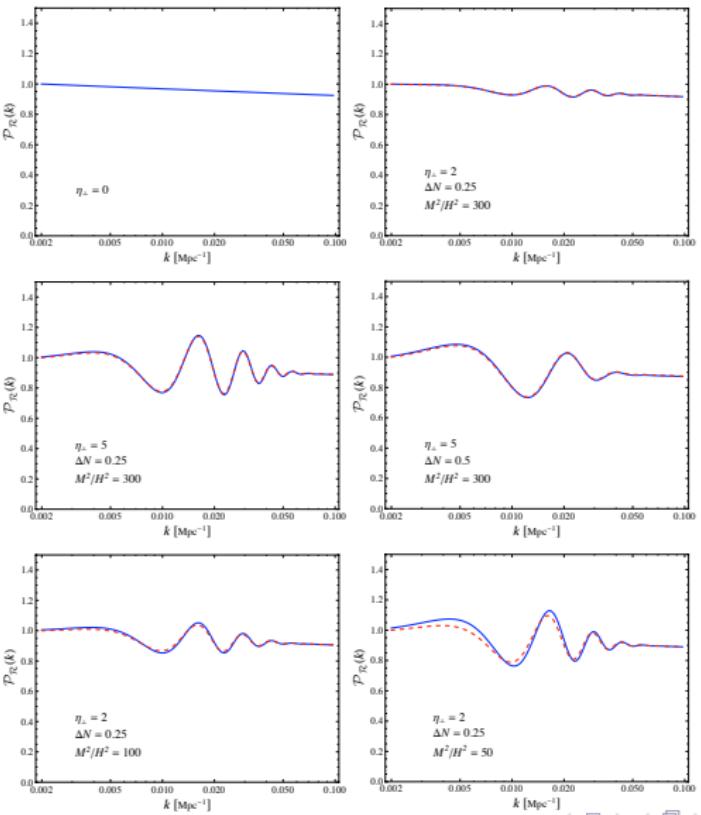
$$\eta_{\perp} = \frac{\eta_{\perp}^{(\max)}}{\cosh^2 [2(N - N_0)/\Delta N]}$$

A **single turn** between otherwise straight trajectories



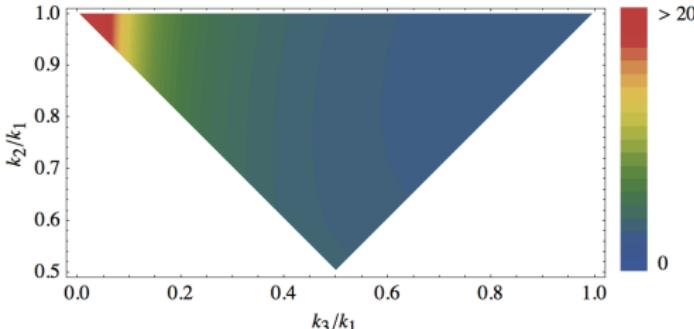
## Features in the correlation functions

# Features in the power spectrum



# Features in the bispectrum

Correlated enhancement of the bispectrum: A strong support for multi-field inflation?



Dimensionless shape function  $(k_1 k_2 k_3)^2 B_{\mathcal{R}}(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3)$ : Local!

$$f_{\text{NL}} \lesssim 15\epsilon\eta_{\perp}^2 \sim \mathcal{O}(3)$$

# How to interpret?

Correlated features in the power and bispectra

## ① Mode mixing between $\parallel$ & $\perp$

- Curvilinear trajectory
- (Iso)curvature modes before  $\neq$  (iso)curvature modes after:  
mixing

## ② Massive particle production

- Excitation of quanta = particle
- Heavy modes  $\rightarrow$  damping away quickly

## ③ Deviation from geodesic

- Departure from min =  $\eta_{\perp}^2 H^2 / M^2$
- Reminiscent of linearized grav wave

# Curvature effects on isocurvature perturbation

Constant curvature of 2D surface  $R^a{}_{bcd} = K(\delta^a{}_c G_{bd} - \delta^a{}_d G_{bc})$

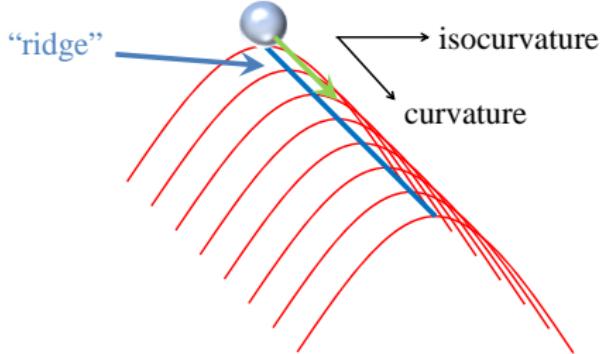


Only **non-zero** component of the Riemann tensor =  $R_{TNTN}$

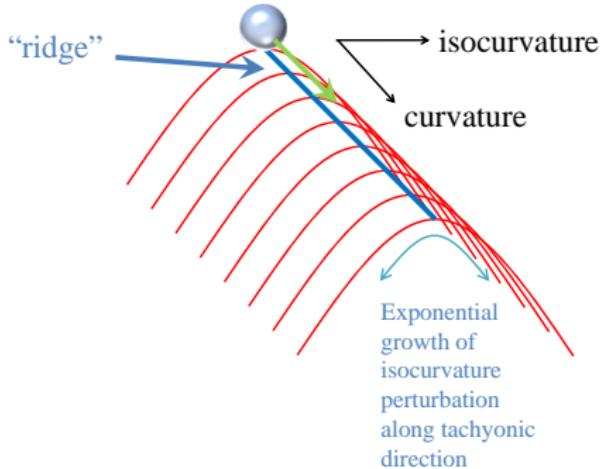
$$S_2 \supset \int d^4x R_{NTTN} \dot{\phi}_0^T \dot{\phi}_0^T Q_N Q_N \sim -K \int d^4x \dot{\phi}_0^2 Q_N^2 \rightarrow \begin{cases} \text{enhanced } (K < 0) \\ \text{suppressed } (K > 0) \end{cases}$$

**No effect** on the curvature perturbation

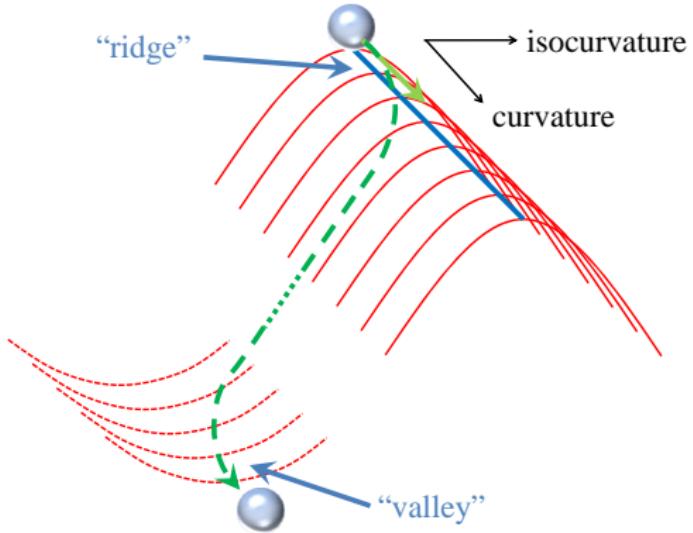
# Enhancing isocurvature perturbation



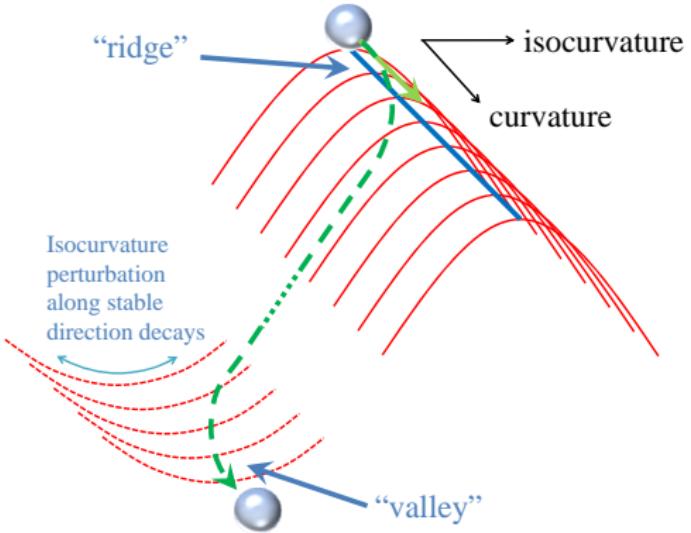
# Enhancing isocurvature perturbation



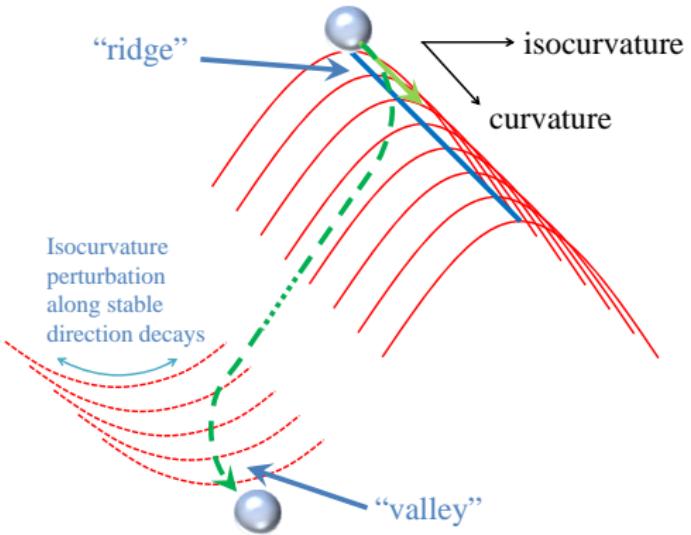
# Enhancing isocurvature perturbation



# Enhancing isocurvature perturbation



# Enhancing isocurvature perturbation



Supporting negative effective mass

$$\left. \begin{array}{l} \text{positive bare mass} \\ \text{negative curvature } m_{\text{cur}}^2 \sim \epsilon m_{\text{Pl}}^2 H^2 \mathbb{R} \end{array} \right\} \rightarrow m_{\text{eff}}^2 < 0 : \text{significant iso?}$$

# Conclusions

## ① Action of general multi-field system

- Flat gauge: physical d.o.f. in field fluctuations
- Covariant derivative w.r.t. field space:  
$$P = P \left[ G_{IJ}, X^{IJ}, f_a^{J_1 \dots J_{n_a}}(\phi^I) \right]$$
- A straightforward approach to obtain higher order action **easily and systematically**

## ② Effective single field inflation

- Integrating out the heavy isocurvature modes
- Non-trivial speed of sound dependence
- Or perturbation from curved trajectory

## ③ Phenomenology of multi-field system

- Oscillation in the power spectrum + correlated enhancement of the bispectrum
- Isocurvature perturbation