NEW PERSPECTIVE ON GALAXY CLUSTERING AND COSMOLOGY: GENERAL RELATIVISTIC EFFECTS

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I. INTRODUCTION:

Galaxies as a Cosmological Probe – What is the Problem?

Dark Energy Surveys

- current and future surveys:
    - Baryonic Oscillation Spectroscopic Survey
    - Dark Energy Survey
    - Panoramic Survey Telescope & Rapid Response System
    - Hobby-Eberly Telescope Dark Energy Experiment
    - Wide Field Multi-Object Spectroscoph
    - Large Synoptic Survey Telescope
- future space missions:
    - EUCLID, Supernova Acceleration Probe
    - Cosmic Inflation Probe
    - Wide-Field Infrared Survey Telescope

Legend of Galaxies

- **current and future dark energy surveys:**
    - ***better precision and larger scales!***

- **galaxies as cosmological probes:**
    - **BAO signature:** D_A, Ω_b/Ω_m, k~0.06 h/Mpc
    - **galaxy power spectrum:** n_s, Ω_k, $\Omega_m h$, k~0.01 h/Mpc
    - **primordial non-Gaussianity:** f_{NL}, n_s, k~0.001 h/Mpc

a "clean" cosmological probe in linear regime

BUT is our faith well founded?

Cosmological Probe

- *precision cosmology!*
- **galaxies trace underlying matter**
  - **biased tracer:** $\delta_g = b\ \delta_m$
  - **z-space distortion:** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **gravitational lensing:** $\delta_g = b\ \delta_m + (5p-2)\ \kappa$
  - **contributions are added in *adhoc* manner!**

is this everything? or are there more contributions?

we need unified treatments!

Relativistic Perspective

- **theoretical inconsistency in galaxy clustering**
  - **standard description:** $\delta_g = b\,\delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **synchronous gauge (e.g., CMBFAST, CAMB)**
    - **free falling frame** $\psi = V = 0$
  - **Poisson equation** $\nabla^2\psi \;\sim \delta_m \sim 0 \;\;?$

galaxy clustering is based on Newtonian description!

Gauge Issues

- **theoretical predictions in cosmology**
  - **compute perturbations such as** $\delta_m, \ \psi, \ P_m(k), \ \cdots$
  - **compare to observable quantities**
  - **perturbations are *gauge-dependent***
  - ***so are many theoretical descriptions!***

- **observable quantities**
  - **gauge-invariance is a necessary condition**

theoretical predictions* vs *observables: gauge issue!

II. LARGE-SCALE STRUCTURE:

General Relativistic Description – New Perspective

Cosmology

- **modern cosmology: general theory of relativity**
- **cosmological framework:**
  - **described by Einstein equations**
  - **homogeneous & isotropic FLRW universe**
  - **inflation, big bang nucleosynthesis, CMB**

- ***galaxies in cosmological framework?***
  - **Newtonian, no GR description**
  - **automatic disqualification?**

Galaxies in General Relativity

- *what are observables?*
- geodesic equations of photons from galaxies
- time component: *Sachs-Wolfe* effect
  - observed redshift

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'\;(\dot\psi-\dot\phi)\right]\,.$$

- spatial component: *gravitational lensing* effect
  - observed position $\hat{n}=(\theta,\phi)$
  - lensing displacement $(\delta r,\;\delta\theta,\;\delta\phi)$
  - magnification $\mu\simeq 1+2\kappa$

Effects on Galaxies

- **construct a galaxy fluctuation field:**
  - **total number of observed galaxies** $N_{\rm tot}$
  - **observed volume** $dV_{\rm obs}$ **given** $(z_{\rm obs}, \hat{n})$
  - **fluctuation field** $\delta_{\rm obs} = \frac{n_{\rm obs}}{\langle n_{\rm obs} \rangle} - 1$

- **relation to *true* number density:**
  - **number conservation** $N_{\rm tot} = n_{\rm true}\, dV_{\rm true} = n_{\rm obs}\, dV_{\rm obs}$
  - **volume element** $dV_{\rm obs} = \frac{r^2(z_{\rm obs})}{H(z_{\rm obs})}\, dz_{\rm obs}\, d\Omega_{\rm obs}$
  - **note** $z_{\rm true} \neq z_{\rm obs}$, $d\Omega_{\rm true} \neq d\Omega_{\rm obs}$

Unified Treatment

- **observable:** $N_{\rm tot} = n_{\rm true}\; dV_{\rm true} = n_{\rm obs}\; dV_{\rm obs}$
- **volume effects:**
  - **redshift-space distortion:** $\frac{\partial z}{\partial z_{\rm obs}} \simeq \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **lensing magnification:** $\frac{\partial \Omega}{\partial \Omega_{\rm obs}}\frac{\partial f}{\partial f_{\rm obs}} \simeq \frac{1}{\mu^2} = 1 - 4\;\kappa$
- **source effects:**
  - **magnification bias:** $\bar n_{\rm obs}(f_{\rm obs}) \simeq \bar n(f_{\rm obs}/\mu)$
- ***complete description of different effects***

Subtle Issues

- ***what are "true" volume and number density?***
- **true number density:** $n_{\rm true} = n_{\rm phy}$
  - **number density in local inertial frame**
- **true volume:** $dV_{\rm true} = \frac{r^2(z_{\rm true})}{H(z_{\rm true})}\, dz_{\rm true}\, d\Omega_{\rm true}$ **?**
  - $z_{\rm true}$, $\hat{s}$ **in FLRW universe** ***not really "true!"***
  - **at what coordinate system (*gauge*)?**

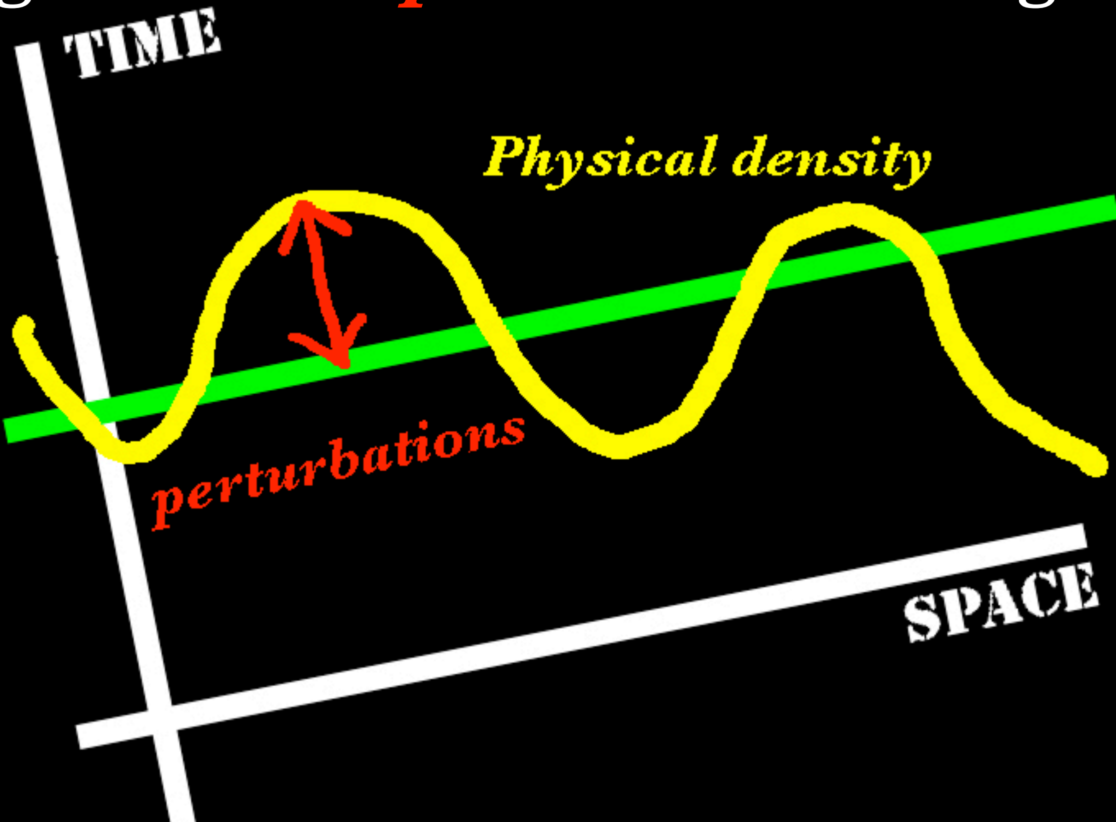
    e.g., $z_{\rm true} = z(t)$ **,** $\bar{\rho}_m = \bar{\rho}_m(t)$
- **fully relativistic theory:**
  - **quest for true volume**
  - **better understanding of gauge issues**

Correspondence

- **cosmological perturbations:**
  - **inhomogeneous physical spacetime**
  - **homogeneous *fictitious* background**

Gauge Freedom

- general covariance in GR:
  - free to choose a coordinate system
  - change in *correspondence* to background



Gauge Issues

- **gauge-dependent quantities**
  - **change its value depending on coordinates**
  - ***cannot be directly associated with observables!***

- **theoretical descriptions of observables**
  - **should be gauge-invariant**
  - ***but surprisingly not for most!***

- **Newtonian limit**
  - **unambiguous hypersurface of simultaneity**

More Subtle Issues

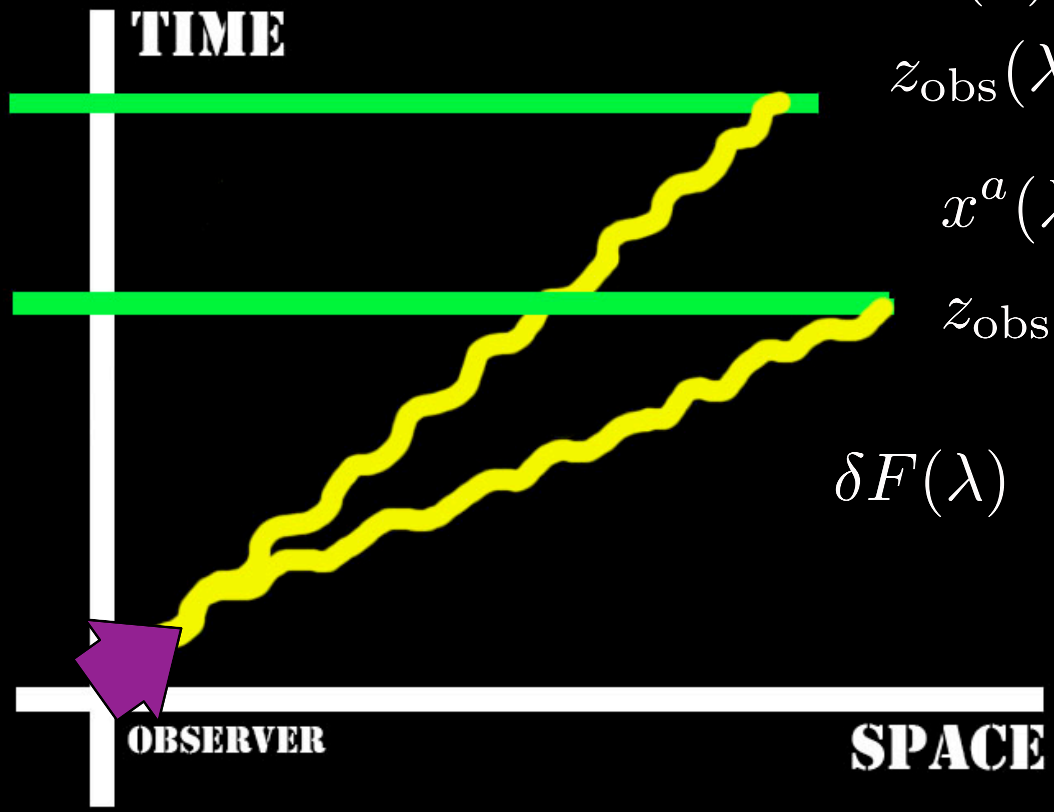
- **observed mean** vs **ensemble average**
  - observed mean is *gauge-invariant*
  - ensemble average is *gauge-dependent*
- spatial gauge transformation (*unphysical*)
  - mapping is invariant, time is fixed
- **observer position**
  - also gauge-dependent

Complications

- **perturbations along photon geodesic:**
  - **observed angle and redshift**

$$\bar{x}^a(\lambda)$$

$$z_{\rm obs}(\lambda) = z_{\rm src} = z_h$$



$$x^a(\lambda) = \bar{x}^a(\lambda) + \delta x^a(\lambda)$$

$$z_{\rm obs}(\lambda) \neq z_{\rm src} \neq z_h$$

$$\begin{aligned} \delta F(\lambda) &= F[x^a(\lambda)] - \bar{F}[\bar{x}^a(\lambda)] \\ &\neq \delta F[x^a(\lambda)] \end{aligned}$$

Sasaki, MNRAS, 1987

Observed Redshift

- **observed redshift:**

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'\,(\dot\psi-\dot\phi)\right].$$

- **observer's point of view:**
  - **true redshift *z* from observed redshift *$z_{\rm obs}$***

- **new perspective:**
  - **true redshift is *fictitious & gauge-dependent***
  - **in another coordinate $z, \quad V, \quad \psi$ *change!***

Observed Redshift

- **observed redshift:**

$$1+z_{\rm obs}=(1+z)\left[1+V(z)-V(0)-\psi(z)+\psi(0)-\int_0^r dr'\ (\dot\psi-\dot\phi)\right]\ .$$

$$=(1+z^s)\left[1+\frac{1}{6}\int_0^r dr'\ (\dot h+3\ \dot h^{\parallel}_{\alpha\beta}e^\alpha e^\beta)\right]$$ **(synchronous)**

- **gauge transformation:**
  - **true redshift** $z\ \neq\ z^s$
  - **true volume element** $\frac{r^2(z)}{H(z)}\neq\frac{r^2(z^s)}{H(z^s)}$
  - **observed redshift is *gauge-invariant*** $z_{\rm obs}\ =\ z^s_{\rm obs}$

Gravitational Lensing

- **observed position:** $\hat{n} = (\theta, \phi)$
- **lensing displacements:** $(\delta r,\ \delta\theta,\ \delta\phi)$ **&** $\delta\tau$
- **true position:** $\hat{s} = (\theta + \delta\theta,\ \phi + \delta\phi)$
- **magnification:** $\mu \simeq \left|\frac{d^2\hat{n}}{d^2\hat{s}}\right| = 1 + 2\ \kappa$

- **gauge-dependent quantities:** $\delta r,\ \delta\theta,\ \delta\phi,\ \hat{s},\ \kappa$
    - ***standard weak lensing is gauge-dependent!***

Magnification

- **definition is *inaccurate!***
- **coordinate-independent definition:**
  - **luminosity in local inertial frame** L
  - **measure flux & redshift** $f_{\rm obs}$ & $z_{\rm obs}$
  - **magnification is *physical!***

$$\mu = f_{\rm obs} \left(\frac{L}{4\pi D_L^2(z_{\rm obs})} \right)^{-1}$$

  - **in a homogeneous universe using** $z_{\rm obs}$**, not** $z_{\rm true}$

$$D_L(z_{\rm obs}) = (1 + z_{\rm obs})\, r(z_{\rm obs})$$

Luminosity Distance

- **observed flux and intrinsic luminosity**:
  - **includes Sachs-Wolfe and lensing effects**

$$\mathcal{D}_L(z_{\rm obs}) = \sqrt{\frac{L}{4\pi f_{\rm obs}}} \equiv D_L(z_{\rm obs})(1+\delta\mathcal{D}_L)$$

Sasaki, MNRAS, 1987

$$= D_L(z_{\rm obs})\left[1+\frac{\delta\lambda}{r_s}-\frac{1}{2}\int_0^{r_s} dr\delta\theta\right]$$

  - **shift in affine parameter** $\delta\lambda \sim \delta z$
  - **distortion in wave vector expansion** $\delta\theta \sim \kappa$
  - **gauge-invariant** $\delta\mathcal{D}_L$

Luminosity Distance

- **relation to magnification:**

$$\mu = f_{\rm obs}\left(\frac{L}{4\pi D_L^2(z_{\rm obs})}\right)^{-1} = \left(\frac{D_L(z_{\rm obs})}{\mathcal{D}_L(z_{\rm obs})}\right)^2$$

$$= 1 - 2\,\delta\mathcal{D}_L \simeq 1 + 2\,\kappa$$

- **prevalent gauge issues in cosmology:**
  - **observed magnification is *physical***
  - ***but* usual parametrization is *gauge-dependent***
  - **source effect** $\bar{n}_{\rm obs} = \bar{n}[f_{\rm obs}(1 + 2\delta\mathcal{D}_L)]$

Observed Number of Galaxies

- *we still need "true" volume!*
- **total number of observed galaxies:**
  - **observables** $N_{\rm tot}, \ \hat{n} = (\theta, \phi), \ z_{\rm obs}$

$$N_{\rm tot} = \int \, dz_{\rm obs} d\Omega_{\rm obs} \ n_{\rm obs} \frac{r^2(z_{\rm obs})}{(1+z_{\rm obs})^3 H(z_{\rm obs})}$$

$$= \int n_{\rm phy} dV_{\rm phy}$$

- **physical volume:** $dV_{\rm phy}$
  - **occupied by observed galaxies**
  - ***trace backward photon geodesic!***

Matias's Magic

- **integral of 3-form in 4D spacetime manifold:**
  - **observables** $z_{\rm obs}\,,\ \theta_{\rm obs}\,,\ \phi_{\rm obs}$
  - **photon geodesic path** $x^a(\lambda)=\bar x^a(\lambda)+\delta x^a(\lambda)$
  - **Sachs-Wolfe and gravitational lensing effects**
  - **distortion in local Lorentz frame**
  - ***manifestly gauge-invariant***

$$N_{\rm tot}=\int\sqrt{-g}\;n_{\rm phy}\;\varepsilon_{abcd}\;u^d\;\frac{\partial x^a}{\partial z_{\rm obs}}\frac{\partial x^b}{\partial\theta_{\rm obs}}\frac{\partial x^c}{\partial\phi_{\rm obs}}\;dz_{\rm obs}\;d\theta_{\rm obs}\;d\phi_{\rm obs}$$

Levi-Civita symbol ε_{abcd} **, comoving velocity** u^a

Observed Number of Galaxies

- ***fun*** **and/or** ***pain*** **in perturbation expansion!**

$$N_{\rm tot} = \int \sqrt{-g}\, n_{\rm phy}\, \varepsilon_{abcd}\, u^d \frac{\partial x^a}{\partial z}\frac{\partial x^b}{\partial \theta}\frac{\partial x^c}{\partial \phi}\, dz\, d\theta\, d\phi$$

$$= \int n_{\rm phy} \frac{r^2 \sin\theta}{(1+z)^3 H}\, dz\, d\theta\, d\phi \left[1 + 3D + V + 2\frac{\delta r}{r} + H\frac{\partial}{\partial z}\delta r + \left(\cot\theta + \frac{\partial}{\partial\theta}\right)\delta\theta + \frac{\partial}{\partial\phi}\delta\phi + \frac{\bar r^2}{r^2} H \frac{\partial \bar r}{\partial z}\right]$$

$$\equiv \int n_{\rm obs} \frac{r^2\sin\theta}{(1+z)^3 H}\, dz\, d\theta\, d\phi$$

- **subscript "*obs*" is omitted!**

Observed Number Density

- **so far, we have**
  - **volume effects:** $n_{\rm obs} = n_{\rm phy}\left(1 + \sum_\mu \delta_\mu\right)$
  - **source effects:** $\bar n_{\rm obs} \rightarrow \bar n_{\rm phy}\left[f_{\rm obs}(1 + 2\delta\mathcal{D}_L)\right]$

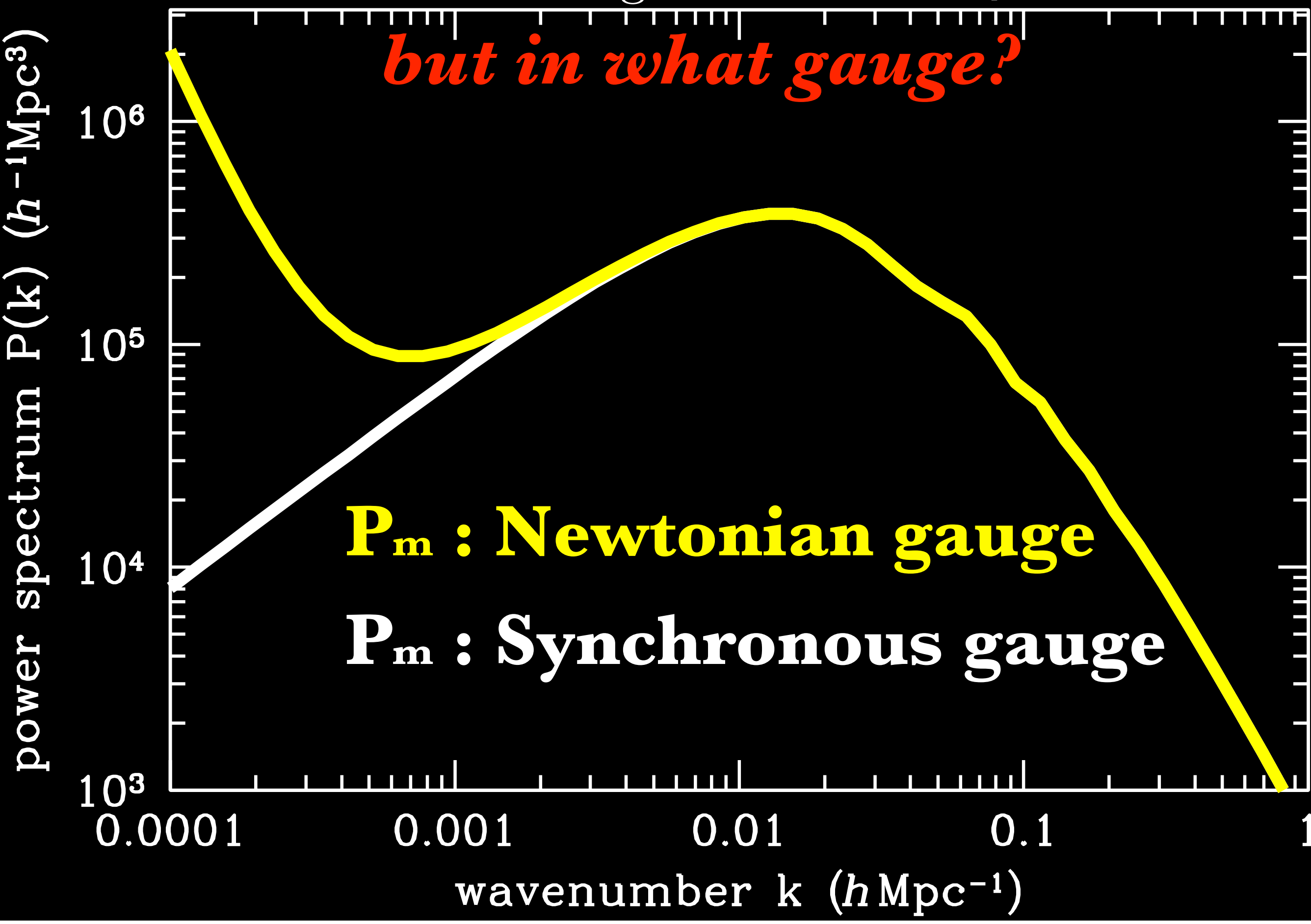
$dn_{\rm phy}/dL \propto L^{-s}$ $\qquad = \bar n_{\rm phy}(L_{\rm thr})(1 - 5p\;\delta\mathcal{D}_L)$

$p = 0.4\;(s-1)$

BUT ***why do we care about galaxies?***

Galaxy Bias

- **galaxies trace underlying matter!**
  - **standard relation** $\delta_{\rm gal} = b\ \delta_m$ (δ_m **: perturbation)**



Galaxy Bias

- **galaxy formation**
  - **described in a local coordinate (proper time)**

$$n_{\rm phy} = F[\rho_m] \quad \rightarrow \quad n_{\rm phy} = F[\rho_m, t_p] \ ,$$

- **time slicing in observation** $1 + z_{\rm obs} = (1+z)(1+\delta z)$
  - ***observed redshift*** **defines simultaneity**
  - **measured at observed redshift**

$$n_{\rm phy} = \bar{n}_{\rm phy}(z_{\rm obs})[1 + b\ \delta_m^{(v)} - e\ \delta z^{(v)}]$$

$$e = \frac{d\ln\bar{n}_{\rm phy}}{d\ln(1+z)} \ , \qquad 1 + z_{\rm obs} = (1+z)(1+\delta z)$$

Cosmological Probe

- *accurate relation to underlying matter*
  - **most cases :** $\delta_g = b\ \delta_m$
  - **prudent work :** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r}$
  - **best efforts so far :** $\delta_g = b\ \delta_m - \frac{1+z}{H}\frac{\partial V}{\partial r} + (5p-2)\ \kappa$
  - **this work :**

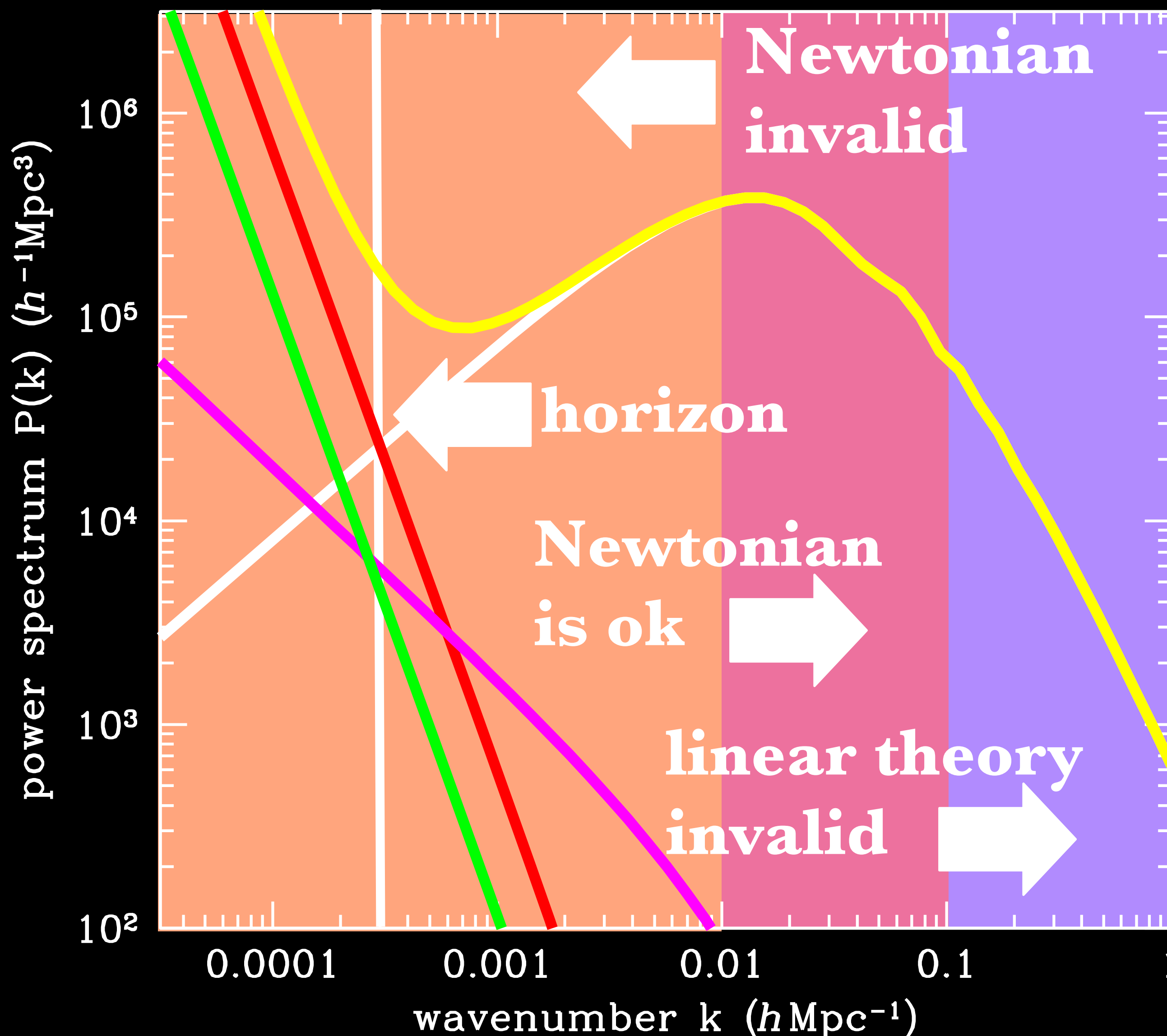
$$\delta_g = b\ \delta_m^{(v)} - e\ \delta z^{(v)} + \alpha_\chi + 2\ \varphi_\chi + V - C_{\alpha\beta}\ e^\alpha e^\beta + 3\ \delta z_\chi + 2\ \frac{\delta\mathcal{R}}{r} - H\frac{\partial}{\partial z}\left(\frac{\delta z_\chi}{\mathcal{H}}\right) - 5p\ \delta\mathcal{D}_L - 2\ \mathcal{K}\ ,$$

Yoo, Fitzpatrick, Zaldarriaga, PRD, 2009

Yoo, PRD, 2010

Scales of Interest

- **Synchronous**
  (CMBFast, CAMB)
  $P_\delta(k)$
  $P_\eta(k)$
- **Newtonian**
  $P_\delta(k)$
  $P_\psi(k)$
  $P_v(k)$



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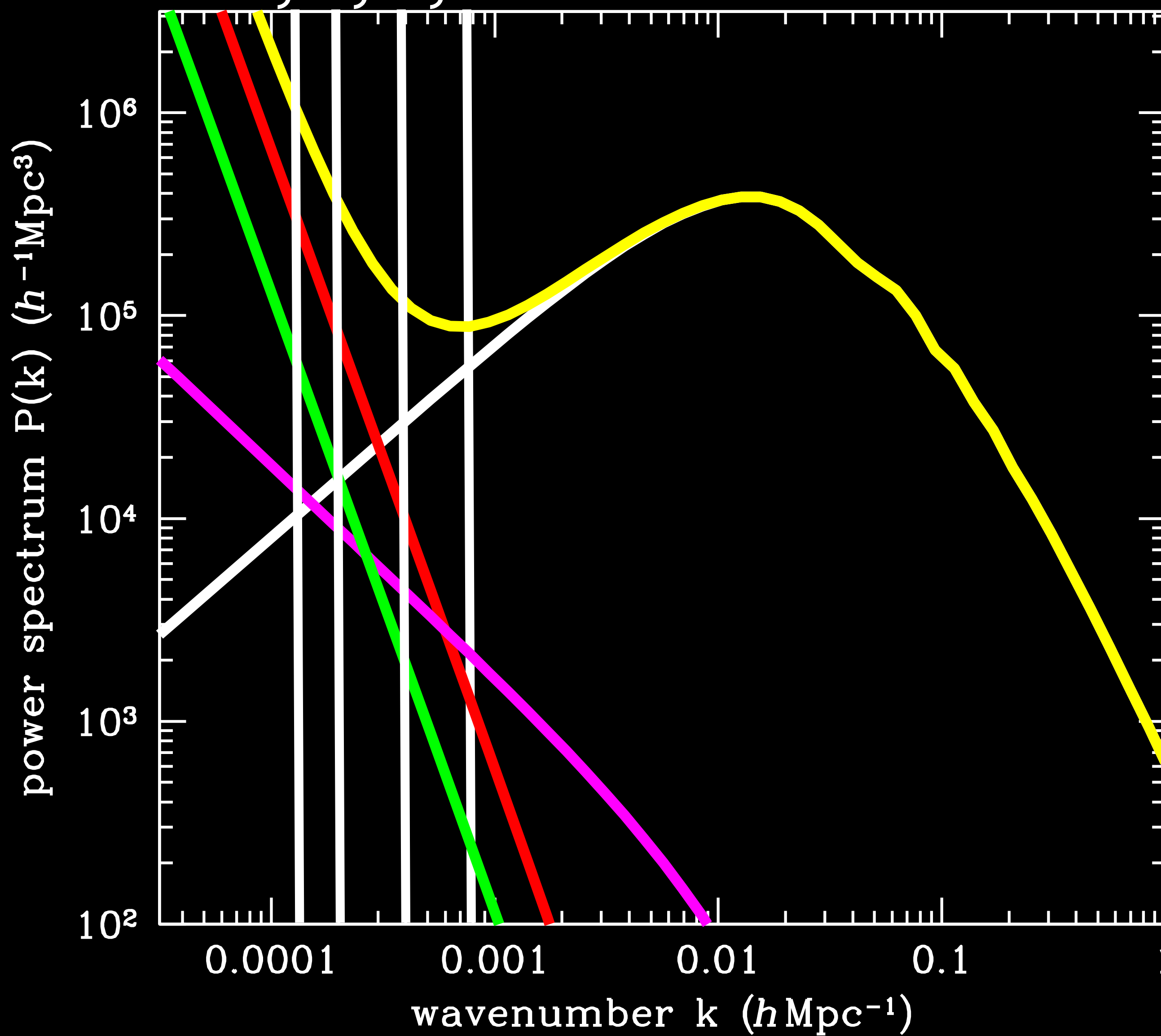
  $P_v(k)$

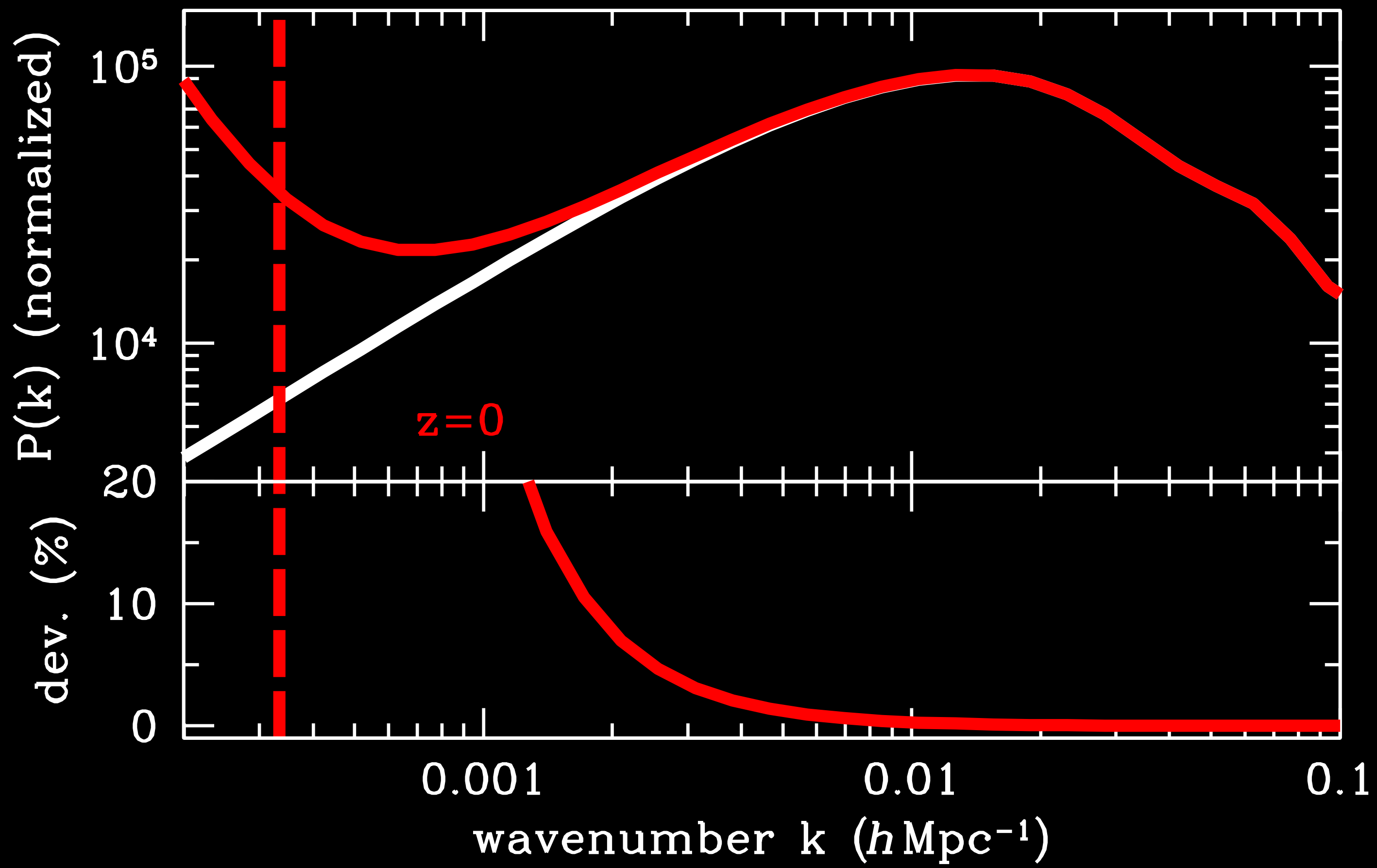
z=6, 3, 1, 0.5



Hubble Horizon

- **horizon scale ~ 3 Gpc/h (today), ~ 3 deg. (recomb.)**

relativistic effects are order one at horizon scale!



Summary

- *fully gauge-invariant general relativistic description*

- **standard method:** *gauge-dependent!*
  - **galaxy clustering**
  - **gravitational lensing**

- **in Newtonian limit:** *Do not worry!*
  - **general relativity reduces to Newtonian**

III. APPLICATIONS:

Impacts on Current Surveys – Why Bother?

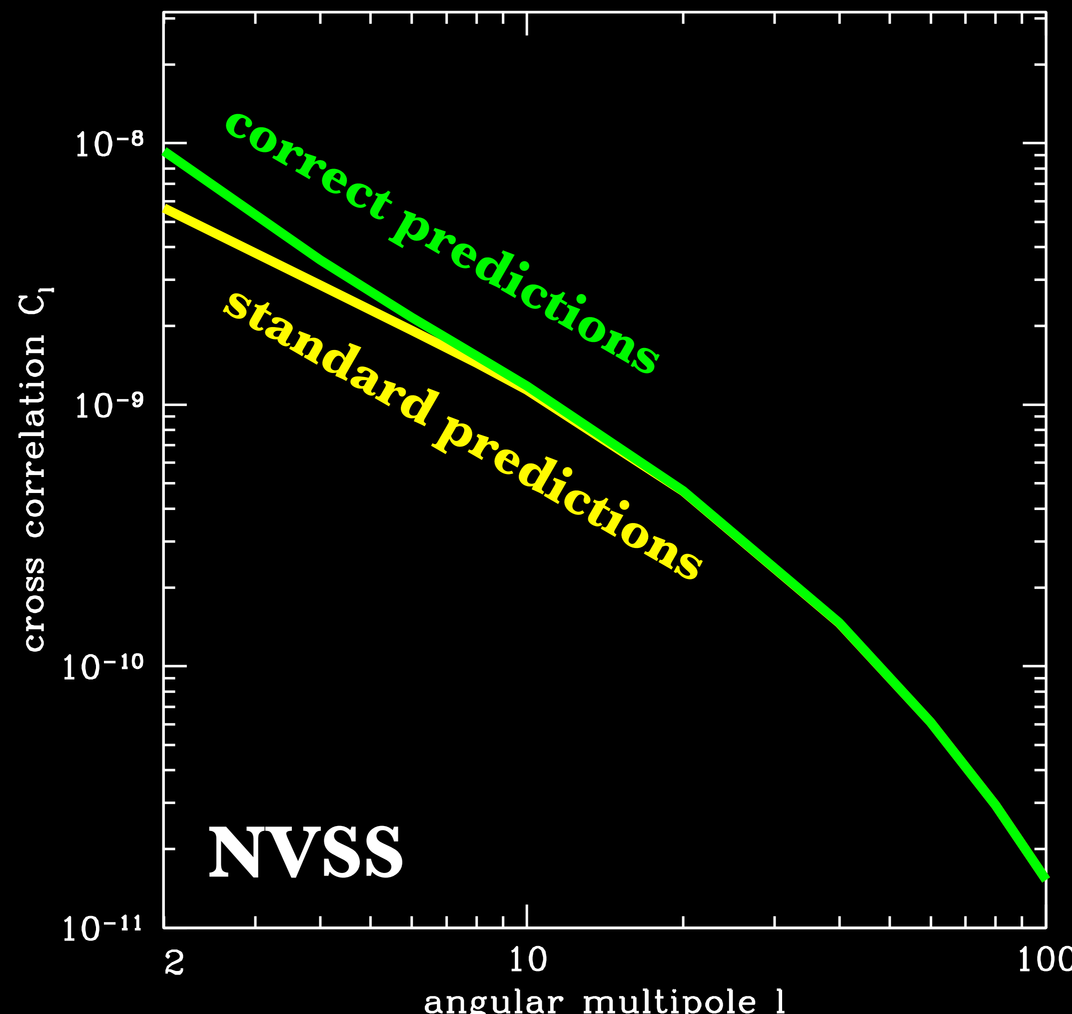
Systematic Errors

- **theoretical predictions:**
  - **new cal.** (***correct***)
  - **standard** (***incorrect***)

- **standard method:**

$$\delta_g = b\ \delta_m^{sync} + (5p - 2)\ \kappa$$

- ***underestimate*** **the observed signals by *a factor two* at low multipoles**

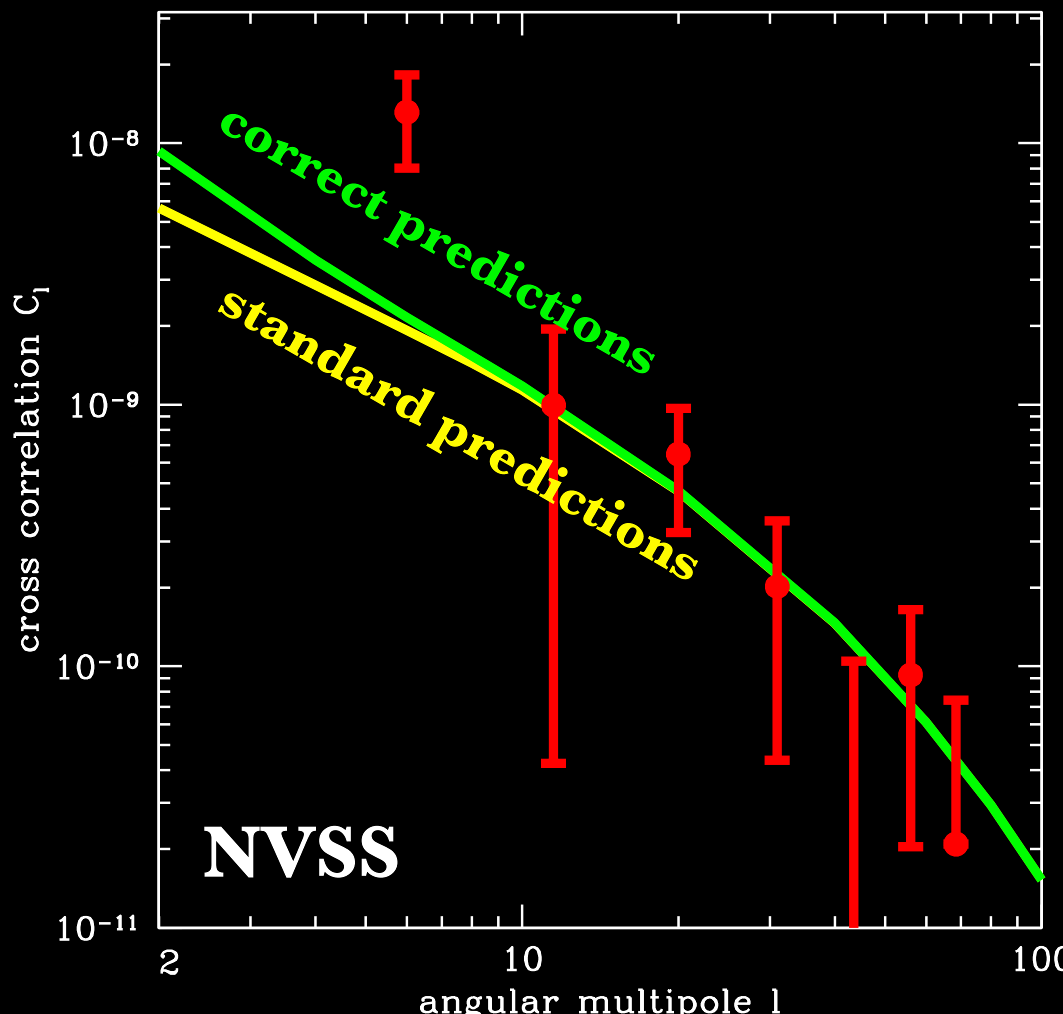


Systematic Errors

- **theoretical predictions:**
  - **new cal.** (***correct***)
  - **standard** (***incorrect***)

- **3.7-σ detection, but *observed signal is larger by 2 at low multipoles* when all tracers are combined**

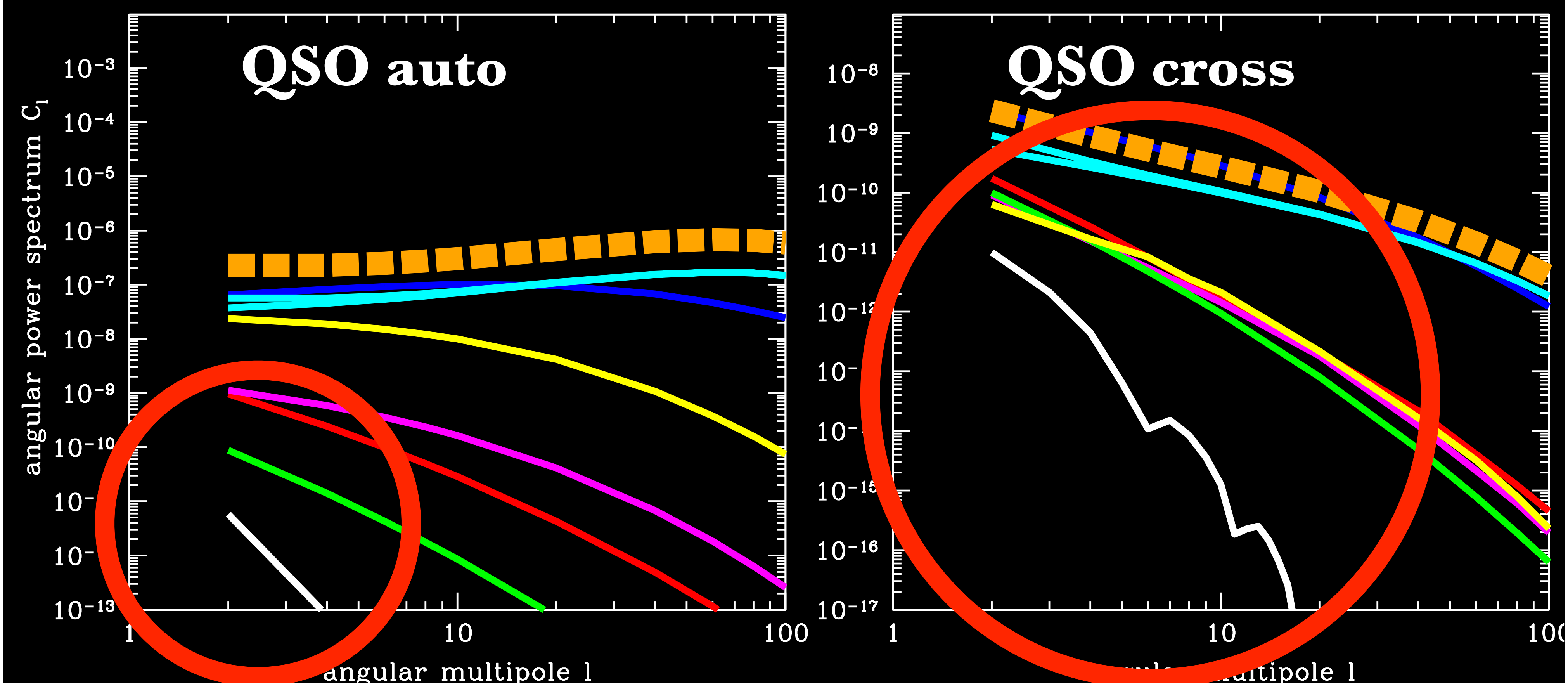
- ***anomalous large signal***



Ho, Seljak et al., PRD, 2008

Primordial Gravity Waves

- integrated Sachs-Wolfe effect *for* gravity waves
  - tensor-to-scalar ratio *r=0.1*



Galaxy Power Spectrum

- **matter fluctuation:** $\rho_m = \bar{\rho}_m(t)[1+\delta_m] = \bar{\rho}_m(z_{\rm obs})[1+m_\delta]$
  - **gauge-dependent** δ_m $\qquad 1+z_{\rm obs} = (1+z)(1+\delta z)$
  - **gauge-invariant, *observable*** $m_\delta = \delta_m - 3\,\delta z$
  - **Bardeen's gauge-invariant** $\epsilon_m,\ \epsilon_g$ $\quad (\epsilon_m \neq \epsilon_g \neq m_\delta)$

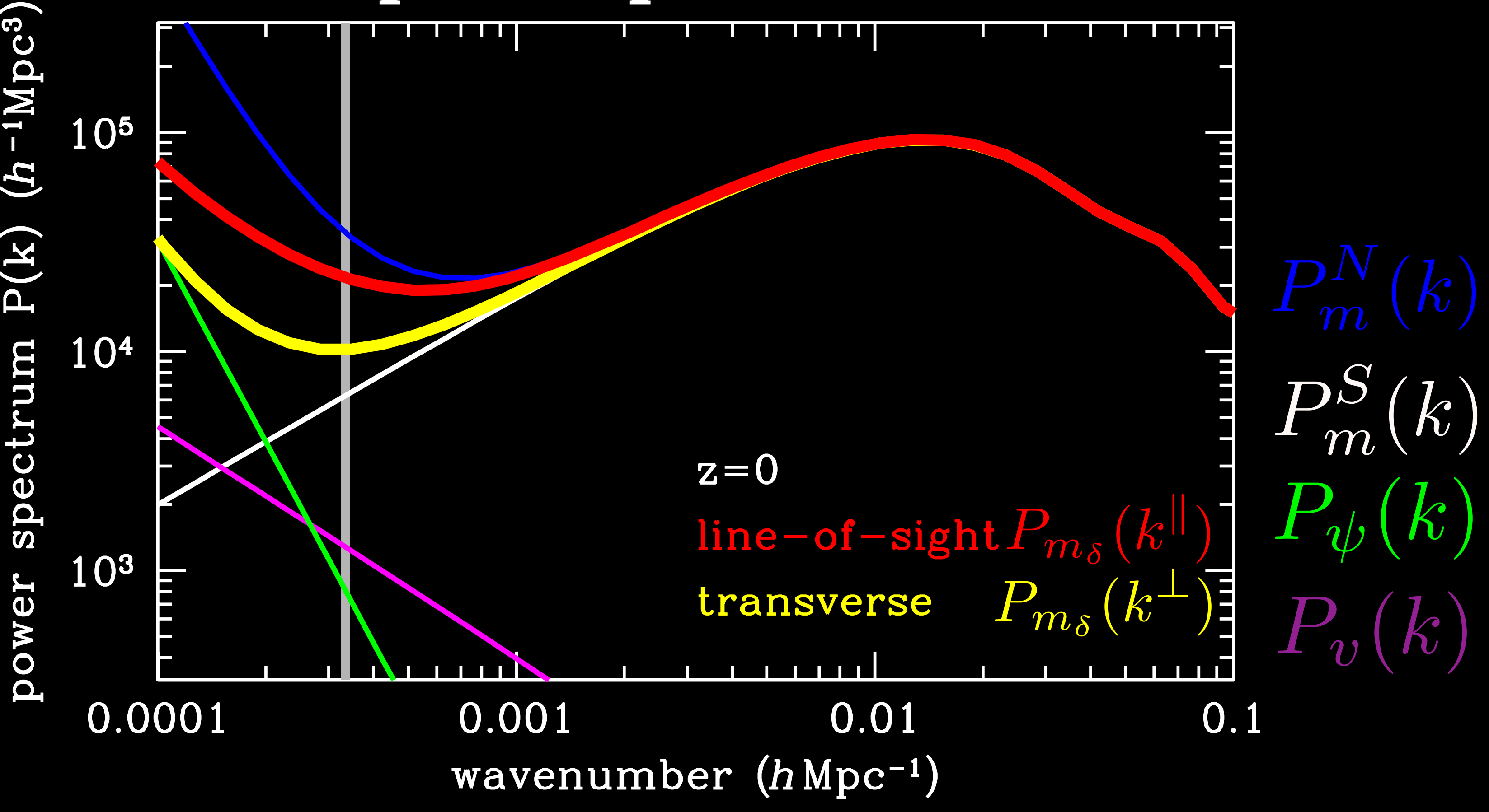
correct "real-space" matter fluctuation!

- **time slicing (hypersurface of simultaneity)**
  - **constant coordinate time for** δ_m**,** $\bar{\rho}_m(t)$
  - **observed redshift** $z_{\rm obs},\ \bar{\rho}_m(z_{\rm obs})$
  - **matter rest frame, zero-shear frame**

Yoo, PRD, 2010

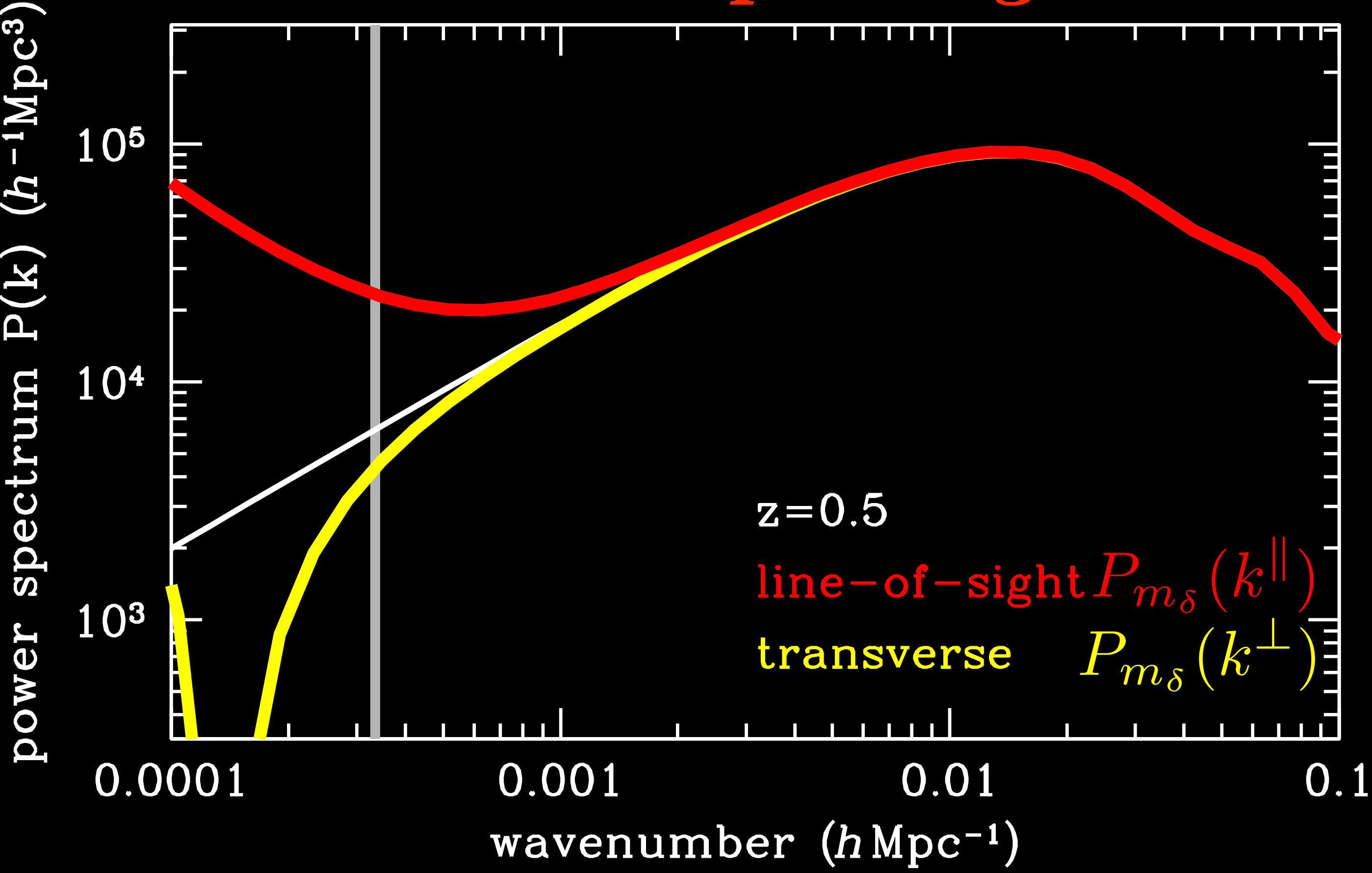
“Real-Space” Power Spectrum

- ***no longer isotropic***, **neither** $P_m^S(k)$ **, nor** $P_m^N(k)$
- **real-space matter power spectrum** $P_{m_\delta}(k, \mu_k)$



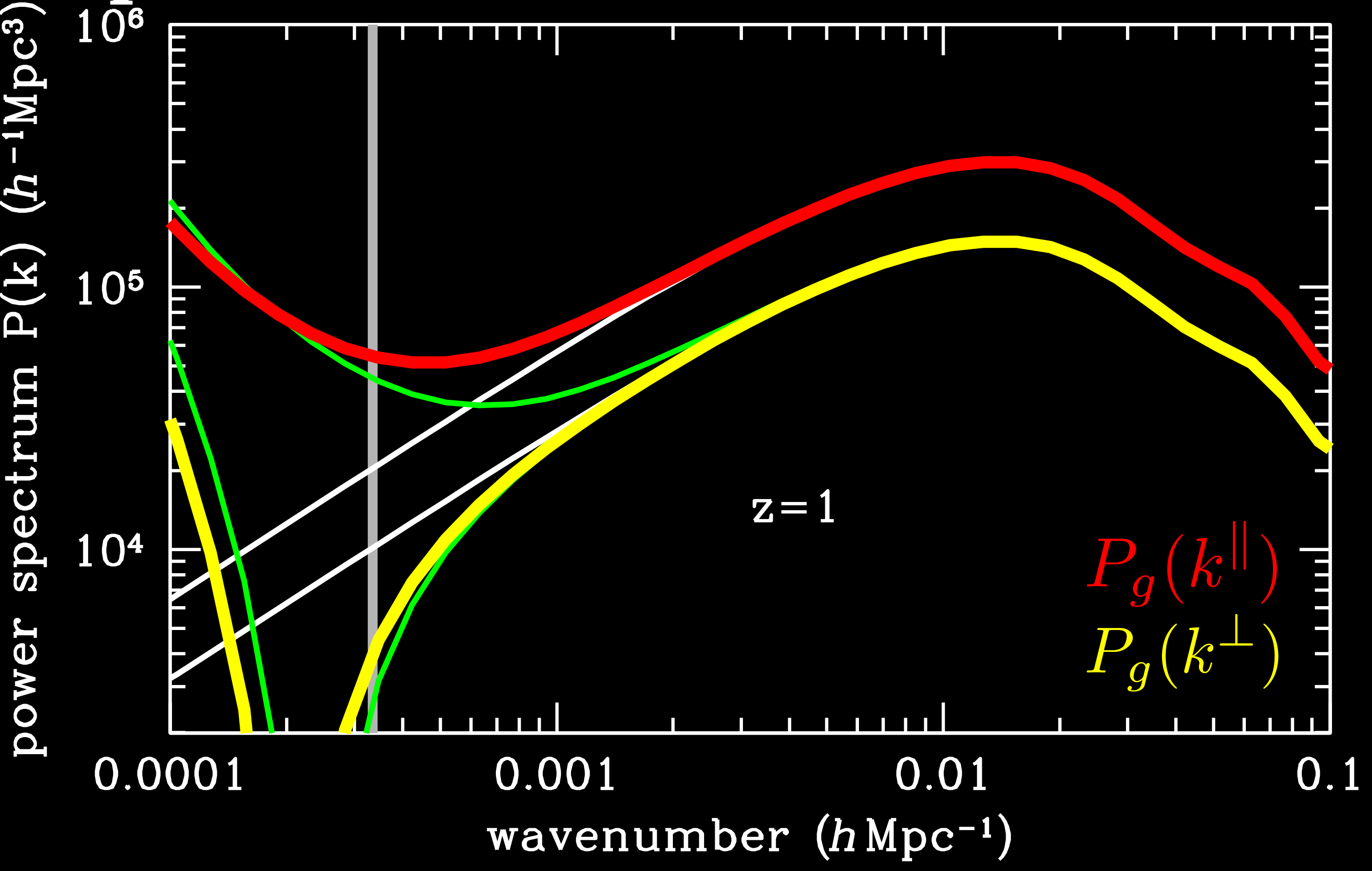
"Real-Space" Power Spectrum

- **appear as an underdense region on large scales**
- ***characteristic scale*** **and** ***shape change***



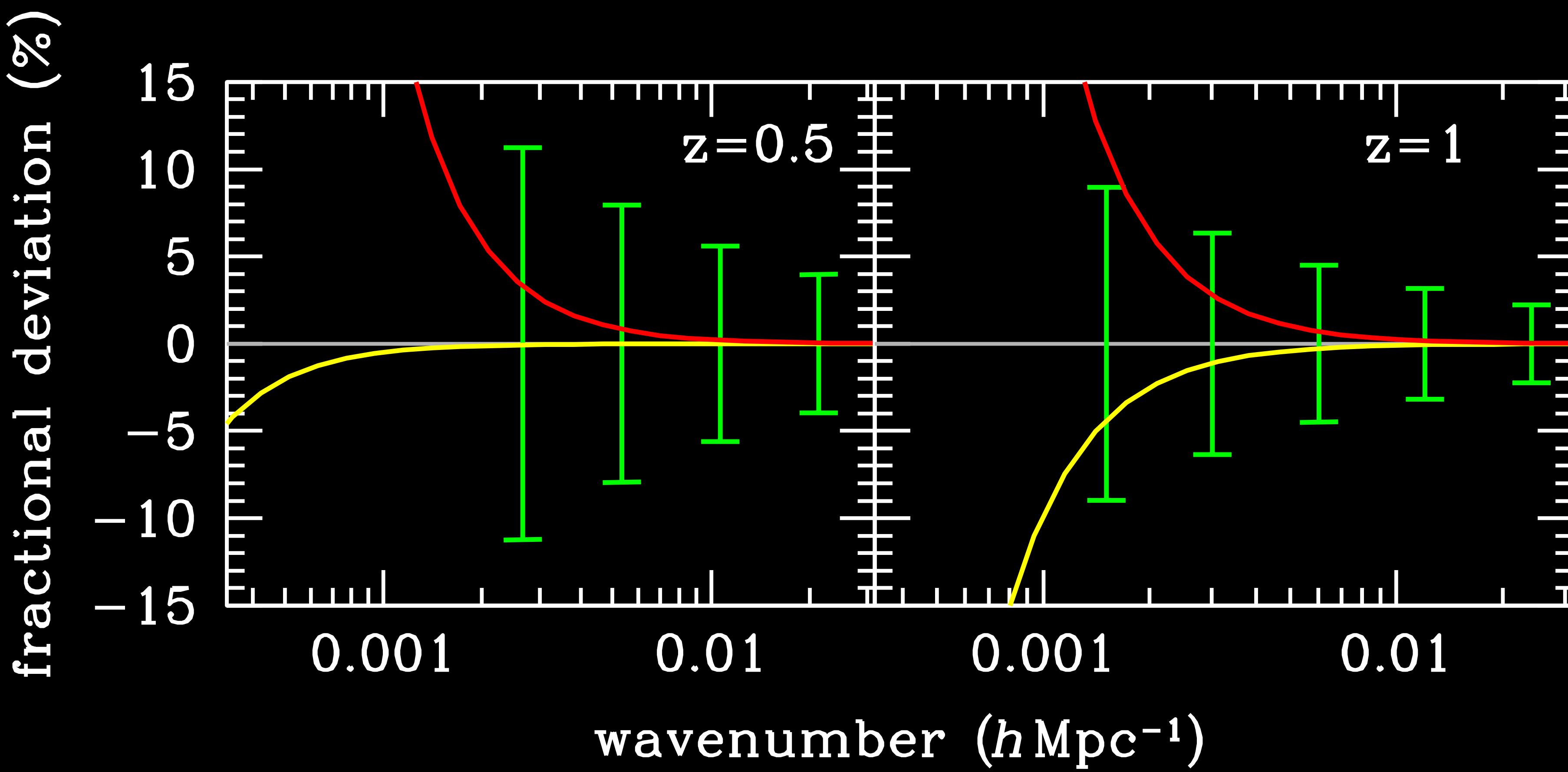
Full Galaxy Power Spectrum

- largely similar to real-space matter power spectrum
- redshift-space distortion boost



Systematic Errors

- **Baryonic Oscillation Spectroscopic Survey (BOSS)**



IV. SUMMARY AND PROSPECTS:

What do We Learn from This?

New Perspective on GC

- *new perspective* on
  galaxy clustering as a cosmological probe
- unified treatment:
  - redshift-space distortion, magnification bias, …
- subtle gauge issues: *observables*
  - redshift, magnification, galaxy bias
- general relativistic description:
  - gauge-invariant formalism
  - *tensor* as well as scalar contributions

Relativistic Effects

- **with optimal weighting and multi-tracer method**
  GR effect in galaxy clustering *reality & measurable*

- **future work:**
  - **constraints on *modified gravity* on horizon scales**
    complementary to the Solar system constraints
  - **constraints on *dark energy* models with superhorizon perturbations**

New Perspective on Cosmology

- galaxy clustering as a worked example

- *many* theoretical predictions in cosmology
    - gauge-dependent, unobservable
    - how to remedy other probes?
    - *follow observational procedure*

New Perspective on Galaxy Clustering and Cosmology:

General Relativistic Effects

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