Themes

	Theme
Monday	SLE/Math Physics
Tuesday	Complex/Real/Harmonic Analysis
Wednesday	Probability/Applied Math
Thursday	Harmonic Analysis/PDE
Friday	Complex/Real/Harmonic Analysis

Schedule

	Monday	Tuesday	Wednesday	Thursday	Friday
09:45 - 10:00	Opening Remarks				
10:00 - 10:45	Smirnov	Garnett	Burdzy	Katz	Bonk
11:00 - 11:45	Duplantier	Azzam	Gonzalez	Binder	Ward
12:00 - 12:45	Lunch		Maggioni	Lunch	
13:00 - 13:55	Discussion and Poster Session		Lunch	Discussion and Poster Session	
14:00 - 14:45	Makarov	Iosevich	Free Time	Steinerberger	Poltoratski
14:50 - 15:35	Rohde	Pereyra	Free Time	Logunov	Rogers
	Coffee Break and Discussion		Free Time	Coffee Break and Discussion	
16:05 - 16:50	Zhan	Benedicks	Free Time	Mayboroda	Müller
17:00 - 17:45	Saksman	Sodin	Free Time	Schlag	Tolsa
19:00 - 21:30		Banquet			

Monday: SLE/Math Physics

	Speaker	Title
09:45 - 10:00		Opening Remarks
10:00 - 10:45	Smirnov	Clusters, loops and trees
11:00 - 11:45	Duplantier	CLE Exteme Nesting and Liouville Quantum Gravity
		Lunch and Discussion
14:00 - 14:45	Makarov	Etudes for the inverse spectral problem
14:50 - 15:35	Rohde	Conformal laminations and trees
		Coffee Break and Discussion
16:05 - 16:50	Zhan	SLE loop measures
17:00 - 17:45	Saksman	On Gaussian multiplicative chaos and the Riemann zeta function

Clusters, loops and trees

Stanislav Smirnov

Université de Genève, St. Petersburg University

Abstract

We will report on our ongoing work with Antti Kemppainen aimed at describing geometrically the scaling limit of the critical Ising model.

CLE Exteme Nesting and Liouville Quantum Gravity

Bertrand Duplantier

Institute for Theoretical Physics, Paris-Saclay University

Abstract

We describe recent advances in the study of Schramm-Loewner Evolution (SLE), a canonical model of non-crossing random paths in the plane, and of Liouville Quantum Gravity (LQG), a canonical model of random surfaces in 2D quantum gravity. The latter is expected to be the universal, conformally invariant, continuum limit of random planar maps, as weighted by critical statistical models. SLE multifractal spectra have natural analogues on random planar maps and in LQG. An example is extreme nesting in the Conformal Loop Ensemble (CLE), as derived by Miller, Watson and Wilson, and extreme nesting in the $O(n)$ loop model on a random planar map, as derived recently via combinatorial methods. Their respective large deviations functions are shown to be conjugate of each other, via a continuous KPZ transform inherent to LQG.

Joint work with Gaëtan Borot and Jérémie Bouttier.

A Global Research Symposium
Geometry, Analysis and Probability

Etudes for the inverse spectral problem

Nikolai Makarov

Caltech

Conformal laminations and trees

Steffen Rohde

University of Washington

Abstract

Conformal maps f of the unit disc D have a continuous extension to the circle if (and only if) the boundary of the image $f(D)$ is locally connected. This extension induces an equivalence relation on the circle by declaring that $x \sim y$ if $f(x) = f(y)$. Which equivalence relations on the circle arise in this way? After a brief discussion of the history and motivation, I will present a characterization under the additional assumption that $f(D)$ is a John domain whose complement has empty interior.

SLE loop measures

Dapeng Zhan

Michigan State University

Abstract

We use Minkowski content (i.e., natural parametrization) of SLE to construct several types of SLE_κ loop measures for $\kappa \in (0, 8)$. First, we construct rooted SLE_κ loop measures in the Riemann sphere $\hat{\mathbb{C}}$, which satisfy Möbius covariance, conformal Markov property, reversibility, and space-time homogeneity, when the loop is parameterized by its $(1 + \frac{\kappa}{8})$-dimensional Minkowski content. Second, by integrating rooted SLE_κ loop measures, we construct the unrooted SLE_κ loop measure in $\hat{\mathbb{C}}$, which satisfies Möbius invariance and reversibility. Third, we extend the SLE_κ loop measures from $\hat{\mathbb{C}}$ to subdomains of $\hat{\mathbb{C}}$ and to two types of Riemann surfaces using Brownian loop measures, and obtain conformal invariance or covariance of these measures. Finally, using a similar approach, we construct SLE_κ bubble measures in simply/multiply connected domains rooted at a boundary point. The work answers a question raised by Greg Lawler.

On Gaussian multiplicative chaos and the Riemann zeta function

Eero Saksman

University of Helsinki

Abstract

We recall some basic properties of Gaussian multiplicative chaos and describe our recent results on its connection to the functional statistics of the Riemann zeta function on the critical line (and to that of random unitary matrices). The talk is based on joint work with Christian Webb (Aalto University).

Tuesday: Complex/Real/Harmonic Analysis

	Speaker	Title
10:00 - 10:45	Garnett	Many Theorems and Few Stories
11:00 - 11:45	Azzam	The Analyst's Traveling Salesman Theorem for large dimensional objects
		Lunch and Discussion
14:00 - 14:45	Iosevich	Group actions, Mattila integral and simplexes inside fractal sets
14:50 - 15:35	Pereyra	Weighted inequalities and dyadic harmonic analysis
		Coffee Break and Discussion
16:05 - 16:50	Benedicks	Almost sure continuity along curves traversing the Mandelbrot set
17:00 - 17:45	Sodin	Translation-invariant probability measures on entire functions

Many Theorems and Few Stories

John Garnett

UCLA

The Analyst's Traveling Salesman Theorem for large dimensional objects

Jonas Azzam

University of Edinburgh

Abstract

The classical Analyst's Traveling Salesman Theorem of Peter Jones gives a condition for when a subset of Euclidean space can be contained in a curve of finite length (or in other words, when a "traveling salesman" can visit potentially infinitely many cities in space in a finite time). The length of this curve is given by a sum of quantities called beta-numbers that measure how non-flat the set is at each scale and location. Conversely, given such a curve, the sum of its beta-numbers is controlled by the total length of the curve, giving us quantitative information about how non-flat the curve is. This result and its subsequent variants have had applications to various subjects like harmonic analysis, complex analysis, and harmonic measure. In this talk, we will introduce a version of this theorem that holds for higher dimensional objects other than curves. This is joint work with Raanan Schul.

Group actions, Mattila integral and simplexes inside fractal sets

Alexander Iosevich

University of Rochester

Abstract

We will describe how a simple group action viewpoint can be used to derive the classical Mattila integral and to study the distribution of simplexes inside compact subsets of Euclidean space of a given Hausdorff dimension. More general configuration are studied as well where the various notions of rigidity come into play along with Sard's theorem. A variety of interesting model multi-linear variants of generalized Radon transforms arise in this context and we shall discuss some steps towards the general theory of these objects.

Weighted inequalities and dyadic harmonic analysis

Maria Cristina Pereyra

University of New Mexico

Abstract

In this talk we survey the interplay between dyadic techniques and weighted inequalities. This interplay has led, among others, to the solution of the A2 conjecture by Hytönen and the two weight problem for the Hilbert transform by Lacey, Sawyer, Shen, and Uriarte-Tuero.

Almost sure continuity along curves traversing the Mandelbrot set

Michael Benedicks

KTH Royal Institute of Technology

Abstract

We study continuity properties of dynamical quantities while crossing the Mandelbrot set through typical smooth curves. In particular, we prove that for almost every parameter c_0 in the boundary of the Mandelbrot set M with respect of the harmonic measure and every smooth curve $\gamma : [-1, 1] \mapsto \mathbb{C}$ with the property that $c_0 = \gamma(0)$ there exists a set $\mathcal{A}_\gamma$ having 0 as a Lebesgue density point and such that that $\lim_{x \to 0} \mathrm{HDim}(J_{\gamma(x)}) = \mathrm{HDim}(J_{c_0})$ for the Julia sets J_c. This is joint work with Jacek Graczyk.

Translation-invariant probability measures on entire functions

Mikhail Sodin

Tel Aviv University

Abstract

I shall speak about a somewhat unexpected and wild object: the probability measures on the space of entire functions (of one complex variable) which are (a) invariant with respect to the action of the complex plane by translations, and (b) do not charge the constant functions. The existence (and even an abundance) of such measure was discovered by Benjy Weiss.

The talk will be based on the joint work with Lev Buhovsky, Adi Glucksam, Alexander Logunov, arXiv:1703.08101.

Wednesday: Probability/Applied Math

	Speaker	Title
10:00 - 10:45	Burdzy	On the number of collisions of billiard balls
11:00 - 11:45	Gonzalez	A limit theorem for random games
12:00 - 12:45	Maggioni	Multiscale Geometric Methods for high dimensional data near low-dimensional sets
		Lunch

On the number of collisions of billiard balls

Krzysztof Burdzy

University of Washington

Abstract

In lieu of an abstract I offer a problem. Consider three billiard balls of the same radius and mass, undergoing totally elastic reflections on a billiard table with no walls (the whole plane). All three balls can be given non-zero initial velocities. What is the maximum (supremum) possible number of collisions among the three balls? The supremum is taken over all initial positions and initial velocities. I will discuss this problem and its generalization to any finite family of balls in one, two and higher dimensions. Joint work with Mauricio Duarte.

A limit theorem for random games

Maria Jose Gonzalez

University of Cádiz

Abstract

We will apply techniques which arise in the context of Fourier Analysis on Boolean functions to study the behavior of the iterates of a special kind of functions, called selectors, acting on random variables. Our main motivation comes from game theory, however selectors appear in many different contexts, such as elections or noisy computations. Joint work with F. Durango, J.L. Fernandez and P. Fernandez.

Multiscale Geometric Methods for high dimensional data near low-dimensional sets

Mauro Maggioni

Johns Hopkins University

Abstract

We discuss a family of ideas, algorithms, and results for analyzing various new and classical problems in the analysis of high-dimensional data sets. These methods we discuss perform well when data is sampled from a probability measure in high-dimensions that is concentrated near a low-dimensional set. They rely on the idea of performing suitable multiscale geometric decompositions of the data, and exploiting such decompositions to perform a variety of tasks in signal processing and statistical learning. In particular we will discuss the problem of dictionary learning, of regression, of learning the probability measure generating the data, and efficiently computing optimal transportation plans between probability measures. These are joint works with W. Liao, S. Vigogna and S. Gerber.

Thursday: Harmonic Analysis/PDE

	Speaker	Title
10:00 - 10:45	Katz	Improved Hausdorff dimension bounds for Kakeya sets in $\mathbb{R}^3$
11:00 - 11:45	Binder	Uniqueness and almost periodicity in time of solutions of the KdV equation with certain almost periodic initial conditions
		Lunch and Discussion
14:00 - 14:45	Steinerberger	Brownian motion and PDEs
14:50 - 15:35	Logunov	0.01% improvement of the Liouville property for discrete harmonic functions on $\mathbb{Z}^2$
		Coffee Break and Discussion
16:05 - 16:50	Mayboroda	The hidden landscape of localization of eigenfunctions
17:00 - 17:45	Schlag	Long term dynamics of nonlinear wave equations

Improved Hausdorff dimension bounds for Kakeya sets in $\mathbb{R}^3$

Nets Hawk Katz

Caltech

Abstract

We show that for a uniform $\epsilon > 0$, Kakeya sets have Hausdorff dimension $5/2 + \epsilon$. This is joint work with J. Zahl.

Uniqueness and almost periodicity in time of solutions of the KdV equation with certain almost periodic initial conditions

Ilia Binder

University of Toronto

Abstract

In 2008, P. Deift conjectured that the solution of KdV equation with almost periodic initial data is almost periodic in time. I will discuss the proof of this conjecture (as well as the uniqueness) in the case of the so-called Sodin-Yuditskii type initial data, i.e. the initial data for which the associated Schrödinger operator has purely absolutely continuous spectrum which satisfies some regularity conditions. In particular, it applies to small analytic quasiperiodic initial data with Diophantine frequency vector. This is a joint work with D. Damanik (Rice), M. Goldstein (Toronto) and M. Lukic (Rice).

Brownian motion and PDEs

Stefan Steinerberger

Yale University

Abstract

Classical Brownian motion sheds an interesting light on the theory of elliptic partial differential equations – I will discuss various recent new results that strenghten/simplify/improve some classical results and point towards new problems. The main point of the talk is that all the Brownian motion arguments are quite simple and flow naturally (Brownian motion helps you drift in the right direction).

0.01% improvement of the Liouville property for discrete harmonic functions on $\mathbb{Z}^2$

Aleksandr Logunov

Tel Aviv University, Chebyshev Laboratory

Abstract

Let u be a harmonic function on the plane. The Liouville theorem claims that if $|u|$ is bounded on the whole plane, then u is identically constant. It appears that if u is a discrete harmonic function on the lattice $\mathbb{Z}^2$, and $|u| < 1$ on 99.99% of $\mathbb{Z}^2$, then u is a constant function.

Based on a joint work (in progress) with L. Buhovsky, Eu. Malinnikova, and M. Sodin.

The hidden landscape of localization of eigenfunctions

Svitlana Mayboroda

University of Minnesota

Abstract

Numerous manifestations of wave localization permeate acoustics, quantum physics, mechanical and energy engineering. It was used in construction of noise abatement walls, LEDs, optical devices, to mention just a few applications. Yet, no systematic methods could predict the exact spatial location and frequencies of the localized waves.

In this talk I will present recent results revealing a new criterion of localization, tuned to the aforementioned questions, and will illustrate our findings in the context of the boundary problems for the Laplacian and bilaplacian, $divA\nabla$, and (continuous) Anderson and Anderson-Bernoulli models on a bounded domain. Via a new notion of "landscape" we connect localization to a certain multi-phase free boundary problem and identify location, shapes, and energies of localized eigenmodes. The landscape further provides estimates on the rate of decay of eigenfunctions and delivers accurate bounds for the corresponding eigenvalues, in the range where both classical Agmon estimates and Weyl law may fail.

Long term dynamics of nonlinear wave equations

Wilhelm Schlag

University of Chicago

Abstract

We will review some of the work over the past decade aiming at a complete description of the asymptotic structure of solutions with finite energy to certain semilinear wave equations. A goal of this talk is to demonstrate the interplay between dynamical system ideas and methods (invariant manifolds in infinite dimensions), calculus of variations, and the solution theory of dispersive equations based on classical harmonic analysis (restriction theory of the Fourier transform and Strichartz estimates). We will also present new work on a damped wave equation, but with damping that vanishes asymptotically. The latter is joint work with Nicolas Burq and Geneviève Raugel at Orsay, France.

Friday: Complex/Real/Harmonic Analysis

	Speaker	Title
10:00 - 10:45	Bonk	Quasisymmetric rigidity for Sierpinki carpets
11:00 - 11:45	Ward	Product Hardy spaces associated to operators with heat kernel bounds on spaces of homogeneous type
		Lunch and Discussion
14:00 - 14:45	Poltoratski	Spectral gaps and the size of uncertainty
14:50 - 15:35	Rogers	Settings in which many Sobolev spaces are not algebras
		Coffee Break and Discussion
16:05 - 16:50	Müller	Davis and Garsia inequalities for Hardy martingales in Banach spaces
17:00 - 17:45	Tolsa	Harmonic measure, Riesz transforms, and uniform rectifiability

Quasisymmetric rigidity for Sierpinki carpets

Mario Bonk

UCLA

Abstract

Sierpinski carpets exhibit surprising rigidity under quasisymmetric maps. This phenomenon appears in various contexts in geometric group theory and complex dynamics. In my talk I will give a survey of some recent results in this area.

Product Hardy spaces associated to operators with heat kernel bounds on spaces of homogeneous type

Lesley Ward

University of South Australia

Abstract

Much effort has been devoted to generalizing the Calderón–Zygmund theory from Euclidean spaces to metric measure spaces, or spaces of homogeneous type. Here the underlying space $\mathbb{R}^n$ with Euclidean metric and Lebesgue measure is replaced by a set X with a general metric or quasi-metric and a doubling measure. Further, one can replace the Laplacian operator that underpins the Calderón–Zygmund theory by more general operators L satisfying heat kernel estimates.

I will present recent joint work with P. Chen, X.T. Duong, J. Li and L.X. Yan along these lines. We develop the theory of product Hardy spaces $H^p_{L_1,L_2}(X_1 \times X_2)$, for $1 \leq p < \infty$, defined on products of spaces of homogeneous type, and associated to operators L_1, L_2 satisfying Davies–Gaffney estimates. This theory includes definitions of Hardy spaces via appropriate square functions, an atomic Hardy space, a Calderón–Zygmund decomposition, interpolation theorems, and the boundedness of a class of Marcinkiewicz-type spectral multiplier operators.

Spectral gaps and the size of uncertainty

Alexei Poltoratski

Texas A&M University

Abstract

I will talk about a recent solution to the so-called gap problem in Fourier analysis and its applications in complex analysis and spectral theory for differential operators.

Settings in which many Sobolev spaces are not algebras

Luke Rogers

University of Connecticut

Abstract

Let (X, μ) be a measure space on which there is a non-negative definite self-adjoint Markovian operator $\mathcal{L}$ with dense domain in $L^2(\mu)$. The cannonical example is the (non-negative) Laplacian on Euclidean space, and in other contexts it is known that natural examples of such operators $\mathcal{L}$ can be used in place of the Laplacian to define differential equations for physical phenomena such as wave and heat propagation on the space X. The corresponding Sobolev spaces are $W^{\alpha,p}_{\mathcal{L}} = \{f \in L^p : \mathcal{L}^{\alpha/2} \in L^p\}$.

In the Euclidean setting with $\mathcal{L} = -\Delta$ it is well-known that the space $W^{\alpha,p}_{-\Delta} \cap L^\infty$ is an algebra when $p \in (1, \infty)$ and $\alpha > 0$. There are extensions of this result to Lie groups with polynomial volume growth and manifolds with positive injectivity radius and non-negative Ricci curvature. In joint work with Thierry Coulhon we give bounds on the range of p and α for which such results can be true in a general space (X, μ) by exhibiting a class of fractal spaces and operators for which the algebra property fails.

Davis and Garsia inequalities for Hardy martingales in Banach spaces

Paul F. X. Müller

Johannes Kepler Universität Linz

Abstract

We present Davis decompositions for Hardy martingales with values in Banach spaces and use them to derive Davis and Garsia inequalities for vector valued Hardy martingales. We relate the underlying isomorphic invariants to the concept of H^1 uniform convexity.

Harmonic measure, Riesz transforms, and uniform rectifiability

Xavier Tolsa

ICREA - Universitat Autònoma de Barcelona

Abstract

In this talk I will review some recent results on harmonic measure where the connection between Riesz transforms and rectifiability plays an essential role. In particular, I will recall a converse of the Riesz brothers theorem and the solution of the so-called two phase problem. I will also explain a more recent result on a characterization of uniform rectifiability in terms of Carleson estimates for bounded harmonic functions which is based on a corona decomposition involving harmonic measure. Some of these results are collaborations with J. Azzam, J. Garnett and M. Mourgoglou.