Shot Noise and Orbital Entanglement in Mesoscopic Structures

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Recent developments

Orbital entanglement

Samuelsson, Sukhorukov, Buttiker, PRL 91, 157002 (2003)

Zero-frequency measurement (Bell Inequality)

X. Maitre, W. D. Oliver, Y. Yamamoto, Physica E6, 301 (2000)
N.M. Chtchelkatchev et al., Phys. Rev. B 66, 161320 (2002)
P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 91, 157002 (2003)

Normal components

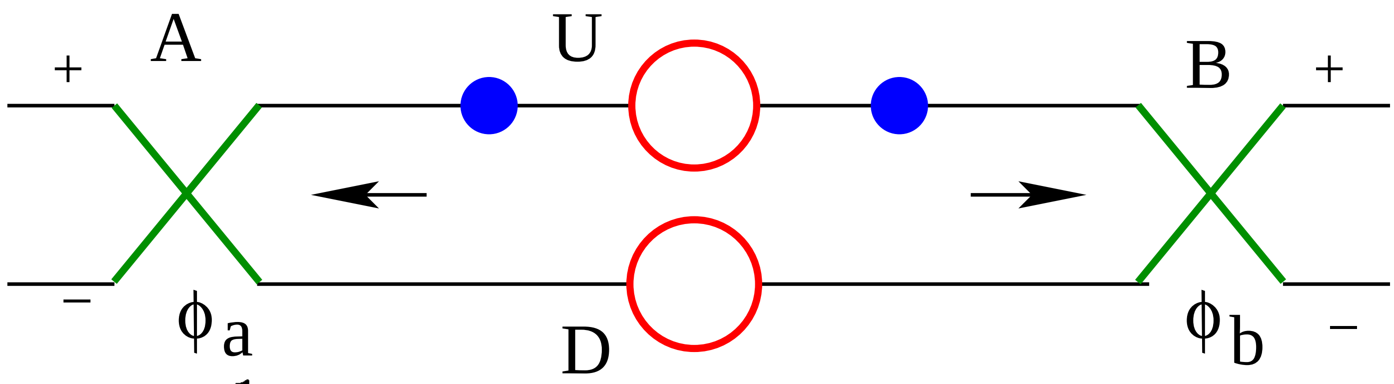
C.W.J. Beenakker et al, PRL 91, 147901 (2003);

P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 92, 026805 (2004).

Controllable geometries

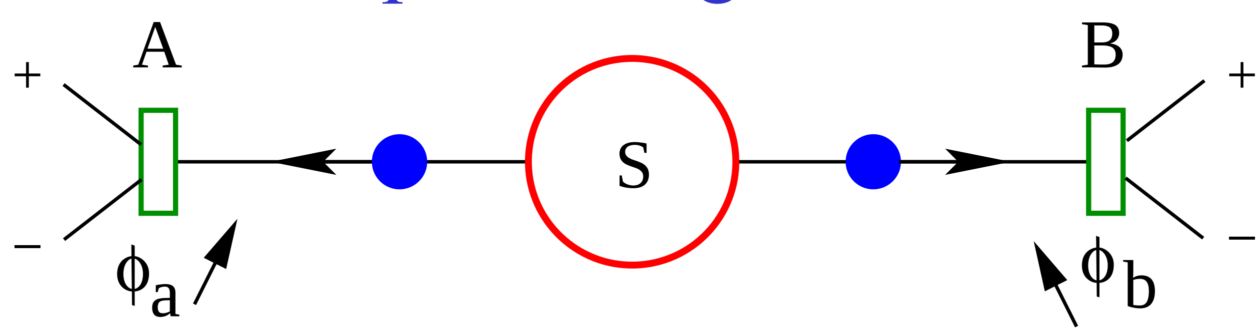
P. Samuelsson, E.V. Sukhorukov and M. Buttiker, PRL 92, 026805 (2004)

Orbital entanglement



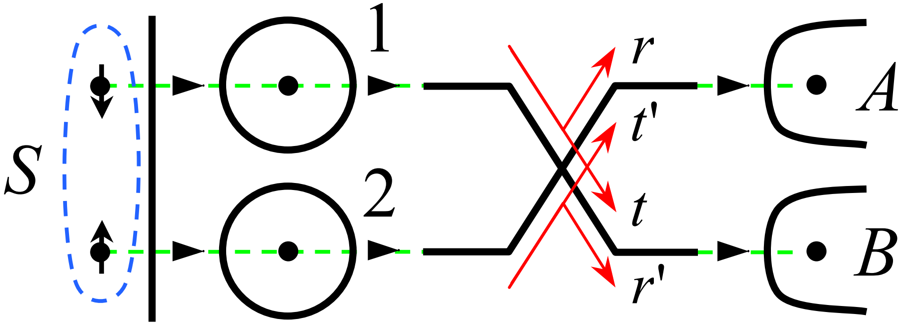
$$\Psi = \frac{1}{\sqrt{2}} [\Psi_U(A)\Psi_U(B) + \Psi_D(A)\Psi_D(B)]$$

Spin entanglement

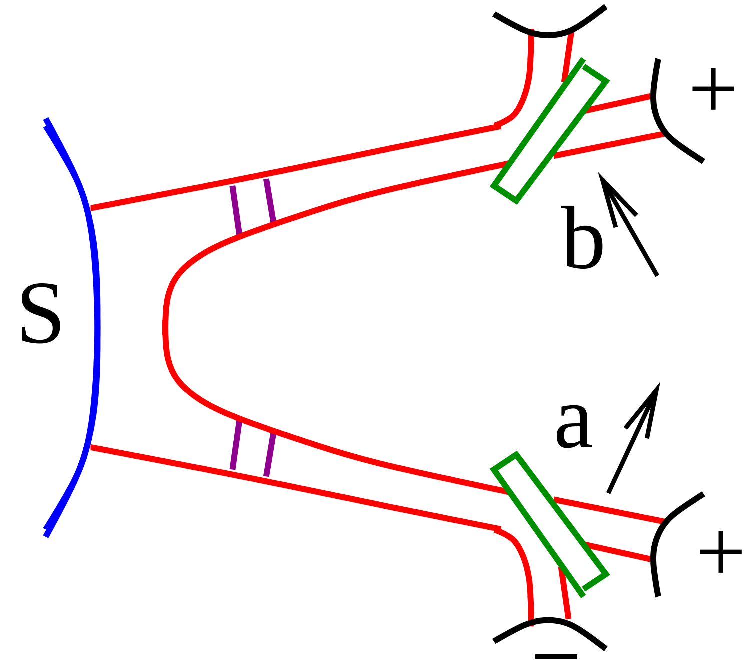


$$\Psi = \frac{1}{\sqrt{2}} \left[\Psi_\uparrow(A)\Psi_\downarrow(B) - \Psi_\downarrow(A)\Psi_\uparrow(B)\right]$$

Spin entanglement proposals



Recher, Sukhorukov, Loss,
PRB 63, 165314 (2001);
Burkard, Sukhorukov, Loss,
PRB 61, 16303 (2000).



Lesovik, Martin, Blatter,
EPJP 24, 287 (2001);
Chtchekaltchev et al,
PRB 66, 161320 (2002).

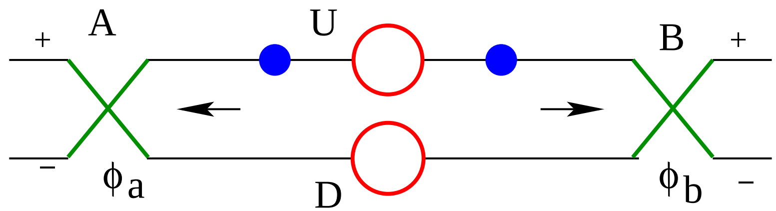
Combined system:

Samulesson, Sukhorukov, Buttiker,
PRB 73, 115330 (2004).

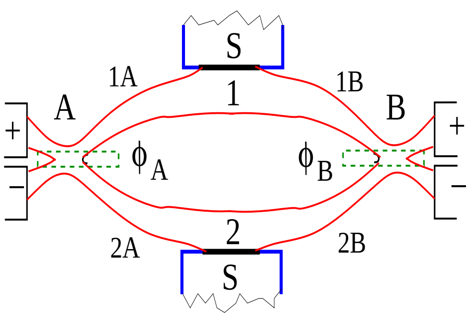
Advantages: long spin coherence times

Disadvantages: read-out, single spin manipulation

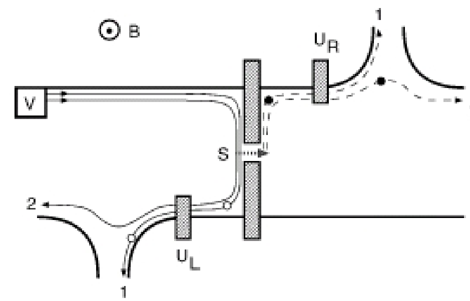
Orbital entanglers

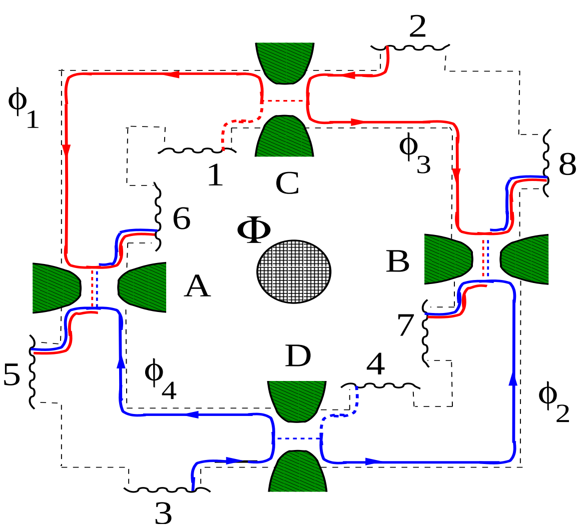


Two particle injection from two contacts

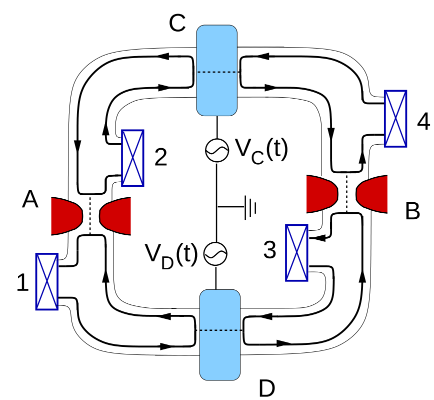


Electron-hole injection from a barrier





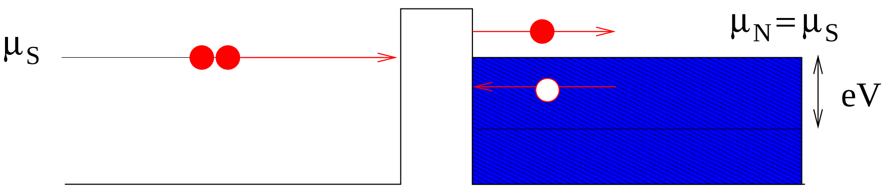
Dynamic generation of orbtial entanglement



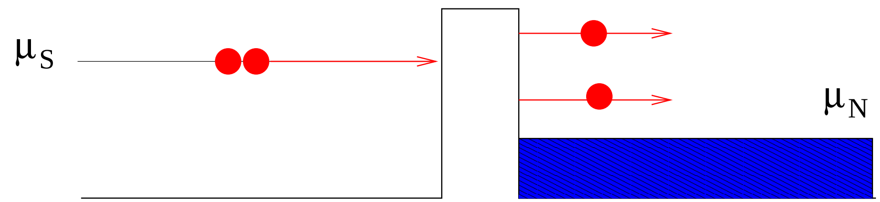
Source of orbital two-particle entanglement

Superconducting-normal hybrid structures

Bogoliubov-de Gennes picture



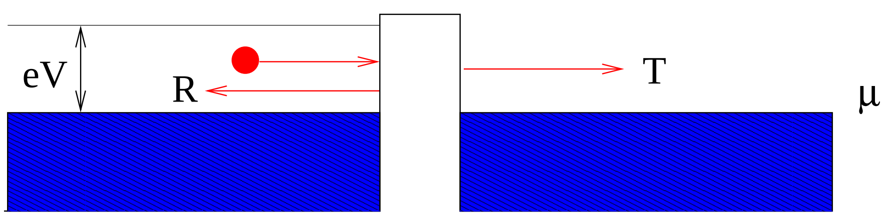
Pair-tunneling picture



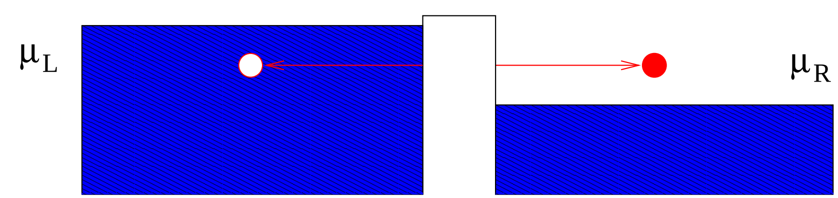
Samuelsson, Sukhorukov and Buttiker,
PRL 91, 157002 (2003)

Normal-conductors

Electron picture of tunneling



Electron-hole picture



Beenakker et al ,
PRL 91, 147901 (2003)

Shot noise

Classical shot noise:

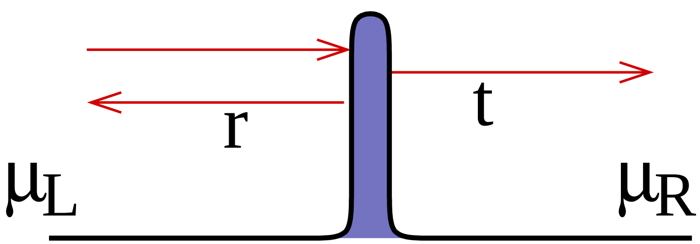
W. Schottky, Ann. Phys. (Leipzig) 57, 541 (1918)

$$\langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle$$

Quantum Shot Noise:

Khlus (1987), Lesovik (1989), Yurke and Kochanski (1989), Buttiker (1990), Beenakker (1991)

$$|\Psi>_{inc} = e^{ikx}$$
$$|\Psi>_{ref} = r e^{-ikx}$$
$$|\Psi>_{tra} = t\, e^{ikx}$$



$\Rightarrow$

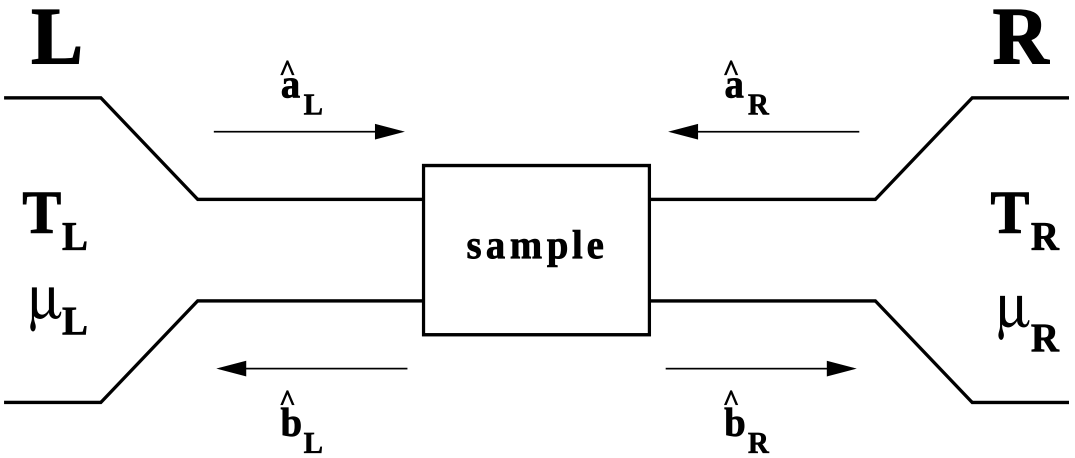
$$\langle (\Delta n_T)^2 \rangle = T(1-T)$$

$$\langle (\Delta I)^2 \rangle_\nu = 2e\langle I \rangle (1-T)$$

Scattering Theory

Central object: scattering matrix

$$s = \begin{pmatrix} r & t \\ t & r \end{pmatrix}$$



Eigen channels: $t^\dagger t \quad \Rightarrow \quad T_n$

Conductance

$$G = \frac{e^2}{h} Tr(t^\dagger t) = \frac{e^2}{h} \sum_n T_n$$

Shot noise

$$S = 2e\frac{e^2}{h}|eV|Tr(r^\dagger r t^\dagger t) = 2e\frac{e^2}{h}|eV|\sum_n T_n(1-T_n)$$

Buttiker, 1990

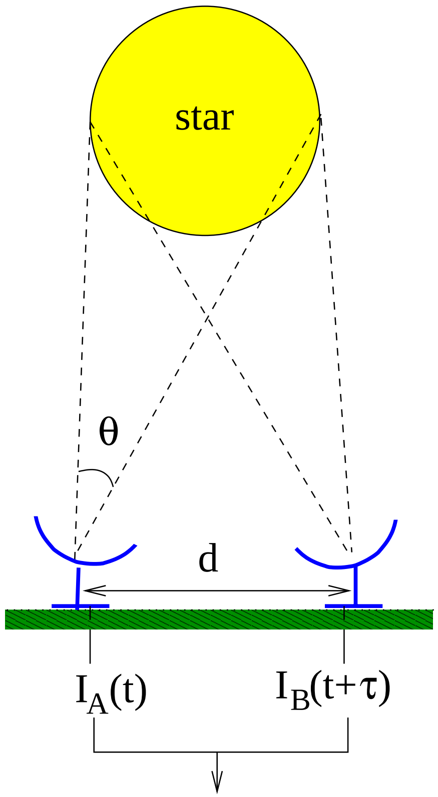
HBT-Intensity Interferometer

Hanbury Brown and Twiss, Nature 177, 27 (1956)

Interference not of amplitudes but of intensities
Optics: classical interpretation possible
Quantum mechanical explanation:
Purcell, Nature 178, 1449 (1956)
Indistinguishable particles:

Statistics, exchange amplitudes

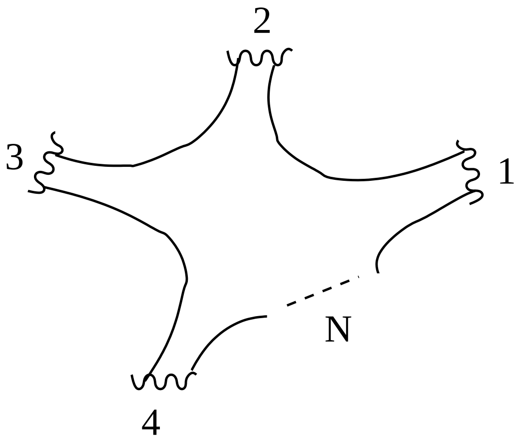
$$\int d\tau \langle \Delta I_A(t) \Delta I_B(t+\tau) \rangle = f\left(\frac{d\theta}{\lambda}\right)$$



Scattering theory of mesoscopic transport

Buttiker, PRL 65, 2901 (1990)

Mesoscopic conductor with N contacts



$$S = \begin{pmatrix} s_{11} & s_{12} & s_{13} & \cdots & s_{1N} \\ s_{21} & s_{22} & s_{23} & \cdots & \\ s_{31} & s_{32} & s_{33} & & \\ \vdots & \vdots & & \ddots & \\ s_{N1} & & & & s_{NN} \end{pmatrix}$$

At kT = 0,

$$G_{\alpha\beta} = 2\frac{e^2}{h}\mathrm{Tr}\left[s^{\dagger}_{\alpha\beta}s_{\beta\alpha}\right]$$

$$S_{\alpha\beta} = 2\int dt \langle \Delta\hat{I}_{\alpha}(t)\Delta\hat{I}_{\beta}(0)\rangle$$

At kT = 0, M contacts with $f_{\gamma} = f$, N-M contacts at $f_{\delta} = f_0$

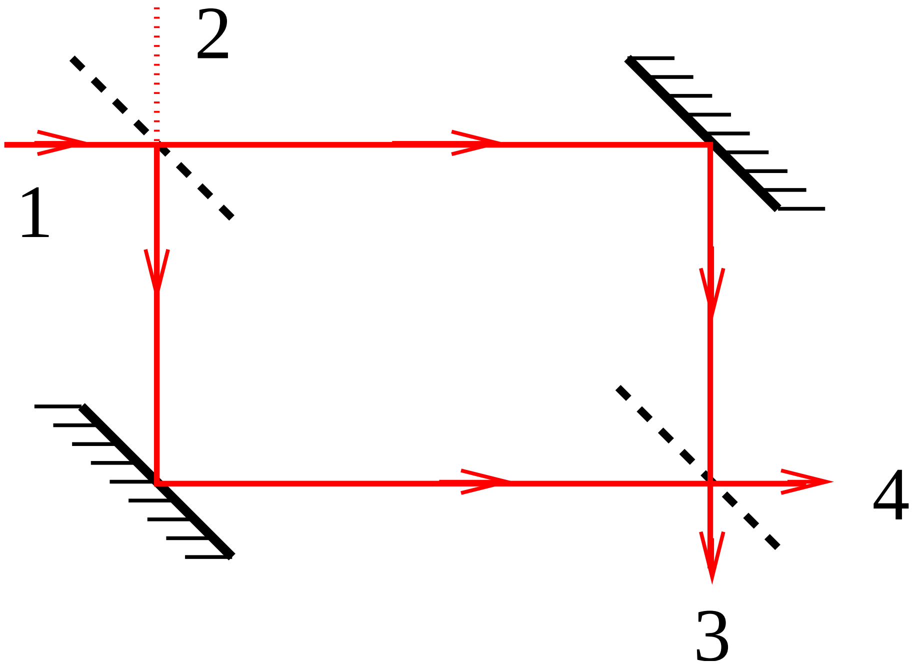
$$S_{\alpha\beta} = 2\frac{e^2}{h}\int dE\,\mathrm{Tr}\left[B^{\dagger}_{\alpha\beta}B_{\beta\alpha}\right], \qquad B_{\alpha\beta} = \sum_{\gamma=1}^{M} s_{\alpha\gamma}s^{\dagger}_{\beta\gamma}(f_{\gamma} - f_0)$$

M=1, partition noise

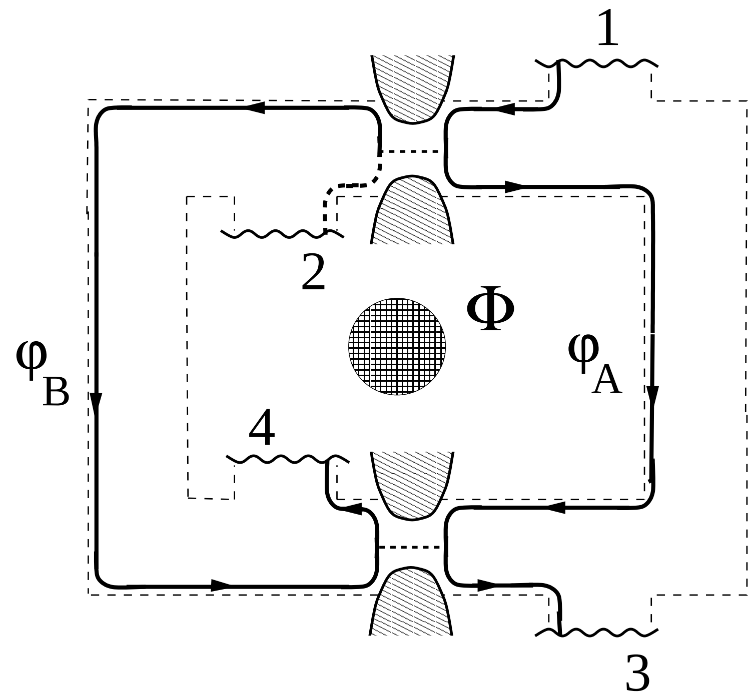
M > 1, relative phase of scattering matrix elements becomes important

Exchange interference effects: Buttiker, PRL 68, 843 (1992)

Optical and Electrical Mach-Zehnder-Interferometer



One particle Aharonov-Bohm effect



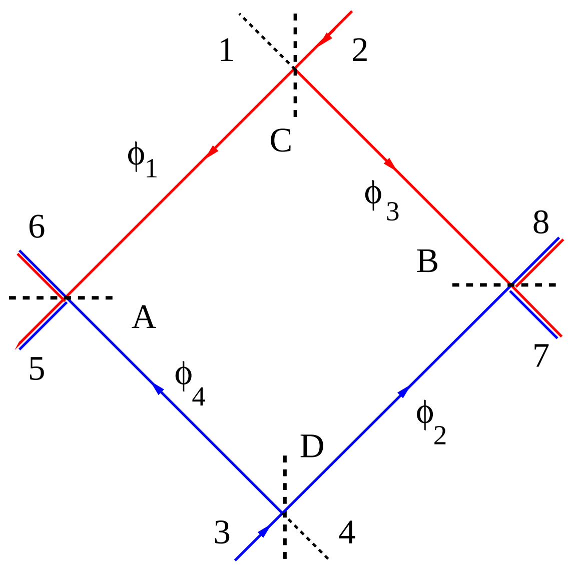
Ji et al, Nature 422, 415 (2003)

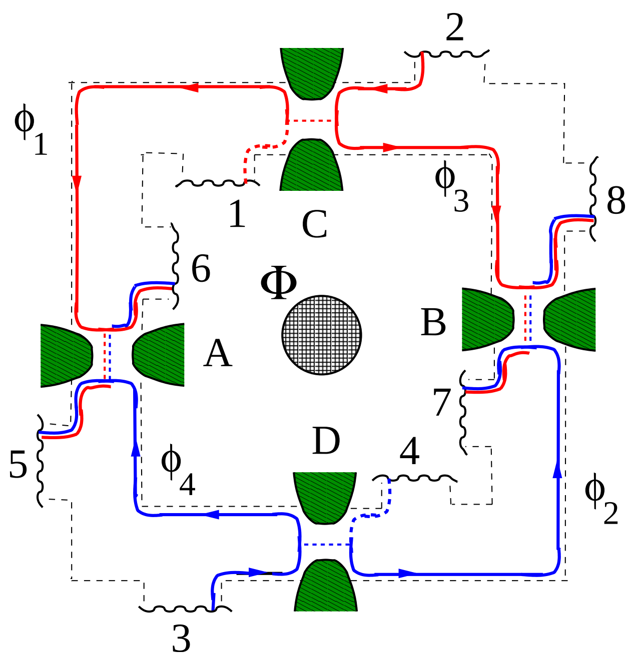
$$s_{31} = \frac{1}{2}\left[e^{i(\phi_A - \chi_1)} + e^{i(\phi_B - \chi_2)}\right] \qquad \chi_2 - \chi_1 = 2\pi\Phi/\Phi_0$$

$$G_{31} = \frac{e^2}{2h}\left[1 + \cos(\phi_A - \phi_B - 2\pi\Phi/\Phi_0)\right]$$

Electrical HBT Interferometer

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)





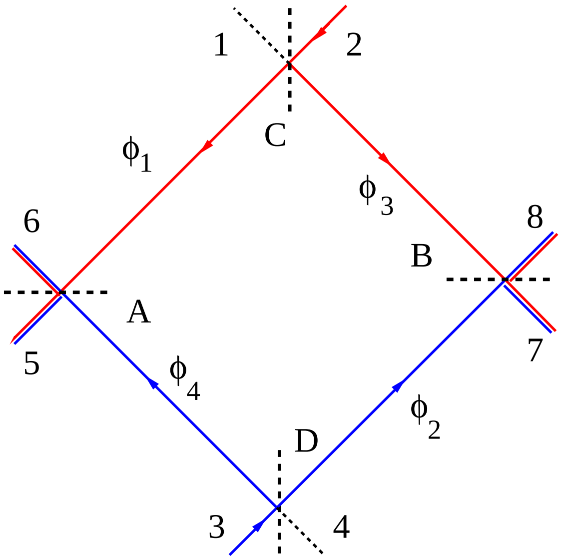
$$s_{52} = T_A^{1/2} e^{i(\phi_1 + \chi_1)} T_C^{1/2}$$

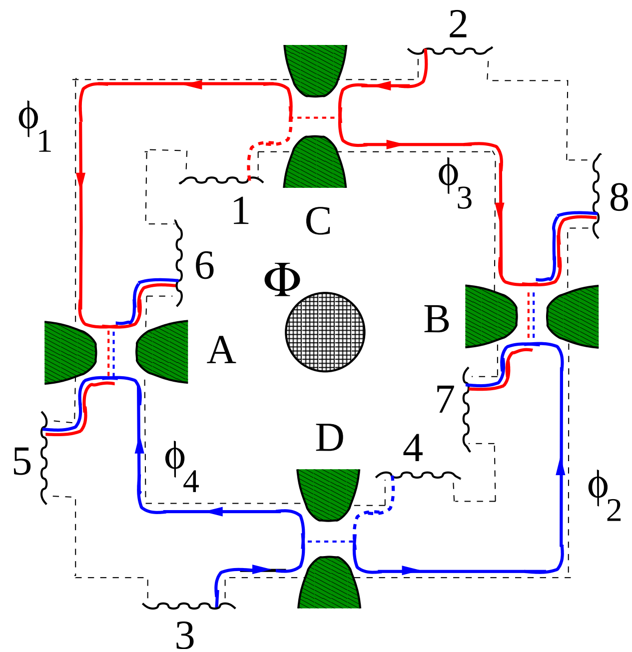
$$G_{52} = -\frac{e^2}{h} T_A T_C$$

All elements of the conductance matrix are independent of AB-flux

Two-particle Aharonov-Bohm Effect

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)





Fourth-order interference: Current-current correlation

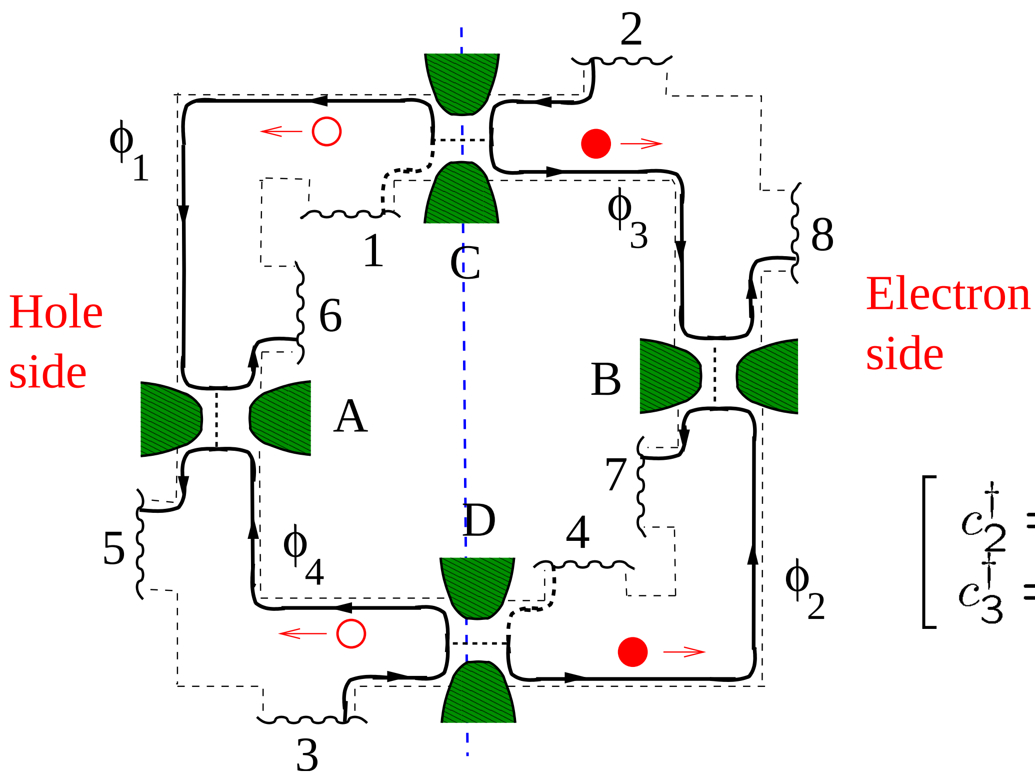
$$S_{58} = -2\frac{e^2}{h}\int dE|s^*_{52}s_{82} + s^*_{53}s_{83}|^2(f - f_0)^2$$

For $T_A = T_B = T_C = T_D = 1/2$;

$$S_{58} = -\frac{e^2}{4h}|eV|\left[1 + \cos\left(\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\frac{\Phi}{\Phi_0}\right)\right]$$

Two-particle entanglement

Samuelsson, Sukhorukov, Buttiker, PRL 92, 026805 (2004)



$$\begin{bmatrix} c_2^\dagger = t_C c_{2A}^\dagger + r_C c_{2B}^\dagger \\ c_3^\dagger = r_D c_{3A}^\dagger + t_D c_{3B}^\dagger \end{bmatrix}$$

$R_C = T_D = R \ll 1 \; ; \tau_C = \hbar/eV \; ; \tau \sim \hbar/eVR$, tunneling limit

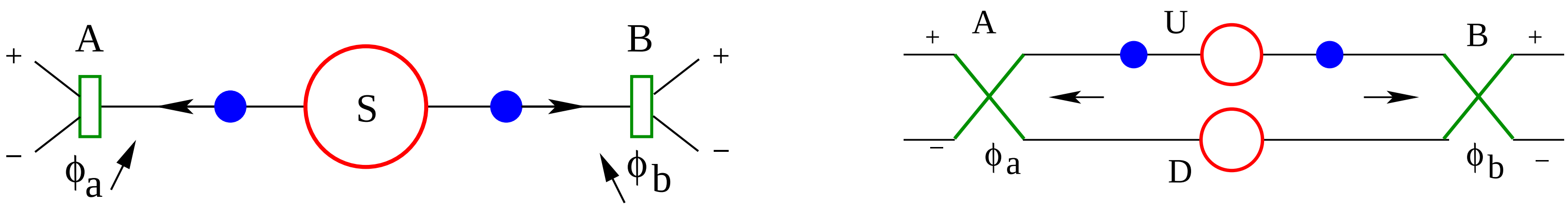
$|\Psi_{in}\rangle = \prod_{0<E<eV} c_2^\dagger(E) c_3^\dagger(E)|0\rangle$ incident state

$|\Psi\rangle = |\bar{0}\rangle + \sqrt{R}\int_0^{eV} dE \left[c_{3B}^\dagger c_{3A} + c_{2B}^\dagger c_{2A}\right]|\bar{0}\rangle + O(R)$ orbitally entangled e-h-state

Bell Inequality

Comparison of a classical local theory with quantum mechanical predictions

Here: entanglement test



Bell Inequality: Clauser et al. , PRL 23 , 880 (1969)

$$S_B = |E(\theta_A,\theta_B) - E(\theta'_A,\theta_B) + E(\theta_A,\theta'_B) + E(\theta'_A,\theta'_B)| \leq 2$$

$$E(\theta_A,\theta_B) = P_{++} + P_{--} - P_{+-} - P_{-+} = \frac{\langle (I_{A+} - I_{A-})(I_{B+} - I_{B-}) \rangle}{\langle (I_{A+} + I_{A-})(I_{B+} + I_{B-}) \rangle}$$

$$P_{\alpha\beta}(\theta_A,\theta_B) = (1 + \alpha\beta\cos[2(\theta_A - \theta_B)])/4$$

$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t+\tau) b_\alpha(t+\tau) b_\beta(t) \rangle \qquad (\tau\Delta\omega \leq 1)$$

spin

$\alpha = \uparrow, \downarrow,$ $\qquad$ θ_A , θ_B $\qquad$ angles of spin filters, polarizers

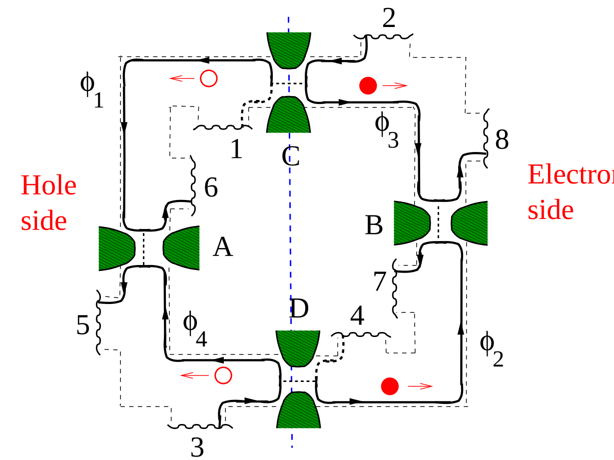
orbital

$\alpha = U, D,$ $\qquad$ θ_A , θ_B $\qquad$ rotation angles: splitter

Entanglement test: Bell Inequality

With $\phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0 = 2\pi$

$$S_{A/B} = \begin{pmatrix} \cos\theta_{A/B} & -\sin\theta_{A/B} \\ \sin\theta_{A/B} & \cos\theta_{A/B} \end{pmatrix}$$



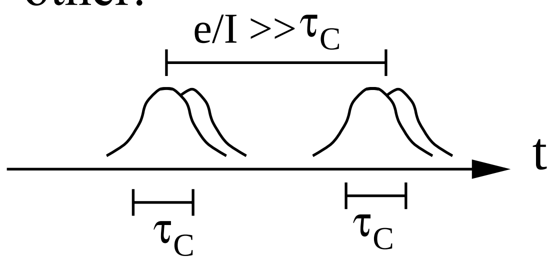
using e-h-state

$$P_{\alpha\beta} = (1 + \alpha\beta\cos[2(\theta_A - \theta_B)])/4$$

Noise correlators

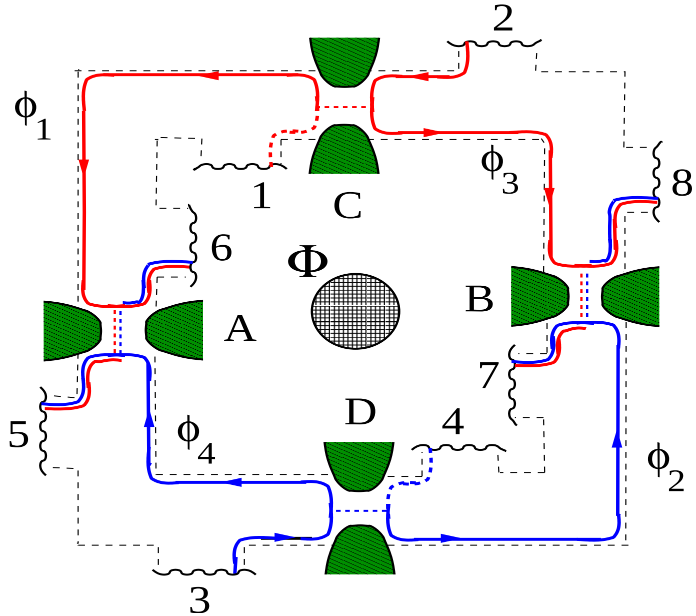
$$S_{58} = S_{67} = -S_0 P_{++}, \quad S_{57} = S_{68} = -S_0 P_{+-}, \quad S_0 = -(4e^2/h)|eV|R$$

In the tunneling limit $R_C = T_D = R \ll 1$; $\tau_C = \hbar/eV$; $\tau = e/I = \hbar/eVR$, measuring the noise cross-correlation is equivalent to coincidence detection in a long time interval: Only two particles within a pair are correlated with each other.



Dephasing

(tunneling limit)



Spatially seprated sources: qubit protectet against relaxation:

$$|\rho\rangle = |UU\rangle\langle UU| + |DD\rangle\langle DD| + \gamma(|UU\rangle\langle DD| + |DD\rangle\langle UU|)/2$$

$$S_{58} = -\frac{e^2}{4h}|eV|\left[1 + \gamma^2 \cos(\phi_0)\right]$$

$$S_B^{max} = 2\sqrt{1 + \gamma^2 \cos^2 \phi_0}, \quad \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$

Electron-electron entanglement trough postselection

Symmetric interferometer $T, R \approx 1/2$

Electron-hole picture not appropriate

Incident electron state is a product state: no intrinsic entanglement

Two-particle effects nevertheless persists

A Bell Inequality can be violated

Explanation: Entanglement through ``postselection" (measurement)

Joint detection probability

$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t) b_\alpha(t) b_\beta(t)\rangle = (h^2/e^2)[(1/2\tau_c)S_{\alpha\beta} + I_\alpha I_\beta]$$
$$= |s_{\alpha 3} s_{\beta 2} - s_{\alpha 2} s_{\beta 3}|^2$$

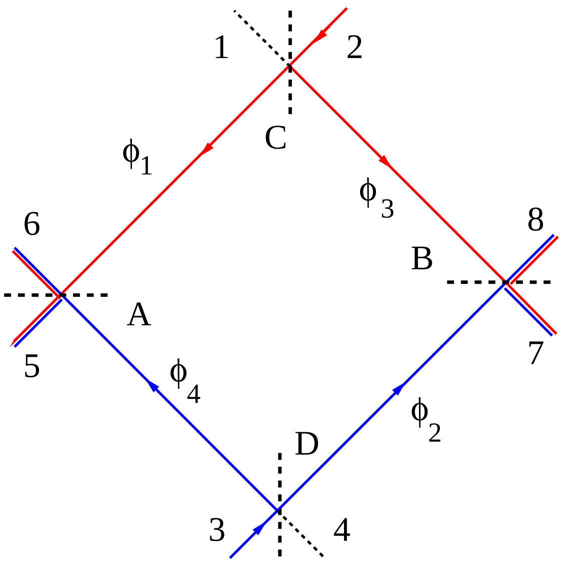
$$\langle I_\alpha \rangle = \frac{e^2}{h} V(|s_{\alpha 2}|^2 + |s_{\alpha 3}|^2)\,, \qquad \tau_c = \hbar/eV$$

Bell parameter (Bell Inequality):

$$S_B^{max} = 2\sqrt{1+\cos^2\phi_0}, \;\; \phi_0 = \phi_1 + \phi_2 - \phi_3 - \phi_4 + 2\pi\Phi/\Phi_0$$

Short time statistics: Pauli principle leads to injection of at most one electron in a short time interval: only two-particle transmission probability enters

Thermal photon sources



Sources: black body

Energy window: narrow band filters $\Delta\omega = 2\pi/\tau_C$

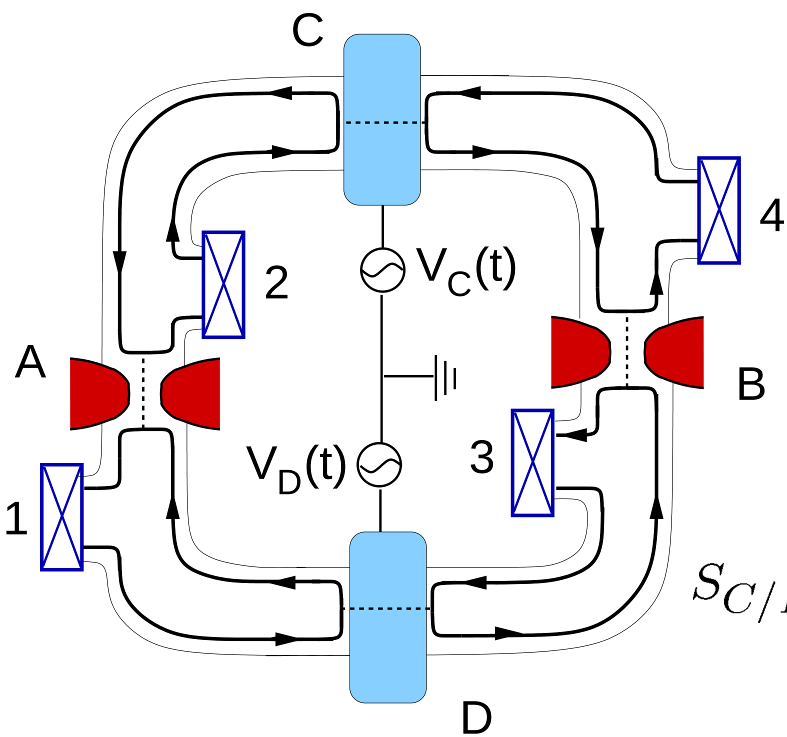
$$P_{\alpha\beta} \propto \langle b^\dagger_\beta(t) b^\dagger_\alpha(t) b_\alpha(t) b_\beta(t)\rangle \propto [(1/2\tau_c)S_{\alpha\beta} + I_\alpha I_\beta] \ ,$$

$$S_B^{max} = (2/3)\sqrt{1+\cos^2\phi_0} \ ,$$

No violation: In contrast to electron injection through a single quantum channel where in each time-slot only one particle is injected, in the bosonic case, many particles can be injected.

Dynamic orbital entanglement generation

Samuelsson, Buttiker, cond-mat/041010581



Contact potentials 1-4 at equilibrium

$V_C(t)$

$V_D(t)$ oscillating with $\omega = \frac{2\pi}{\tau}$

$$\hbar\omega << kT << \delta E$$

Floquet S-matrix

$$S_{C/D}(E_n, E) = \begin{pmatrix} r_{C/D}(E_n, E) & t'_{C/D}(E_n, E) \\ t_{C/D}(E_n, E) & r'_{C/D}(E_n, E) \end{pmatrix}$$

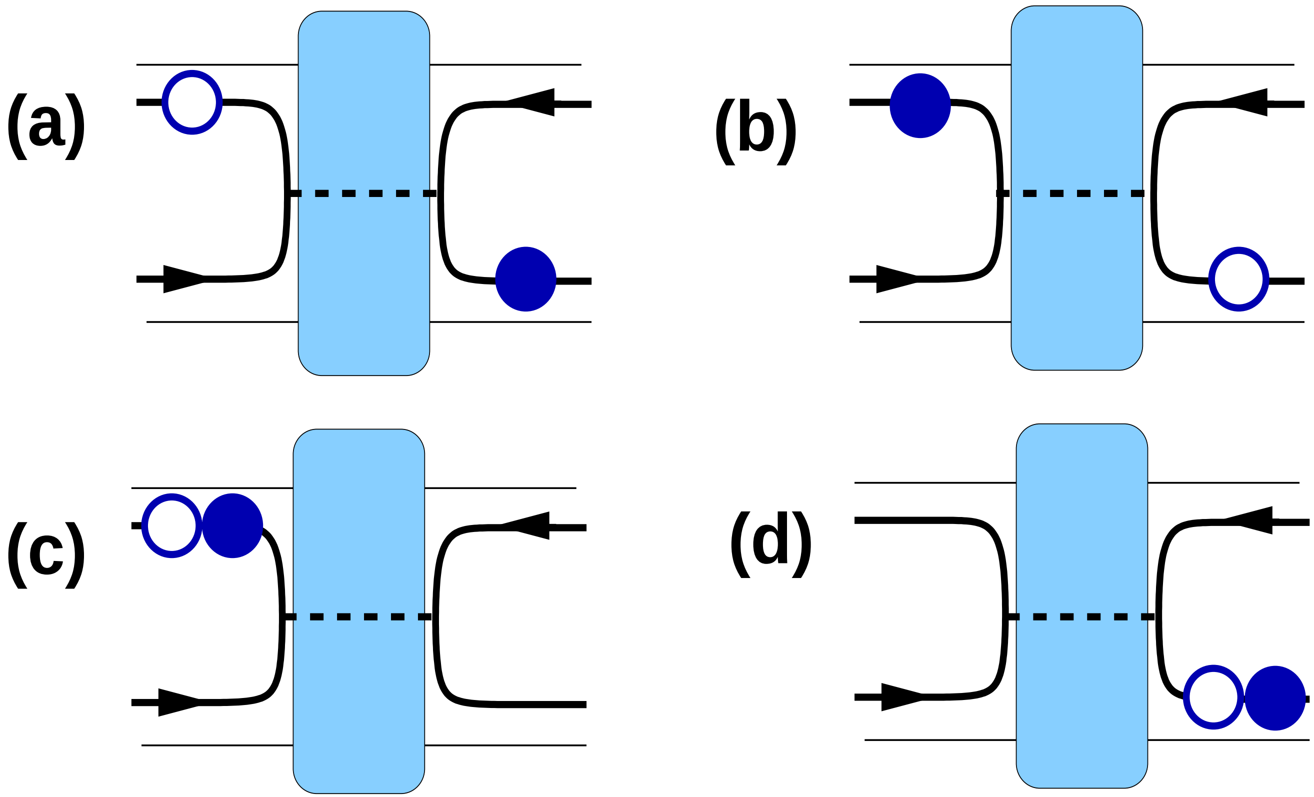
$$E_n = E + n\hbar\omega$$

small amplitude limit: only one side-band

$$V_{C/D}(t) = V_{C/D} + \delta V_{C/D} \cos(\omega t + \phi_{C/D})$$

$$t_D^0 \equiv t_D(E, E) \;\; , \delta t_D^+ \equiv t_D(E_1, E)$$

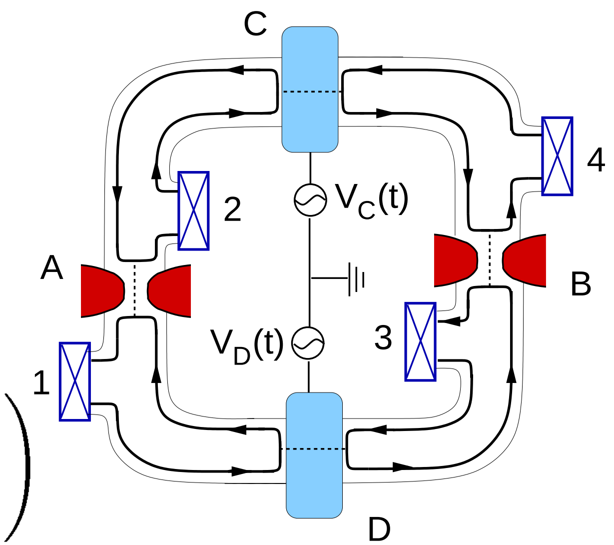
Electron-hole processes



$$(|CC\rangle + |DD\rangle) \otimes |\bar{\Psi}\rangle$$

orbitally entangled electron-hole state

The quantum state



incident state

$$|\Psi_{in}\rangle = \prod_{j=1}^{4} \prod_{E} a_j^\dagger(E)|0\rangle$$

outgoing state

$$\begin{pmatrix} b_{AC}(E) \\ b_{BC}(E) \end{pmatrix} = \sum_{n=0,\pm 1} S^C(E, E_n) \begin{pmatrix} a_2(E_n) \\ a_4(E_n) \end{pmatrix}$$

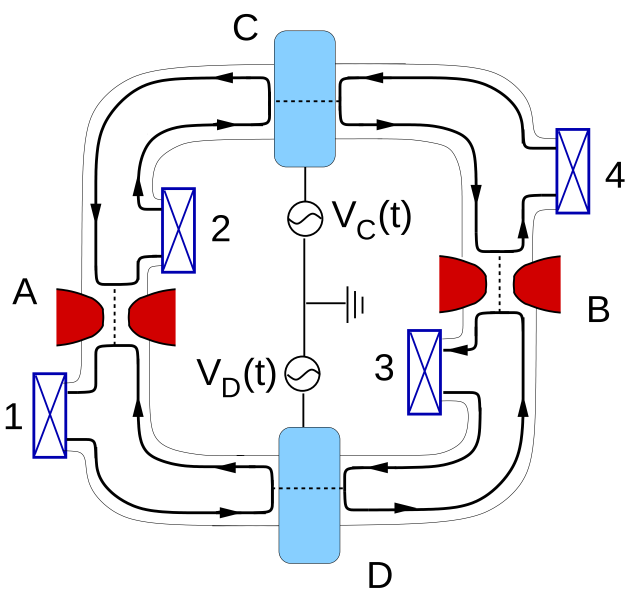
$$|\Psi_{out}\rangle = |\bar{0}\rangle + \int_{-\hbar\omega}^{0} dE \left(|\Psi_{out}^C(E)\rangle + |\Psi_{out}^D(E)\rangle \right)$$

$$|\Psi_{out}^C(E)\rangle = \sum_{\alpha,\beta=A,B} f_{\alpha\beta}^C \; b_{\alpha C}^\dagger(E_1) \; b_{\beta C}(E)|\bar{0}\rangle$$

$$f_{AA}^C = \delta r_C^+ r_C^* + \delta t_C'^+ t_C'^*, \;\; f_{BB}^C = \delta t_C^+ t_C^* + \delta r_C'^+ r_C'^*,$$
$$f_{AB}^C = \delta r_C^+ t_C^* + \delta t_C'^+ r_C'^*, \;\; f_{BA}^C = \delta t_C^+ r_C^* + \delta r_C'^+ t_C'^*$$

$$|\bar{0}\rangle = \prod_E b_{AC}^\dagger(E) b_{BC}^\dagger(E) b_{AD}^\dagger(E) b_{BD}^\dagger(E)|0\rangle$$

Entanglement test/Bell Inequality



$$\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_A & \sin\theta_A \\ -\sin\theta_A & \cos\theta_A \end{pmatrix} \begin{pmatrix} b_{AC} \\ b_{AD} \end{pmatrix}$$

similarly at B with θ_B

current noise spectrum

$$S_{ij}(t) = 2e \int dt' \langle \Delta I_i(t+t'/2) \Delta I_j(t-t'/2) \rangle$$

equal scatterers at C and D

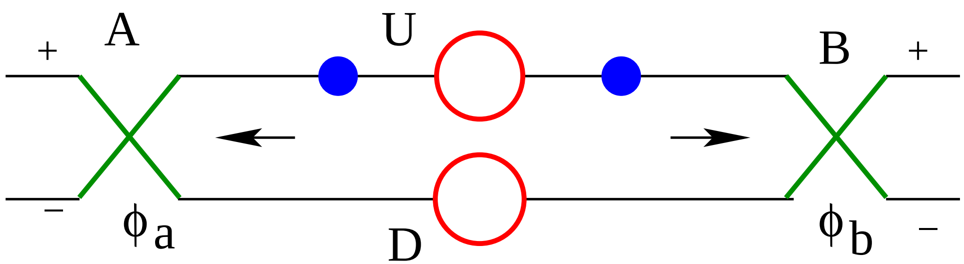
$$S_{13}^{dc} = S_0 \left[\cos^2\theta_A \cos^2\theta_B + \sin^2\theta_A \sin^2\theta_B + 2\gamma \cos\theta_A \cos\theta_B \sin\theta_A \sin\theta_B\right]$$

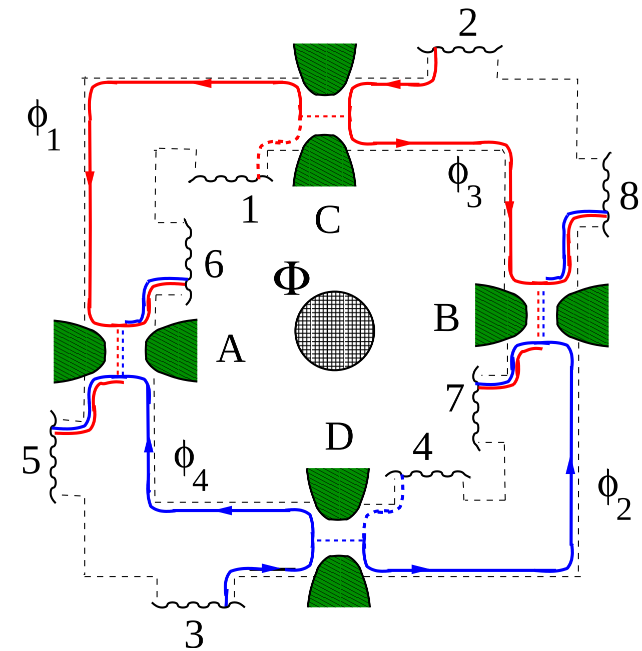
$$S_0 = (e^2/\tau)|\delta r t^* + \delta t' r'^*|^2 \; ; \quad \gamma = \cos\varphi \;\; \cos(\phi_C - \phi_D)$$

$$P_{ij}^{eh}(t,t') \propto \langle b_i^{e\dagger}(t) b_j^{h\dagger}(t') b_j^h(t') b_i^e(t) \rangle$$

- $P_{ij}(t,t) \propto S_{ij}^{dc}$ optimal angles $\Longrightarrow 2\sqrt{1+\gamma^2} < 2$ BI

Summary





Principle of orbital entanglement interferometers

dc-interferometers

NS: e-e-emission

Normal conductor: e-h-emission

two-particle Aharonov-Bohm effect

violation of Bell-Inequality: zero-frequency noise measurements

controllable geometry

ac-interferometers

motivation: time-controlled entanglement generation and detection