

## ABSTRACTS

**Scott Ahlgren** (University of Illinois, USA)

Title: Congruences for the modular forms of weights two and four

Abstract: We discuss some conjectures and some results which describe the possible congruences between modular forms of prime level in weights two and four.

**Daniel Bump** (Stanford University, USA)

Title: Metaplectic forms and Weyl group multiple Dirichlet series

Abstract: TBA

**Jan Hendrick Bruinier**(Universität zu Köln, Germany)

Title: Twisted Borcherds products on Hilbert modular surfaces

Abstract: We construct a natural family of rational functions on a Hilbert modular surface from the classical  $j$ -invariant and its Hecke translates. These functions are obtained by means of a multiplicative analogue of the Doi-Naganuma lifting and can be viewed as twisted Borcherds products. We study some properties of their CM-values. (This is joint work with T. Yang.)

**Dohoon Choi** (KIAS, Korea)

Title: Congruence properties for modular forms of half integral weight

Abstract: In this talk, we discuss the following congruence properties for modular forms of half integral weight:

- distribution of Fourier coefficients of a modular form of half integral weight modulo primes,
- $p$ -adic limit of Fourier coefficients of a modular form of half integral weight.

**YoungJu Choie** (POSTECH, Korea)

Title: The first sign change of Fourier coefficients of cusp forms

Abstract: Let  $f$  be a non-zero cusp form of even integral weight  $k > 1$  on the Hecke congruence subgroup  $\Gamma_0(N)$  with real Fourier coefficients  $a(n)$ . Then as is easy to see,  $a(n)$  changes sign infinitely many often. It is therefore quite natural to ask when the first sign change occurs and one would hope for an upper bound for it depending only on  $k$  and  $N$ .

The above problem was taken up by Kohnen and Sengupta(2006) in the special case where  $f$  is a normalized Hecke eigenform of squarefree level  $N$  that is a newform (so  $a(1)=1$  and  $a(n)$  is the  $n$ -th Hecke eigenvalue).

In this talk we shall extend the method given by Kohnen and Sengupta to arbitrary cusp forms  $f$  of squarefree level  $N$ . To the best of our knowledge, this seems to be the first general effective result regarding sign changes of Fourier coefficients of cusp forms. This is a joint work with W.Kohnen.

**Dorian Goldfeld** (Columbia University, USA)

Title: Multiple Dirichlet series associated to cusp forms on  $GL(n)$

Abstract: TBA

**Solomon Friedberg** (Boston College, USA)

Title: Multiple Dirichlet series attached to Weyl groups

Abstract: In this talk I give an introduction to generalizations of the Riemann zeta function called "Weyl group multiple Dirichlet series." Weyl group multiple Dirichlet series are sums in several variables whose coefficients involve Gauss sums and also reflect the combinatorics of a given root system. The earliest examples came from the Mellin transforms of Eisenstein series and have been intensively studied over the last 20 years. These functions and their residues unify and generalize a number of examples which have been previously treated individually, often with applications to number theory. I will give an account of some of the major research to date and the opportunities for the future. This research is all joint with Daniel Bump and Benjamin Brubaker, and part is also joint with Jeffrey Hoffstein and Gautam Chinta.

**Jeffrey Hoffstein** (Brown University, USA)

Title: Some number theoretic applications of multiple Dirichlet series

Abstract: Some of the more mysterious objects in automorphic forms are the Fourier coefficients of theta functions on the  $n$ -fold cover of  $GL(2)$ . Kubota defined  $\tau^{(n)}(m)$ , the  $m^{\text{th}}$  coefficient of the theta function on the  $n$ -cover of  $GL(2)$ , by (very roughly)

$$(1) \quad \frac{\tau^{(n)}(m)}{Nm^{1/(2n)}} = \text{Res}_{s=1/2+1/n} \sum_d \frac{g^{(n)}(m, d)}{Nd^{s+1/2}}.$$

Here  $g^{(n)}(m, d)$  denotes an  $n^{\text{th}}$  order Gauss sum with numerator  $m$  and denominator  $d$ . The ground field is assumed to contain the  $n^{\text{th}}$  roots of 1. When  $n = 2$  this gives the usual quadratic theta function. When  $n = 3$  this gives the cubic theta function, whose coefficients were first determined by Patterson. These proved to be conjugate cubic Gauss sums and the automorphic properties of the cubic theta function were used by Patterson and Heath-Brown to disprove Kummer's conjecture. When  $n \geq 4$  the coefficients  $\tau^{(n)}(m)$  are partially understood, but remain mostly unknown. Nevertheless, they have surprising applications. For example, it has been shown that a cusp form on  $GL(2)$  will have infinitely many non-vanishing  $n^{\text{th}}$  order twists at the center of the critical strip if the

Rankin-Selberg convolution of the form with the theta function on the  $n$ -cover of  $GL(2)$  is non-zero at the point  $1/2 + 1/n$ .

When  $n = 4$ , Patterson conjectured a striking relation between certain Rankin-Selberg convolutions. This reduces to the relation  $\tau^{(4)}(p)^2 = 2g(1, p)/Np^{1/2}$  ( $p$  prime). This conjecture has been proved this for rational function fields, but to this day there has been no obvious generalization to  $n > 4$ .

In this talk we conjecture another relation between Rankin-Selberg convolutions that has as a consequence  $\tau^{(6)}(p^2) = \overline{2g^{(3)}(1, p)}$ . This  $n = 6$  relation is also provable in the case of the rational function field, but unfortunately, just as with Patterson's original conjecture in the  $n = 4$  case, it does not generalize to covers beyond  $n = 6$ . However, unlike the  $n = 4$  case, it comes very close to being generalizable. In this talk I'll discuss the background to this problem the evidence for the conjectures, and the apparent obstacles to finding a generalization.

**Joseph Hundley** (POSTECH, Korea)

Title: Some recent progress in the Rankin-Selberg method

Abstract: TBA

**Bo-Hae Im** (Chung-Ang University, Korea)

Title: Rank of abelian varieties over large fields

Abstract: Let  $K$  be a field of characteristic  $\neq 2$  such that every finite separable extension of  $K$  is cyclic. Let  $A$  be an abelian variety over  $K$ . We show that if  $K$  is infinite, then  $A(K)$  is Zariski-dense in  $A$  and if  $K$  is not locally finite, the rank of  $A$  over  $K$  is infinite. Also, we discuss some related results as a consequence of these results.

**Winfried Kohnen** (Heidelberg University, Germany)

Title: Generalized modular forms and application

Abstract: We will report on recent joint work with G. Mason in which "arithmetic" properties of generalized modular functions (i.e. those with a character not necessarily unitary) are investigated. This has applications to  $q$  series formed with graded traces of certain vertex operator algebras.

**DongUk Lee** (KIAS, Korea)

Title: Some recent results on  $p$ -adic monodromy of Picard moduli scheme

Abstract: Let  $E$  be an imaginary quadratic number field and let  $m, n$  be distinct natural numbers. Suppose a rational prime  $p$  splits in  $\mathcal{O}_E$ . We discuss some recent results on the naive  $p$ -adic monodromy of the ordinary locus of good reduction over  $\overline{\mathbb{F}}_p$  of Picard moduli scheme of signature  $(m, n)$ , including application to certain bad reduction of Picard moduli scheme with parahoric level structure and reductiveness in the general case.

**Byeong-Kweon Oh** (Sejong University, Korea)

Title: Almost regular and almost universal quaternary quadratic forms

Abstract: The purpose of this talk is to show several results describing properties of the almost universal or, more generally, almost regular positive definite integral quaternary quadratic forms. In particular, we give an effective method for deciding whether a positive definite integral quaternary quadratic form is almost universal, that is, whether it represents all large positive integers. In this way we obtain an effective and definitive solution to an old problem first addressed and investigated by Ramanujan 90 years ago.

**Soogil Seo** (Yonsei University, Korea)

Title: Truncated Euler Systems

Abstract: We will introduce the so-called truncated Euler systems and using these we will conjecture that the structures of a certain graded module of the quotient  $E_K/C_K$  of the global units  $E_K$  by the circular units  $C_K$  of an abelian field  $K$  and the class group are related. Using characters, we will refine the structures of these groups.

**Jaebum Sohn** (Yonsei University, Korea)

Title: Koshliakov's formula and Guinand's formula in Ramanujan's lost notebook

Abstract: On two pages in his lost notebook, Ramanujan recorded several theorems involving the modified Bessel function  $K_\nu(z)$ . These include Koshliakov's formula and Guinand's formula, both connected with the functional equation of nonanalytic Eisenstein series, and both discovered by these authors several years after Ramanujan's death. Other formulas, including one by K. Soni and two particularly elegant new results, are stated without proof by Ramanujan. In this talk, we prove all the formulas claimed by Ramanujan on these two pages.